OPTIONS - ALLOCATION FUNDS -

TRANSACTION COSTS

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Abstracts

We study the efficiency of a Buy and Hold strategy, incorporating some options and seeking to super-duplicate a standard allocation policy. The replication strategy allows reducing transaction cost effects. The replication means optimizing two objective functions: MSE (Mean-squared Errors) and WMSE (Weighted Mean-squared Errors). Tests on portfolio efficiency concern, at first time, a long-term investor, and options are OTC (Out-The-Country) and strike prices are approximate by a multiplicative binomial tree. At second time, the empirical evidence poses the case of a short-term investment, on CAC40 index and VX6 options of terms 6 months.

Results prove the presence of Buy and Hold portfolios more efficient than an active allocation strategy. The optimal behaviour of the economic agent is a function of the number and the type of options introduced in the optimization problem.

Keywords: Buy and Hold, replication, standard Allocation, transaction costs, and options.
I - Introduction

The actual implementation of optimal portfolio decisions involves a series of challenges. One first problem to overcome is the impact of transaction costs.

In costless economy, the standard allocation requires and results in an infinite turnover of stock in any finite time interval. For instance, if an investor starts up with an optimal portfolio, he will gradually suffer a deviation from the target proportion as asset prices fluctuate, the initial allocation can be maintained by performing continuous trading (Merton and Samuelson, 1969). In this context, the continuous trading will infinitely prove to be expensive (Jung and Gennotte, 1992, 1994; Leland, 1999). Consequently, transaction costs may transform a trading strategy into a costly-suboptimal trading allocation (Constantinides, 1976, 1979, 1986; Dumas and Luciano, 1991; Jung and Gennotte, 1992, 1994; Leland, 1999).

We look for an outstanding strategy to minimize the deviation from theoretically-derived optimal asset allocation in a real economy. The real setting is related to the presence of transaction costs. In this context, we develop and test an unconventional Buy and Hold strategy that considers financial options as investment instruments.

The options have been an excellent innovation to cover investors from bad anticipations. They entail a reduction of transaction costs and improve the investor’s economic welfare (Yates and Kopprash, 1980; Merton, Scholes and Gladstein, 1982; Bookstalar and Clarke, 1984, 1985; Leland, 1985). However, these yields may be very restrictive. The options can profitably contribute to various financial mechanisms. Furthermore, they can substitute the
roles of the institutions and bring about a harmonious efficient global financial system (see Merton, 1995).

Including derivatives in the optimal portfolio decision is important. In this context, Ross (1976) has proved that the request for options is usually justified in terms of their potential profit. In contrast to the complete market setting, derivative securities are not redundant (Merton, Scholes et Gladstein, 1982; Morard et Naciri, 1990). Statically, the daily rates of option return are very important to their subjacent. The rise of the option return compensates for their high risk (Coval and Shumway, 2001). Therefore, the options can be viewed as an excellent investment instrument. In this context, Carr et al. (2000) have solved the asset allocation problem in an economy where derivatives are required to complete the market. Moreover, Carr and Madan (2001) have considered a single-period model, where the investor allows his wealth between the stock, bonds and European options with a range of strike prices. Liu and Pan (2002) studied the efficiency of options in a dynamic optimal allocation funds. According to these authors, the options could potentially expand the dimension of risk and return tradeoffs. In our research, we look to reveal option-characteristics in unconventional Buy and Hold problem.

By definition, the Buy and hold strategy is a conservative investment strategy. This strategy allows the disappearance of transaction costs, but it entails the loss of capital, as a result of error accumulation. We assume that options can stabilise the Buy and Hold strategy. This central idea leads to the following proposal: For certain characteristics of options, there is a Buy and Hold strategy, which can duplicate expectations of a standard allocation policy, and more efficient than an active portfolio management.

This approach must require special attention on the part of researchers and professionals. Specially, we adopt a method that can permit to crack certain complex problems not yet resolute in financial theory: e.g. market imperfections. Our approach seems
to be simple; we propose a level of performance equivalent to expectations based on perfect setting assumption. This logic seems inspired from the model of Black and Scholes (1973): that a dynamic portfolio value can duplicate an instantaneous price of a European call option. In this paper, we attempt to duplicate the standard allocation policy by a static trading strategy consisting of just a fewer options.

We keep two objective functions to illustrate replication principle: the first is by optimizing “Mean-squared Errors”. It involves minimizing errors square, among end-period wealth expectations, in respective to standard and passive strategies. The parameter RMSE (Root Mean-squared Errors) measures the replication cost or error. The second function consists of minimizing mean-squared errors between expectations of end-period wealth, weighted as well by investor’s tolerance. The “Weighted Root Mean-squared Errors” and the equivalent certainly are used to evaluate replication efficiency. In addition, the replication error should be lower than the cost of implementing an active allocation: discontinuous asset allocation.

In this paper, we are also interested in impacts of transaction costs on investor economic welfare. We develop a discontinuous extension to Merton’s standard allocation funds (1969, 1971). In this context, a multiplicative binomial tree approximates assets returns. The transaction costs are only proportional to the stock’s trade amounts. The investor’s preferences among assets (i.e., stock and bond) respect a power utility function. Based on these assumptions, we prove that a region of no-activity characterizes the adjustment portfolio space. Its limits, higher and lower, are the trade boundaries: i.e. sale and purchase assets.

The analytical analysis keeps into account behaviours of both short and long-term investors. Particularly, we propose an explicit closed-form solution to the discontinuous allocation problem for a long-term investor. Our procedure enables us to calculate the boundaries of the no-activity region in a systematic fashion. Once we know
this region, the investor’s problem is solved. Therefore, we keep up a comparison between standard and discontinuous allocation funds (i.e. controlling transaction costs). For a subsequent analysis, we use transaction costs’ expectation for evaluating replication strategies. In disparity for several researches, our results prove that under certain conditions, a portfolio consisting of just a fewer options is an excellent substitute for the dynamic allocation policy. In Parisian place, the empirical evidence supports only the efficiency of the weighted mean-squared strategy. However, the mean-squared replication is not adequate.

The rest of the research is organised as follows: In section 2, we develop, at first, the standard model of Merton (1969, 1971). Second, we concentrate on the exposure of the discontinuous allocation problem defined transaction costs as a control variable. In section 3, we model our orientation, based on the conception of European options as investment instrument. In section 4, we try to test analytically the effectiveness of the duplicated strategies. The results concern long and short-term investors. Conclusion and final comments are presented in the last section.

II- Standard allocation vs. Discontinuous allocation funds

1- Standard allocation problem

Optimal portfolio choice is one of the central subjects in finance. Yet analytical solutions are known only in a few special cases, under restrictive assumptions on the market structure and/or the investor’s utility function. In this current paper, our method applies to the model of Merton (1969, 1971) for an averse-risk investor. Preferences are modelled by a power function. The investment horizon is finite [0, T]. The economy consists of two assets, one is risky (stock) and the other is riskless (bond). Given these assumptions, we can formulate the standard allocation problem as:
\[ \text{Max } EU [W_T] \quad (1) \]

Where \( U(W_t) = \frac{W_t^\gamma}{\gamma} \) and \( \gamma \) is the degree of aversion to risk, strictly inferior to one.

The optimization consists of determining trading strategy \( \omega \), which permits to maximise the objective (1). The riskless security yields an instantaneous return of \( r \ dt \) and with an initial market price of USD 1, the bond price at any date \( t \) is simply \( \exp (rt) \). The risky security price is denoted by \( S_t(t) \) and is typically assumed to satisfy an Itô stochastic differential equation:

\[ dS_t(t) = \mu S_t(t)dt + \sigma S_t(t)dB_t \]

Here, \( B_t \) is a one-dimensional standard Brownian motion defined on a complete probability space \( (\Omega, \mathcal{F}, \mathbb{P}) \), with natural filtration \( \mathcal{F} = \{\mathcal{F}(t), 0 < t < T\} \). The coefficients \( r, \mu \) and \( \sigma \) are constants. Hence, for a setting without imperfections, the investor’s goal is only subject to the following dynamic budgetary constraint:

\[ dW(t) = \left[ r + \omega(t) \left( \mu - r \right) \right] W(t) dt + \omega(t) W(t) \sigma dB_t \quad (4) \]

We have an investor, who can decide, at each time “\( t \)”, how much portion of wealth \( u^{(1)} \) to invest in stock. For Merton (1969, 1971), the standard allocation means holding along the time the following ratio:

\[ \omega^* = \frac{\mu - r}{\sigma^2(1 - \gamma)} \quad (5) \]

The Merton strategy requires continuous trading, and results in an infinite turnover of stock in any finite time. Thus, the investor anticipates an expected end-period wealth as follows:

\[ W^*_T = W_0 \exp \left( rT - \frac{\pi^2 T (2\gamma - 1)}{2(1 - \gamma)^2} + \frac{\pi B_T}{(1 - \gamma)} \right) \quad \text{Where } \pi = \frac{\mu - r}{\sigma} \quad (6) \]
In costly economy, the end-period wealth seems to be different from its expectation (6). In order to minimize expectation deviations, the following analysis will discuss effectiveness of both discontinuous, and Buy and Hold strategies. It is to be highlighted that results derived from standard allocation will be used as a reference value for both strategies.

2 – Discontinuous allocation problem

The financial markets are not safe from imperfection costs. For Brennan (1975), Amihud and Mendelson (1986, 1989), the transaction costs have a major impact on assets diversification. Typically, they can modify investor decisions, and prove standard model inefficiency.

In their frameworks, Smith (1970) and Pogue (1970) develop an improvement of Markowtiz mono-period approach. They considered transaction costs as a control parameter. Based on their results, the revision of portfolio becomes a function not only of mean-variance couple, but also of expected cost’s level.

According to Davis and Norman (1990), Jouini et al. (1997), the strategy of Merton and Samuelson (1969) is suboptimal because asset’s revision is not absolute. In a dynamic allocation problem, these researchers look for an efficient model including transaction cost as a control variable into stochastic return process.

In practice, investors keep up a comparison across benefits and costs before any trading assets. In a same sense, Leland (1999) developed the portfolio problem even with a cost function. In this case, the transaction costs are maintained as a parameter in the objective function.

Jung and Gennotte (1992, 1994) verified that the end-period level of investor’s utility is different from its expectation value. The different results from the liquidation and adjustment cost. Thus, a strong relation exists between maximizing preferences and trade operations. Two
optimum levels should limit, respectively, sale and buy orders. Among these limits, the utility level is suboptimal.

In this area, Kamin (1975) and Constantinides (1976) have initiated the important development. They looked to optimal investment and consumption decisions of an agent seeking to maximise expected utility. For a power utility assumption, the agent should adopt a strategy with respect to no-activity region. The revision of assets is authorized even if the portfolio lies outside this region. However, the agent can conserve shares different from their target ratio.

In Conformity with this last point, we develop a discontinuous extension to the standard allocation problem. Our objective consists, principally, to reveal costs’ impacts on investor’s economic welfare.

In this context, \((x_t^0, x_t)\) denotes respectively instantaneous amounts of bond and stock, before any rebalancing. The couple \((y_t^0, y_t)\) represents there corresponding values obtained after revising decision. We assume that the transaction costs “θ” is only proportional to stock trade quantities. The investor incurs costs for each decision of purchasing or selling stock toward bond as the following:

\[
\begin{align*}
\text{Sale decision} & : & \quad y_t^0 & = x_t - v_t^0 \quad \text{and} \quad y_t^0 & = x_t + v_t^0 \\
\text{Purchase decision} & : & y_t^0 & = x_t + v_t^0 \quad \text{and} \quad y_t^0 & = x_t - v_t^0 - \theta v_t^0
\end{align*}
\]

For \(t = 0 \ldots T-1\), adjusted portfolio gets, at each time, \(x_{t+1} = y_t z_t \quad \text{and} \quad x_t^0 = y_t^0 r\)

Where, \(z_t\) corresponds to the instantaneous return of stock. Let \(v_t^0\) and \(v_t^0\) associate to amounts of stock selling or buying at date \(t\).
\[ (\nu_0, \nu_2, \ldots, \nu_{T-1}) \] represents the vector of controls that permits to hold the following portfolio

vector: \((x^0_0, x_0); (x^0_1, x_1); \ldots; (x^0_{T-1}, x_{T-1})\).

We can define under \( \nu \), the objective function by

\[
\max_{\nu_0, \nu_1 \in [0, T-1]} \mathbb{E}[U(x^0_{t+1}, x_t)] = \max_{\nu_0, \nu_1 \in [0, T-1]} \mathbb{EU}[W_t]
\]

We develop the optimization recursively. Instead of “U”, we apply the notion of indirect utility function, denoted by \( J_t(x^0_t, x_t) \). This last function satisfies some characterizes as:

At \( T \)

\[
J_T(x^0_T, x_T) = U(x^0_T, x_T)
\]

For \( t = 0 \ldots T-1 \)

\[
J_t(x^0_t, x_t) = \max_{\nu_t \in [0, T-1]} \mathbb{E}J_{t+1}(x^0_{t+1}, x_{t+1})
\]

The same principal conduits at \( T = 0 \) to:

\[
J_0(x^0_0, x_0) = \max_{\nu_0 \in [0, T-1]} \mathbb{E}U(x^0_0, x_T)
\]

Including Jung and Gennette (1992, 1994), Boyle and Lin (1997), Monoyios (2000), Lay and Lim (2002), the portfolio space is dividing into three regions: purchase, no-activity and sale region. Subject to the homogeneity of the power function and i.i.d returns, the investor’s trading strategy depend only on the ratio \( x_t / x^0_t \). At each step date, the investor trades the ratio of stock-bond in order to maximise the end-period utility expectation. When the ratio remains within a no-activity region, the investor must not trade. Analytically, this case satisfies \( \nu_t = 0 \).

The couple \( (x^0_t, x_t) \) is the portfolio permitting the maximum of \( J_{t+1} \). The following set \( G_t \) characterises all portfolios belongs in a no-activity region:

\[
G_t = \{ (x^0_t, x_t) \mid \text{for all } \nu_t, \mathbb{E}J_{t+1}(x^0_{t+1}, x_{t+1}) \leq \mathbb{E}J_{t+1}(x^0_{t}, x_t z_t) \}
\]

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By definition, the utility function is concave and gets only a global optimum. By taking into account all trading variables, we transform the problem to maximize the following function \( \phi_t \):
\[
\phi_t(u_t, x_t^0, x_t) = E_t J_{t-1}(x_{t-1}^0, x_{t-1})
\]

Two limits define the zone of no-activity: \((a, b)\). The limits present the optimal solutions of \( \phi_t \), since the ratio moves outside the region of no-transaction. Nevertheless, the optimization respects in limits the next conditions:
\[
\frac{\partial \phi_t}{\partial \phi_t}(0, 1, a) = 0 \quad \text{and} \quad \frac{\partial \phi_t}{\partial \phi_t}(0, 1, b) = 0
\]  
(7)

In respect with above modifications, the portfolio’s problem can be illustrated by the following system:
\[
J_t(x_t^0, x_t) = \begin{cases} 
1_t(\theta_t^+, x_t^0, x_t) & \frac{x_t}{x_t^0} < a_t \\
\theta_t(0, x_t^0, x_t) & a_t \leq \frac{x_t}{x_t^0} \leq b_t \\
\phi_t(\theta_t^- , x_t^0, x_t) & \frac{x_t}{x_t^0} > b_t 
\end{cases}
\]  
(8)

\( b_t \leq \frac{x_t}{x_t^0} \leq a_t \): In this region, the quantity necessary to the revision is null.

\( \frac{x_t}{x_t^0} < a_t \): The problem consists of calibrating the maximum quantity to buy stock in order to bring back the optimal ratio on the nearest limit:
\[
\max_{\phi_t} \phi_t(\theta_t^+, x_t^0, x_t) = \phi_t(\theta_t^+, x_t^0, x_t) = \phi_t(0, y_t^0, y_t^-) 
\]
\( \frac{x_t}{x_t^0} > b_t \): The condition of optimum in this zone consists to the following maximization:
\[
\max_{\phi_t} \phi_t(\theta_t^-, x_t^0, x_t) = \phi_t(\theta_t^-, x_t^0, x_t) = \phi_t(0, y_t^0, y_t^-) 
\]
The limits check the condition $0 < a, b < +\infty$. The lower limit represents the whole solutions of $\phi$, when the ratio is in a zone of purchasing. The superior boundary “$b$” consists to the optimal value of the expected utility, when the ratio among financial assets is in the sale region.

We develop in appendix the problem of a discontinuous asset allocation in a binomial setting. According to Boyle and Lin (1997), we retain the following definition to track this problem: “An utility function $J$ is a piece-wise linear utility function with respect to a utility function $U$, if there is a sequence of increasing numbers $q_j$, $j = 0,1,\ldots,s$ and non-negative constants $a_j$ and $b_j$, with respect to the underlying probability space $\{p_i, i=1,\ldots,l\}$, such that:

$$J(x^0, x) = \sum_{j=0}^{l} U(a_j x^0 + b_j x) \Pr(e_i) \quad \text{for} \quad q_j \leq x < q_{j+1} \quad j = 0, 1, 2,\ldots$$

In this case, the portfolio space is divided into a finite number of cones and on each cone, the indirect utility function $J$ is a convex combination of the utility function $U$”.

Likewise, various authors have been studied the impact of transaction costs on hedge funds strategy. In this context, the presence of a no-activity region is also proved to the option-pricing problem, without costless economy assumption (see, Boyle and Vorst, 1992). However, including Leland (1985), Grannan and Swindle (1996), Toft (1996), Martellini (2000), the efficiency of the hedge funds strategy consists of rebalancing asset-weights periodically regarding to a critical level of asset’s price or to a regular time intervals. Consequently, transaction costs may transform hedging strategy into a discontinuous trading strategy similar to Merton standard allocation.

We denote by TEC, the total cost expectation. When reaching maturity, this expectation is equal to the sum of selling and buying cost expectation, denoted by (CT (-)) and (CT (+)), respectively. (See appendix for further details).

$$T(\mathbb{T}) = \sum_{t} CT_{t}^{(-)} + CT_{t}^{(+)}$$
In comparison to earlier frameworks, our method successfully reveals how one can evaluate the costly economy impacts on investor welfare.

3 – Performance of a discontinuous allocation funds

In order to obtain a close-form solution to a discontinuous allocation funds problem, we use a numerical study. In this context, the investor would like to invest an initial wealth of USD 100,000 for 10 years or 40 trimesters. His preferences are modelled by a power utility function in respect to a constant aversion-risk coefficient of 2. At beginning date, we set $S_1(0)=50$ and $S_0(0)=1$ to the stock and Bond prices, respectively. We also set $\mu=10\%$ and $\sigma=25\%$, which are relative to a mean and a standard deviation of the geometric Brownian motion.

In perfect setting, the optimal fraction of wealth in stock is equal to 0.41302. The Merton ratio has a value 0.7036. The expected end-period wealth designed by (7) implies 200,800 UM. In this case, the optimal allocation of wealth between both assets does not affect the investor’s welfare as long as the total wealth is held constant. This is because with costless economy assumption, stock positions can be transformed without cost into bond. When there are transaction costs, this will no longer be the case. In reality, the terminal wealth is inconsistent with its expectation value. A great deal of errors is according to transaction costs.

In this level, we suppose that transaction costs are of 1%. Given the binomial assumption, the investor revises his portfolio each trimester (i.e., $t=0.25$ year). Table 1 presents optimum ratios or no-activity region boundaries for each step time: i.e. a $(t)$ and b $(t)$. These ratios satisfy the optimum conditions (7): first-order partial derivatives.

The entry “NA” means that the boundaries at the corresponding dates are not found on our tree, or the ratios acquire an infinite value. We work recursively backwards from the last period.
At “t=0”, the investor shares his initial wealth as Merton standard allocation. Following table 1, the Merton ratio lies usually within the no-activity region. Despite, the no-activity region is characterized by a no constant boundaries. During the first twelve trimesters, the boundaries converge to a constant width. The width boundaries correspond to inferior and superior limits, respectively. For example, if the ratio is equal to 0.2, then the investor should buy enough of stock to reach a nearest boundary 0.5814. These results coincide to the conclusion of Jung and Gennotte (1992, 1994): the no-activity region narrows and converges to a constant width when the time to the maturity date increases. The constant width is also consistent with studies assumed an infinite horizon (Constantinides, 1986; Dumas and Lociano, 1991; and David and Norman, 1990). Furthermore, if transaction costs alter the volume of securities, Jung and Gennotte (1994) proved that the no-activity region has a cone form, whose width narrows for two years behind starting date. In addition, the convergence of boundaries is a function of some parameters: the region of no-activity converges well as time increases, transaction costs decrease, volatility increases and relative risk coefficient decreases.

We show that the no-activity region tends to widen considerably as we approach as maturity. According to Jung and Gennotte (1994), as the time to maturity decreases, the expected return earns over the remaining time period decreases. If transaction costs are different to zero, the incremental return is minimal near the maturity date. This implies that at near maturity date, the investor does not trade. Equivalently, the no-activity region widens without bounds. With transaction costs on only the stock, the investor will incur transaction costs proportionate to his terminal stock holding. Because transaction costs are offset by a reduction in expected transaction costs, an investor with a large position in stock may reduce his stock position at all times up to maturity. This point explains why upper boundary shifts downward.
Our algorithm is programmed into Matlab. Its technique capacity permits us to resolve high dimension vectors: e.g., $2^{40}$ or $1.0995 \times 10^{12}$ elements. Table 2 surveys some difference in investor’s welfare, related to perfect and costly market assumptions. In consequence of the presence of a no-activity region, the investor supports expected costs equal to USD 22,130. This amount affects expected end-period wealth as well as the terminal level of utility. Therefore, adopting a costless economy assumption, the investor calls for a certainty equivalent (CE) standard allocation more important than a discontinuous one.

Finally, the investor is in a suboptimal situation in costly economy. Two consequences can explain this situation: the premium is relative to the payment of transaction costs at each step. The second is related to the presence of no-activity region; the investor holds as a rule asset weights different from the optimal standard allocation.

In order to reduce these consequences on investor’s economic welfare, the subsequent section expose a replication investment strategy.

**III – Replication strategy performance**

In finance theory, a large number of published works deals with the analysis of hedging portfolio efficiency (see, Yates and Kopprash, 1980; Merton, Scholes and Gladstein, 1982; Bookstalar and Clarke, 1984, 1985; Leland, 1985…). However, few works are interested with including options as asset class in a portfolio problem. In our model, we adopt two possible modifications to the portfolio problem. The first consists of including European type options as investment instrument into the optimization algorithm. This allows us to develop an unconventional Buy and Hold strategy that is covered toward capital loss. The second modification is to apply the replication principle to the definition of the objective-function. In this context, expectations derived from standard allocation are considered as a reference value.
1- Replication strategy problems

The replication seems fundamental, as well in the modern financial theory as in certain scientific phenomena. The financial authorities take advantage of the activity of certain markets to reach some objectives. For Merton (1995): the put bonds function can substitute a dynamic open market rules. In financial theory, the best mechanism based on replication is the model of Black and Scholes (1973). Thus, the value of a dynamic portfolio can duplicate the instantaneous price of a European call option.

In this context, we allow two functions for the replication principle. The first consists of optimising a “Mean-squared Errors”. The portfolios derive from the minimization of errors among terminal wealth expectations: \( W_t^* \) and \( V_T \) correspond respectively to standard and Buy and Hold strategies. We use the measure RMSE (Root mean-squared Errors) for the selection. This measure permits to report the square root of errors to the terminal wealth expectation derived from the standard allocation. The investor opts for portfolio having a smallest "RMSE". A better replication has a RMSE equal to zero. Alternatively, we formulate the problem as follow:

\[
\min_{[a,b,c_1,p_j]} E[(W_t^* - V_T)^2]
\]

We illustrate the RMSE as:

\[
\text{RMSE} = \frac{1}{W_t^*} \sqrt{\sum_i (W_t^* - V_T)^2}, \quad i=1, 2, \ldots
\]

\( i = 0 \ldots n_1 \): number of call options
\( j = 0 \ldots n_2 \): number of put options

According to Merton (1969, 1971), the investor is only interested in the weight of stock to hold. However, to get a terminal wealth \( V_T \) high close to \( W_t^* \), the investor looks for monetary quantities relative to the overall assets: stock (a), bond (b) and call or put options (c or p). The subsequent points can justify such objective function:
The optimal weights derive from the maximization of a concave and differentiable function. The optimal portfolio corresponds to a global solution. This portfolio permits to receive an expected terminal wealth, which maximised investor’s preferences. The function "MSE" takes into account this level;

The absence of preferences in "MSE" permits to imply investment policy as a function of market parameters and independent of the behaviour of investors;

This approach can be applied to other problems of optimization, which do not retain the utility in the objective function: e.g., dollar cost averaging.

The second objective function consists of minimising the errors among terminal wealth expectations, weighted by the degree of aversion at the risk. This function shows "hybrid" by introducing the aversion at the risk in the optimization. It takes into account the limits of the direct approach and the objective function "MSE":

\[
\min_{\{a,b,c_i,p_j\}} E[- U'(W_T^*)(W_T^* - V_T)^2] \tag{11}
\]

The appreciation of the replication is attributed to a measure "Weighted root mean-squared errors. We use also certainty equivalent as a second measure to involve investor’s indifference between standard-allocation and weighted mean-squared Buy and Hold strategy. If the \((CE_S)\) toward \((CE_R)\) is of 100%, then the investor will be indifferent.

\[\text{EU}(V_i) = U(CE_R)\]

The optimal portfolio satisfies the properties of financial assets and options. Behind the maturity date, the potential payoff of options is illustrated as follows:

**Call option**

\[D_{1i} = (P_{T_i} - K_i)^+ = \max (0, P_{T_i} - K_i)\]

**Put option**

\[D_{2i} = (E_j - P_T)^+ = \max (0, E_j - P_T)\]

\(E_j\) : Strike price of a put option \(j\)

\(K_i\) : Settlement price of a call option \(i\)
Our orientation permits to adopt a passive allocation. The optimal buy-and-hold problem is an interesting one for several reasons. First, it is currently impossible to trade continuously, and even if it were possible, market frictions would render continuous trading infinitely costly. Following the last section, we were see that investor spend USD 22,130 as a cost of rebalancing portfolios each trimester. However, Merton’s (1973) insight suggests that it may be possible to approximate a continuous-time trading strategy in a different manner, i.e. by including a few well-chosen options in the portfolio at the outset and trading considerably less frequently. Indeed, Merton (1995) retained that derivatives could be an effective substitute for dynamic open-market operations of central banks seeking to engage in interest-rate stabilization policies. In costly economy, derivative securities may be an efficient way to implement optimal dynamic investment policies. Thus, we suppose that under certain conditions, a buy-and-hold portfolio consisting of options is an excellent substitute for a standard investment policy. Second, the optimal buy-and-hold portfolio can be used to develop a measure of the risks associated with the corresponding dynamic investment policy that the buy-and hold portfolio is designed to duplicate. In fact, the financial theory proposed several measurement of risk in a static context: e.g. the market beta from the Capital Asset Pricing Model. However, there is no consensus regarding the proper measurement of risk for dynamic investment strategies. Market betas are notoriously unreliable in a multi-period setting. By developing a correspondence between a dynamic investment strategy and a buy-and-hold portfolio, it will be possible to accept so static measures in optimization problem.

By implementing a passive strategy, the budgetary equations (4) should correspond to linear equations’ system as follow:

**Terminal wealth :**

\[
V_T = aS_0(0)\exp(rT) + bS_1(T) + \sum_{i=0}^{n_1} ciD_i + \sum_{j=0}^{n_2} p_j D_{2j}
\]  

(12)
Initial wealth:

\[ W(0) = \exp(-rT)E^Q(V_T) \quad (13) \]

The operator \( E^Q[.] \) represents the conditional expected of the terminal wealth \( V_T \) in \( Q \) equivalent to \( P \): \( P \) is the space of probability relative to the standard strategy. The calculation of the current value of assets requires the knowledge of \( Q \) and not that of \( P \). However, we adopt the similar \( Q \) relative to the neutral risk space, as supposed in the model of Black and Scholes (1973). In this case, the terminal value of the risky asset becomes:

\[ \tilde{S}_t(t) = S_t(0)\exp(\sigma W_t - \sigma^2 t / 2) \quad \text{With} \quad W_t = B_t + \frac{\mu - r}{\sigma} t \quad (14) \]

In solving each sub-problem, we proceed to simulate the martingale equation (14). Each point is introduced by a probability 1/4000. We consider also the model of Black and Scholes (1973) to evaluate options. To facilitate the optimization, we limit variables by fixing the exercise prices. A binomial tree simulates the option’s strike price. This simulation generates 41 strikes. Among strike’s series, only three in maximum are employed in the optimal Buy and Hold portfolios.

In each case, the following solvency constraints must be imposed along with the budget constraint to ensure on-negative wealth:

<table>
<thead>
<tr>
<th>A. Options d'achat</th>
<th>B. Options de vente</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 ≤ a. \exp(rT)</td>
<td>0 ≤ a. \exp(r.T)</td>
</tr>
<tr>
<td>0 ≤ a. \exp(rT) + b K_1</td>
<td>0 ≤ a. \exp(r.T) + b E_i</td>
</tr>
<tr>
<td>0 ≤ a. \exp(rT) + (b + c_1) K_2 - c_i K_1</td>
<td>0 ≤ a. \exp(r.T) + (b - p_1) E_2 + p_1 E_i</td>
</tr>
<tr>
<td>0 ≤ a. \exp(rT) + (b + c_1 + c_2) K_3 - (c_1 K_1 + c_2 K_2)</td>
<td>0 ≤ a. \exp(r.T) + E_i (b - p_1 - p_2) + (p_1 E_i + p_2 E_2)</td>
</tr>
<tr>
<td>0 &lt; K_i &lt; K_2 &lt; K_3</td>
<td>0 &lt; E_i &lt; E_2 &lt; E_3</td>
</tr>
</tbody>
</table>

The Buy and Hold portfolio is defined by a straightforward nonlinear optimization problem with a concave objective function and linear constraints. We adopt Kuhn-Tucker (KT) solution for solving this problem. In first time, the KT optimisation asks for an examination of optima for every type of solution determined by a premium order condition. In
second time, the Algorithm programmed by “Matlab” compares all local solutions and selects only a global optimum. All results derived from a number of iteration of 1000 times.

2- A Long-term investor case

In each optimization, we propose $w_i$ as a target for the optimal Buy and Hold portfolio. Table 3 reports the mean-squared Buy and Hold portfolios containing between one to three options, each one is selected from a range of 41 strike prices. Particularly, we add three option strategies in the classical Buy and Hold strategy: (i) call, (ii) put and (iii) straddle.

The table 3 presented only the portfolios having a replication cost close to zero. The first panel corresponds to the case of adding a call strategy. In this case, we remark that if the number of options increases, the RMSE decreases, but the expected end-period utility does not increase regularly.

Results reported by table 3 describe asset quantities for each opportunity. We remark that asset positions of standard allocation are different to mean-squared Buy and Hold portfolios. In MSE optimization, the investor’s wealth is concentrated on bonds. The investment on stock is weak.

The weights of securities fluctuate with the number of options added. In case of three calls, the optimal Buy and Hold portfolio allows different call types. The super-replication consists of shorting only two calls. Given a current stock price of USD 50, these two calls are deeply out-of-the-money, hence there prices are extremely close to zero. Moreover, we should remember that these are 10-year European options. Thus, a strike price should be compared with the expected stock price at maturity: USD 136. Therefore, even taking into account this price, the strike prices of calls are still high.

Further, we show a large discrepancy on allocation wealth with added strategy profile. Particularly, the portfolios including calls are different to those with puts. The second panel of
Table 3 presents the results of adding between one to three puts. The excellent opportunity consists of being long in-the-money puts. The hold quantities of at-the-money and out-of-the-money puts are mainly weak. The comparison between the two high panels of Table 3 reveals no quite difference of stock and bond positions. However, the amount allowed to stock is important, even if the replication is based on straddle strategy. In this case, the investor opts to sell, simultaneously, call and put options. The options are on the whole in-the-money relative to the expected terminal price of stock.

These last results lead to conclude that option characteristics are fuzzy. Thus, investing on a unique option seems to be sufficient for constructing an efficient Buy and Hold portfolio. However, Liu and Pan (2002) proved that the optimal number of options is a function of price jumps. In geometric Brownian setting, Carr and Madan (2000, 2001) suggested that two puts seems to be sufficient in the optimal allocation problem. Our paper falls in line with this suggestion: with just weak number of options, the investor can apply an efficient mean-squared Buy and Hold portfolio.

The analysis of Table 3 is not based on the certainty equivalent measure, because of the non-appearance of investor’s preferences in the objective function. Portfolios summarized in Table 4 take into account this situation, where the optimization is defined by a WMSE objective function.

The first panel of Table 4 exposes the efficient portfolios including only calls. In contrast with Table 3, there is a weak investment on bonds. The investor puts the original wealth and the premium from shorting calls, on stock. The positions holding among the different assets correspond to the hedge funds strategy. In this last, the investor takes a long position on stock and a short one on puts: i.e., the investor profits from option-insurance and thus, he is exposed to the stock risk.
Following the second panel of table 4, we note that if the added put number increases, the “WRMSE” decreases and tends gradually to zero. The super-duplicate strategy is relative to a portfolio with three puts. This portfolio corresponds to an important certainty equivalent. Therefore, a perfect concordance exists between both accepted measures, for the replication evaluation.

Characteristics of options are different for each optimization problem. A comparison between tables 3 and 4 highlights this fact. In spite of all differences, the portfolios reported in these tables are efficient and confirm our proposition: the options are an excellent investment instrument.

The unique problem relative to the application of a replication portfolio is at the outset of the investment horizon. For instance, the analysis of table 4 results proves the presence of many efficiency portfolios. Among these portfolios, the super-replication consists of adding three puts options defined by small strike prices. The fact to conclude an option with strike price of USD5 appears very theoretical, but it seems to be possible in OTC market.

At each optimization, MSE and WMSE, the investor can define a set of Buy and Hold portfolios that allow expectation no longer different from standard allocation policy. Before any decision, the replication errors can be compared with the expected total transaction costs (Second section). In fact, we showed that the super-replication requires certain particularities of options. A modification of these characteristics can generate an extremely replication cost. As mean of consequence, the portfolio having a replication cost inferior to the active allocation cost is efficient in this case.

Table 5 summarizes the number of Buy and Hold more profitable to the active allocation in terms of costs. Particularly, we show that the number of scenarios increases significantly with the number of added options. In addition, scenarios based on calls are superior to puts. A large number of efficient strategies is relative to the three option case.
Finally, it seems to be important that the investor keeps up a comparison between all expectations derived from discontinuous and replication strategies before any decision. To fix his decision, the investor chooses, after the negotiation with his banker, one of the strategies that maximises his preferences.

3. A Short-term investor case

The analysis is based on options having the CAC40 index as a subjacent. These options are traded in the organization market: MONEP. In this place, we assume also the presence of a bond with an initial price equal to Eur 1 and a return rate of 5%. The daily data of the CAC 40 index and the options (PXL) are from Euronext-Paris, for the period of 3 January 2000 to 29 December 2006.

Statistically, we find a correlation coefficient of “-0.13” between the daily returns of the call options (VX6) and the CAC40 index. This negative relation proves that options can be a profitable investment instrument in a portfolio problem. According to Markowitz (1952), the negative correlation between assets generally engenders the reduction of a global risk portfolio. In 2002, the rate returns of options (VX6) are characterised by an annual mean of 18% and a standard deviation of 54%. Then, the options seem to be profitable, but too riskily.

In this section, two different optimization problems are studied. First, the investor maximizes the expected utility on perfect setting assumption. Second, he looks to duplicate standard allocation. The replication consists of minimizing two objective functions: MSE and WMSE. In this context, we assume that the daily CAC40 index returns satisfy an Itô stochastic process. The historic mean and volatility of these daily returns are of 13% and 23%, respectively. We use these last values as estimators of \( \mu \) and \( \sigma \) relatives to Itô stochastic process.
Furthermore, results are also based on assumptions of the first section: e.g., aversion-risk coefficient, $\gamma$, is of 2 and initial wealth is of Eur 100 000. For computational reasons, these results assume the presence of a set of eight options on CAC 40 index, as described in table 6. The number of option added is only 1 or 2. The options studied terms at six months, which coincides to horizon investment: from 28 February through 31 August 2006. At starting date, the CAC40 index is equal to Eur 5084.

All mean-squared Buy and Hold portfolios are a local solution. Therefore, table 7 reports results derived only from WMSE optimization. In this context, we show that the wealth is generally concentrated on CAC40 index and options. When compared to the standard allocation fund results, we note that the bond positions are transformed into option instruments in the considered problem.

The super-replication strategy is relative to two calls. This case has a WRMSE close to critical value, and a 99.87% of standard allocation equivalent certainly. At 31 August 2006, the CAC40 index is of Eur 5196. When the investor chooses to apply a super-replication strategy, he realises a terminal wealth of Eur 108 468. Despite the difference between terminal wealth and its expectation, the replication strategy is profitable. This difference can result from the inefficiency of the Itô stochastic process to approximate the CAC40 index return.

**IV- Conclusion**

The present paper examines in general, the case of an investor who is seeking for optimising his allocation in costly economy. The recent advancement in a modern portfolio theory permits to envisage the profitability of the active allocation. In this context, most studies suggest that the active allocation corresponds in practice to a hedge portfolio or a discontinuous allocation. In this context, our development admits the conception of options as investment instruments and especially consists in replicating the standard allocation rules.
We can draw the following conclusions:

- For certain characteristics of options, the active allocation is not necessary more profitable than a Buy and Hold strategy. Maintaining a passive strategy while purchasing some options, the investor can attain a situation optimizing his preferences independently of market imperfections;
- The options are an excellent instrument of insurance, speculation and investment.

For several reasons, the results of this paper have some limits. In fact, our analysis does not support the variations of some economic determinants: the degree of aversion of risk and the volatility of securities. In addition, the different results derive essentially from the standard model of Merton (1969). However, the financial literature proposed other more sophisticated models. Our tests consider the integration of European put and call options. Haugh and Lo (2001) solved the problem with Asiatic options.

Finally, the procedures developed in our paper may be useful in considering more general versions of the portfolio problem as well as other situations, such as more general utility functions and the inclusion of intermediate consumption. Another interesting extension would be the case of two or more risky assets. The two-asset case presents formidable problems because in general we do not know what the no-transaction regions look like. On the computational front, we would like to develop an efficient algorithm to implement the method developed in the current paper. These issues are left for further research.
Appendix:

In a binomial setting, the rate of return for the risky asset, at each step date, is independent of “t” and has only two states. The price of stock goes up by “u” or down by “d”, that is \( P(z=u)=p, \) \( P(z=d)=1-p \) with \( d<r<u \) and \( 0<p<1 \) (no arbitrage condition).

According to Cox, Ross and Robinstein (1977), the approximation of a geometric Brownian process requires the following notation: \( u = e^{\sqrt{h}}; d = 1/u; \) \( p = \frac{e^{mh} - d}{u - d} \) where \( m \) is a mean; \( r = e^{\delta h} \) and \( \delta \) is the annual interest rate, \( h = 0.25 \) years.

The aforementioned assumptions are used to solve \( J_t, a_t, \) and \( b_t \) recursively for \( t = T \) to \( t = 0 \).

At \( T \), we suppose that the liquidation portfolio tax is of zero. This hypothesis leads to the following equalities at \( T \) and \( T-1 \):

\[
J_T = U(x^0_T, x_T) = \sum_{i=1}^{I} U(r_i x^0_{T-1} + z_{i-1} x_{T-1}), \text{Pr}(e_i)
\]

For the first step date: \( r_{ij} = r \) and \( z_{ij} = u \) or \( d \)

Recall that \( U(W_i) = \frac{W_i^j}{7} \)

- Case 1: \( \psi_{T-1} = 0 \)

In this case, the investor conserves his portfolio until rebalancing.

\[
\phi_{T-1}(0, x^0_{T-1}, x_{T-1}) = \frac{1}{k} \sum_{i=1}^{I} \left[ z_{i-1} x^0_{T-1} + (1 - p) \left( \frac{r_i x^0_{T-1} + u x_{T-1}}{k} \right)^{i} \right]^j \text{Pr}(e_i) = p \left[ \frac{(r_i x^0_{T-1} + u x_{T-1})^j}{k} \right] + (1 - p) \left[ \frac{(r_i x^0_{T-1} + d x_{T-1})^j}{k} \right]
\]

- Case 2: \( \psi_{T-1} = \psi^+_{T-1} \)

In respect to the optimum condition (7), the indirect utility becomes as follows:

\[
J_{T-1}(v_{T-1}, x^0_{T-1}, x_{T-1}) = 1/i \sum_{i=1}^{I} \left[ r_i \left( x^0_{T-1} - (1 + \delta) x_{T-1} \right)^j + z_{i-1} x_{T-1} \right]^j \text{Pr}(e_i)
\]

26
\[ = \phi_{T-1}(0,1,a_{T-1}) \]

The right derivative of \( \phi_{T-1} \) involves the following optimum condition:

\[ \sum_{i} \{ a_{i} + z_{i} \cdot a_{T-1} \}^{\lambda_{i}} \cdot [ z_{i} - (1 + \theta) \cdot n_{i} ] = 0 \quad (15) \]

Alternatively, this condition can be transformed at T-1 as:

\[ p[r \cdot u \cdot r]^{\lambda_{i}} \cdot [ r - (1 + \theta) \cdot u ] \cdot [ r - (1 - \theta) \cdot d ] = 0 \]

At each step date, \( a_{t} \) should verify equation (15) and the domain definition condition that is bounded by \( q_{i} \) and \( q_{i+1} \).

In general, \( a_{t} \) is a global solution for a system of equations that corresponds to a nonlinear optimization problem.

For \( \nu_{T-1} = y_{T-1}^{+} \)

\[ J_{T-1}(x_{T-1}^{0}, x_{T-1}) = \phi_{t}(0, y_{T-1}^{0+}, y_{T-1}^{+}) = \sum_{i} U(n_{i} \cdot y_{T-1}^{0+} + z_{i} \cdot y_{T-1}^{+}) \cdot Pr.(\epsilon_{i}) \]

Recall that,

\[ y_{i}^{+} = x_{i} + v_{i}^{+} \]

\[ y_{0+} = x_{0} - v_{0}^{+} - \theta \cdot v_{0}^{+} \]

The above relations lead to taking out the optimal quantity of stock to buy for reaching \( x_{T-1}^{0+} \):

\[ v_{T-1}^{+} = \frac{x_{T-1}^{0} \cdot a_{T-1} - x_{T-1}}{1 + (1 + \theta) \cdot a_{T-1}} \]

This quantity corresponds to an expected transaction cost equal to:

\[ CT_{T-1}^{+} = \theta \cdot v_{T-1}^{+} \]

Focusing on \( a_{T-1} \) and \( v_{T-1}^{+} \), the adjusted portfolio becomes
Recalling that, at each step date, we use (16) and the definition domain condition to solve $b_{T-1}$. The $b_{T-1}$ is adjusted according to the optimum condition.

In respect with $x_{T-1}$ and $x_{T-1}$,

$$y_{T-1}^0 = \frac{1}{1+\theta} x_{T-1}^0 + \frac{1+\theta}{1+(1+\theta) x_{T-1}^0} x_{T-1}$$

Then, at $T-1$, we have only two states $I=2$ and $i=1, 2$.

Adjusted returns of bond and stock are denoted by:

$$\tilde{r}_j = \frac{r+a_{T-1} u}{1+(1+\theta) a_{T-1}}; \tilde{z}_i = (1+\theta) \tilde{r}_j.$$

In addition,

$$\tilde{r}_j = \frac{r+a_{T-1} u}{1+(1+\theta) a_{T-1}}; \tilde{z}_i = (1+\theta) \tilde{r}_j$$

- Case 3: $\psi_{T-1} = \psi_{T-1}$

$$\phi_{T-1}(y_{T-1}, x_{T-1}, x_{T-1}) = 1/\left[ R I \left[ x_{T-1}^0 (1-I) y_{T-1}^0 \right] z_{ij} (x_{T-1}^0 - y_{T-1}^0) \right] \Pr(\epsilon_{ij}) = \phi_{T-1}(0,1,b_{T-1})$$

The $b_{T-1}$ left derivative of $\phi_{T-1}$ involves the optimum condition (16):

$$\sum_i \left\{ r_{ij} + z_{ij} \cdot b_{T-1} \right\}^{1-1} [z_{ij} + (1-\theta) r_{ij}] = 0 \quad (16)$$

In a binomial setting, the optimum condition (16) can be transformed as follows:

$$p[r+u,b_{T-1}]^{1-1}[r+(1-\theta)u] + (1-p)[r+(1-\theta)d,b_{T-1}]^{1-1}[r+(1-\theta)d] = 0$$

At each step date, we use (16) and the definition domain condition to solve $b_{T-1}$.

Recalling that,
Where \( q \) denotes the region boundaries. Particularly, we calculate the indirect utility function at \( T-1 \) as follows:

\[
y_t^- = x_t - u_t^- \quad \quad \quad y_t^{0,-} = x_t^0 + u_t^- - \theta.u_t^-
\]

The above qualities lead to

\[
u_{T-1}^- = \frac{b_{T-1}x_{T-1}^0 - x_{T-1}}{1 + (1 - \theta)b_{T-1}}
\]

This quantity generates an expected transaction cost defined as:

\[
CT_{T-1} = \left| \theta \right| u_{T-1}^-
\]

Along the horizon investment, the total transaction cost is equal to

\[
TCT(T) = \sum_T TC_t
\]

In respect with last transformations, we can express the indirect utility function by

\[
J_{T-1}(x_{T-1}^0, x_{T-1}) = f_{T-1}(0, y_{T-1}^{0,-}, y_{T-1}^+ + y_{T-1}^-) = \sum_T U\left(\frac{e_i + z_{T-1}^0 b_{T-1}}{1 + (1 - \theta)b_{T-1}}, x_{T-1}^0 + (1 - \theta)x_{T-1}^- - \frac{e_i + z_{T-1}^0 b_{T-1}}{1 + (1 - \theta)b_{T-1}}x_{T-1}^+, \Pr_i(t)\right)
\]

According to binomial setting parameters, we denote the adjusted returns for an up state by:

\[
\tilde{r}_i = \frac{r_i - b_{T-1} - u_i}{1 + (1 - \theta)b_{T-1}}; \quad \tilde{z}_i = (1 - \theta)\tilde{r}_i
\]

For a down state:

\[
\tilde{e}_i = \frac{r_i - b_{T-1} - d}{1 + (1 - \theta)b_{T-1}}; \quad \tilde{z}_i = (1 - \theta)\tilde{e}_i
\]

We can deduce that the space is divided into three regions that are illustrated by juxtaposed cones. Particularly, we calculate the indirect utility function at \( T-1 \) as follows:

\[
J_{T-1}(x_{T-1}^0, x_{T-1}) = p.U(r_{11}, x_{T-1}^0 + z_{11} x_{T-1}) + (1 - p).U(r_{21}, x_{T-1}^0 z_{21} x_{T-1}) \quad q_0 \leq x_{T-1}^0 / x_{T-1}^1 < q_1
\]

\[
J_{T-1}(x_{T-1}^0, x_{T-1}) = p.U(r_{12}, x_{T-1}^0 + z_{12} x_{T-1}) + (1 - p).U(r_{22}, x_{T-1}^0 z_{22} x_{T-1}) \quad q_1 \leq x_{T-1}^0 / x_{T-1}^2 < q_2
\]

\[
J_{T-1}(x_{T-1}^0, x_{T-1}) = p.U(r_{31}, x_{T-1}^0 + z_{13} x_{T-1}) + (1 - p).U(r_{32}, x_{T-1}^0 z_{23} x_{T-1}) \quad q_2 \leq x_{T-1}^0 / x_{T-1}^3 < q_3
\]

Where \( q_1, q_2 \) and \( q_3 \) denote the region boundaries.
We note that asset’s returns at T-1 correspond to those at T-2, multiplied by \((r/u)\) and \((r/d)\).

For instance, if at T-1 we have
\[
q_j \leq x_{T-1}^0 / x_{T-1}^0 \leq q_{j+1},
\]
then at T-2, we obtain the following paths:
\[
(r/u)q_j \leq x_{T-2}^0 / x_{T-2}^0 < (r/u)q_{j+1}
\]
\[
(r/d)q_j \leq x_{T-2}^0 / x_{T-2}^0 < (r/d)q_{j+1}
\]
Rearranging all \((r/u)q_j\) and \((r/d)q_j, j=0, 1,.. from the smallest to the largest and denoting them as follows: \(O < \underline{q}_0 < \underline{q}_1 < \ldots\)

The dimension of definition domain of J becomes 2 \(I\), as a result of the presence of only two states.

Following the above transformations, we illustrate the J function at T-2 by
\[
J_{r,2}(x_{T-2}, x_{T-2}^0) = \sum_i U(r_i x_{T-1}^i + z_0 x_{T-1}) \Pr.(ti) \quad \underline{q}_0 \leq x_{T-2}^0 / x_{T-2}^0 \leq \underline{q}_{j+1}
\]
This procedure will be generalized for computing \(J_{t-3}, J_{t-4}, \ldots J_0\).
References


Leland H. Optimal portfolio management with transactions costs and capital gains taxes. Paper series 1999; Haas School of Business- University of California, Berkley.


Table 1: Boundary series of the no-activity region. Results are relative for an investor averse-risk (coefficient of relative risk-aversion is constant: RRA), preferences are modelled by a power function. The investment horizon is finite: 10 years (40 trimesters), the economy is of two assets, one is risky (stock) and the other is riskless (bond), a multiplicative binomial tree simulates stock returns, the bond interest rate is constant. The discontinuous allocation is an improvement to the standard allocation, well it allows the control of transaction costs along the time, the state-space is characterised by a no-activity region, asset’s revision is at each trimester. (Standard deviation = 25 %, transaction costs of 1% L. sup: Superior limit, L. inf: inferior limit).

<table>
<thead>
<tr>
<th>Years</th>
<th>0.25</th>
<th>0.50</th>
<th>0.75</th>
<th>1.00</th>
<th>1.25</th>
<th>1.50</th>
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<td>NA</td>
<td>NA</td>
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<th>8.75</th>
<th>9.00</th>
<th>9.25</th>
<th>9.50</th>
<th>9.75</th>
<th>10.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>L. sup.</td>
<td>0.7102</td>
<td>0.7102</td>
<td>0.7102</td>
<td>0.7102</td>
<td>0.7102</td>
<td>0.7102</td>
<td>0.7102</td>
<td>0.7102</td>
</tr>
<tr>
<td>L. inf.</td>
<td>0.5814</td>
<td>0.5814</td>
<td>0.5814</td>
<td>0.5814</td>
<td>0.5814</td>
<td>0.5814</td>
<td>0.5814</td>
<td>0.5814</td>
</tr>
</tbody>
</table>

Table 2: Summary of results: standard strategy and discontinuous funds allocation
The standard allocation considers Merton model, the investor is averse-risk (relative risk-aversion constant: RRA), preferences are modelled by a power function. The investment horizon is finite: 10 years (40 trimesters), the economy is of two assets, one is risky (stock) and the other is riskless (bond). The bond yields an instantaneous return of \( r_t \) and with an initial market price of USD 1, stock prices satisfy an Itô stochastic differential equation. The discontinuous allocation is an improvement to the standard allocation, it adopts the control of transaction costs along the time; a multiplicative binomial tree simulates the stock returns. The asset’s revision is at each trimester. (Initial wealth = 100 000, transaction costs = 1%, RRA = 2, Standard deviation = 25 %, Horizon = 10 years)

<table>
<thead>
<tr>
<th>Expected Terminal Wealth</th>
<th>Expected costs</th>
<th>Expected Utility</th>
<th>Certainly equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard allocation</td>
<td>200 800</td>
<td>0</td>
<td>-5.4937 e-6</td>
</tr>
<tr>
<td>Discontinuous Allocation</td>
<td>178 620</td>
<td>22 180</td>
<td>-5.7771 e-6</td>
</tr>
</tbody>
</table>
Table 3: Buy and Hold portfolios duplicating standard allocation rules: Objective function «Mean-squared errors». The investor is averse-risk (relative risk-aversion constant: RRA), preferences are modelled by a power function. The investment horizon is finite: 10 years, the economy is of two assets, one is risky (stock) and the other is riskless (bond). The Bond yields an instantaneous return of $r_d$ and an initial market price of USD 1; Stock prices satisfy an Itô stochastic differential equation. In this case, the investor seeks to duplicate the standard allocation policy rules. The replication strategy consists of holding a Buy and Hold portfolio added option strategy. Table 3 reports portfolio efficiency for each strategy: RMSE close to zero. (Initial funds = 100 000, RRA = 2, Standard deviation = 25 %, RMSE: root mean-squared errors, EU: expected utility; CE: Certainty equivalent, E: Call strike price; K: Put strike price).

<table>
<thead>
<tr>
<th>Strike prices</th>
<th>Optimal allocation</th>
<th>EU</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bond</td>
<td>Stock</td>
<td>Call 1</td>
</tr>
<tr>
<td>E1 224</td>
<td>1.131 e5</td>
<td>124.61</td>
<td>-217.63</td>
</tr>
<tr>
<td>E1 1294</td>
<td>1.217 e5</td>
<td>9.388 e-6</td>
<td>-2.383 e6</td>
</tr>
<tr>
<td>E2 475</td>
<td>1.217 e5</td>
<td>16.04 e-5</td>
<td>-7.721 e6</td>
</tr>
<tr>
<td>E1 1662</td>
<td>1.217 e5</td>
<td>475</td>
<td>1.217 e5</td>
</tr>
<tr>
<td>E2 136</td>
<td>1.217 e5</td>
<td>43.579</td>
<td>43.579</td>
</tr>
<tr>
<td>E3 38</td>
<td>2265.1</td>
<td>43.579</td>
<td>43.579</td>
</tr>
<tr>
<td>K1 174</td>
<td>1.217 e5</td>
<td>9.420 e-5</td>
<td>0.000175</td>
</tr>
<tr>
<td>K2 6</td>
<td>1.217 e5</td>
<td>43.579</td>
<td>43.579</td>
</tr>
<tr>
<td>K3 5</td>
<td>1.217 e5</td>
<td>1.095 e-5</td>
<td>-2.672 e7</td>
</tr>
</tbody>
</table>
Table 4: Buy and Hold portfolios duplicating the standard allocation rules. Objective function «Weighted Mean-squared Errors». The investor is averse-risk (RRA), preferences are modelled by a power function. The investment horizon is finite: 10 years, the economy is of two assets, one is risky (stock) and the other is riskless (bond). The Bond yields an instantaneous return of $r_d$ and an initial market price of USD 1; stock prices satisfy an Itô stochastic differential equation. In this case, the investor searches to duplicate the standard allocation policy rules. The replication strategy consists of holding a Buy and Hold portfolio including some options. Option strike prices are simulated by a multiplicative binomial tree, table 3 reports only portfolio efficiency for each option strategy: WRMSE close to zero. (Initial funds = 100 000, Horizon = 6 months, RRA = 2, Standard deviation = 25 %, WRMSE: weighted root-mean-squared errors, EU: Expected utility; CE: Certainty equivalent, E: call strike price, K: Put strike price).
Table 6: Specificities of (VX6) option contracts negotiated on MONEP (Maturity: from 1 Mars 2006 through 30 June 2006; Subjacent: CAC40 index; Type European option).

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Subjacent</th>
<th>Type</th>
<th>Strike price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call (VX6)</td>
<td>6 months</td>
<td>CAC 40</td>
<td>In The money</td>
</tr>
<tr>
<td></td>
<td>(September 2006)</td>
<td></td>
<td>In The money</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>At the money</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Out the money</td>
</tr>
<tr>
<td>Put (VX6)</td>
<td>6 months</td>
<td>CAC 40</td>
<td>At the money</td>
</tr>
<tr>
<td></td>
<td>(September 2006)</td>
<td></td>
<td>In the money</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>In the money</td>
</tr>
</tbody>
</table>

Table 5: Number of Buy and Hold strategy more profitable than a discontinuous allocation funds. The investor is averse-risk; preferences are modelled by a power function. The investment horizon is finite: 10 years, the economy is of two assets, one is risky (stock) and the other is riskless (bond). The bond yields an instantaneous return of \( r_d \) and an initial market price of USD 1, stock prices satisfy an Itô stochastic differential equation. In this case, the investor chooses to duplicate the standard allocation policy rules. The replication strategy consists of holding a Buy and Hold portfolio including some options. Option strike prices are simulated by a multiplicative binomial tree, the objective function is defined by mean-squared errors as well by weighted mean-squared errors. (MSE: mean-squared errors; WMSE: weighted mean-squared errors).
Tableau 7: Replication strategy performance: Parisian place case (CAC40 & VX6 options).

The investor is averse-risk (relative risk-aversion constant: RRA), preferences are modelled by a power function. The investment horizon is as 6 months. The portfolio is a combination of three asset classes: CAC40 index, Bond, VX6 options. The Bond yields an instantaneous return of $r_t$ and an initial market price of Eur 1, CAC40 prices satisfy an Itô stochastic differential equation, statistic analysis allows 13% and 23 % to the mean and the standard deviation, respectively. Option strike prices are standardised by MONEP (see, table 6). In this case, the investor chooses to duplicate the standard allocation policy rules. The replication strategy consists of holding a Buy and Hold portfolio, the objective function is defined by weighted mean-squared errors. (Initial funds = 100 000, Horizon = 6 months, RRA = 2, WRMSE: weighted root mean-squared errors, EU: expected utility; CE_R: Certainty equivalent, E: VX6 Call strike price, K: VX6 Put strike price).

<table>
<thead>
<tr>
<th>Strick Price</th>
<th>Optimal allocation</th>
<th>EU</th>
<th>CE</th>
<th>WRMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bond</td>
<td>CAC40</td>
<td>Call 1</td>
<td>Call 2</td>
</tr>
<tr>
<td>E1 5184</td>
<td>0</td>
<td>20.8391</td>
<td>-14,6541</td>
<td>1.7191</td>
</tr>
<tr>
<td>E2 4984</td>
<td>0</td>
<td>19.4398</td>
<td>2.6175</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Strick Price</th>
<th>Optimal allocation</th>
<th>EU</th>
<th>CE</th>
<th>WRMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bond</td>
<td>CAC40</td>
<td>Put 1</td>
<td>Put 2</td>
</tr>
<tr>
<td>K1 4584</td>
<td>1.0037</td>
<td>19.6443</td>
<td>1.3470</td>
<td>-</td>
</tr>
<tr>
<td>K2 4584</td>
<td>0</td>
<td>19.6443</td>
<td>1.3470</td>
<td>-</td>
</tr>
</tbody>
</table>