A MIXED INTEGER DISJUNCTIVE MODEL FOR TRANSMISSION NETWORK EXPANSION

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Abstract: The classical non-linear mixed integer formulation of the transmission network expansion problem cannot guarantee finding the optimal solution due to its non-convex nature. We propose an alternative mixed integer linear disjunctive formulation, which has better conditioning properties than the standard disjunctive model. The mixed integer program is solved by a commercial Branch and Bound code, where an upper bound provided by a heuristic solution is used to reduce the tree search. The heuristic solution is obtained using a GRASP metaheuristic, capable of finding sub-optimal solutions with an affordable computing effort. Combining the upper bound given by the heuristic and the mixed integer disjunctive model, optimality can be proven for several hard problem instances.

Keywords: transmission planning, combinatorial optimization, heuristics.

I. INTRODUCTION

The problem of determining the optimal set of candidate circuit additions for a power transmission network so as to supply the forecasted loads with minimum cost is usually formulated as a mixed nonlinear program. The nonlinearity is due to constraints related to the linearized power flow equations, where bus voltage angle variables are multiplied by circuit investment binary decision variables. The system generation is supposed capable of supplying the forecasted load, and candidate circuits are informed for all possible network branches, called rights-of-way. The linearized power flow model is composed of Kirchoff's first and second laws, which are linear equations relating node (bus) angles, generations and loads to circuit flows. The linearized power flow equations are usually used in planning studies of high voltage meshed networks, providing good approximations for the circuit flows, and avoids the need to iteratively solve the non-linear power flow equations. Inequality constraints are simple upper bounds on generations and circuit flows.

In many real world large-scale applications, the mathematical model is a large-scale mixed integer problem. Successful solution approaches include decomposition techniques [1,2] and heuristics [7,8], although neither can guarantee optimality of the solution. The decomposition approach, even if capable of solving medium sized problem instances, cannot prove optimality of solution since the nonlinear model is non-convex.

In this work a mixed integer disjunctive formulation will be analyzed, where the nonlinear constraints are avoided by using a disjunctive form to which they are equivalent. This standard disjunctive formulation is solved using the B&B code of the XPRESS [13] solver (release 11). A GRASP [11] based metaheuristic method [9,10], capable of solving large problem instances (but not proving optimality), is used in this work to provide an upper bound to the B&B solver, being described in the appendix.

The standard disjunctive formulation suffers from bad conditioning due to the use of large penalties in the disjunctive constraints. An alternative disjunctive formulation using "optimal" penalty factors and a tighter representation of power flows on candidate circuits will also be presented, providing improved performance.

The following notation will be used throughout this work:
n is the number of nodes (busses);
m is the number of candidate circuits (branches);
Ω^0_i is the set of existing circuits connected to bus i, i=1,n;
Ω_i^+ is the set of candidate circuits connected to bus i, i=1,n;
$\Omega_i = \Omega^0_i \cup \Omega_i^+$;
f is the vector of circuit flows (existing and candidates);
f^{max} is the vector of circuit capacities (existing and candidates);
g is the vector of bus generations;
g^{max} is the vector of bus generation capacities;
d is the vector of bus active loads;
θ is the vector of bus voltage angles (in radians);
r is the vector of bus load curtailment;
x is the binary vector of decision investment on candidate circuits;
c is the vector of candidate circuit unit cost (in million dollars);
γ is the vector of circuit susceptances (the inverse of the reactance);
M is the penalty vector of candidate circuits.

The work is organized as follows. In section II the classical non-linear formulation is presented, as well as the Benders decomposition solution approach. Section III reviews the standard disjunctive formulation, and discusses the alternative formulation. These disjunctive formulations, solved by the B&B code that is used in combination with the heuristic solution provided by GRASP, are compared in Section IV by means of a (benchmark) medium sized problem instance. A

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real world problem instance is solved in Section V and results are shown. Section VI concludes; also future works are discussed.

II. THE CLASSICAL NON-LINEAR FORMULATION OF THE TRANSMISSION EXPANSION PROBLEM

The classical formulation represents the linearized power flow equations, bounds on generations and circuit flows, and integrality constraints for investmente variables, being shown below.

$$\mathrm{Min}_{\{x,f,g,\theta\}} \; c\,x$$

s.t.

(power node balance equation – first Kirchoff's law)

$$\sum_{k=(i,j)\ j\in\Omega_i} f_k + g_i = d_i \quad , \quad i=1,n$$

(2^{nd} Kirchoff's law for existing circuits)

$$f_k - \gamma_k\,(\theta_i - \theta_j) = 0 \quad , \quad k=(i,j) \;,\; j\in\Omega_i^{0} \;, \quad i=1,n$$

(2^{nd} Kirchoff's law for candidate circuits)

$$f_k - x_k\,\gamma_k\,(\theta_i - \theta_j) = 0 \;,\; k=(i,j) \;,\; j\in\Omega_i^{+} \;, \quad i=1,n$$

(existing circuit flow upper and lower bounds)

$$-f_k^{max} \le f_k \le f_k^{max} \;,\; k=(i,j) \;,\; j\in\Omega_i^{0} \;,\; i=1,n$$

(candidate circuit flow upper and lower bounds)

$$-f_k^{max}\, x_k \le f_k \le f_k^{max}\, x_k \;,\; k=(i,j) \;,\; j\in\Omega_i^{+} \;,\; i=1,n$$

(bus generation upper bounds)

$$0 \le g_i \le g^{max} \;,\; i=1,n$$

(integrality constraints of investment variables)

$$x \in \{0,1\}^m$$

(reference bus angle is fixed, other bus angles are free variables)

$$\theta_{ref} = 0$$

Note that a non-linearity appears due to the product of variables q and x in the 2^{nd} Kirchoff's law for candidate circuits. Consider the 2^{nd} Kirchoff's law for a candidate circuit k: if x_k=0, the corresponding flow must be null, while if x_k=1, equality is enforced, as required. This is a mixed integer non-linear program, so it cannot be solved by classical optimization techniques.

The non-convex nature of the continuous relaxation of this formulation can be avoided in two ways: relaxing 2^{nd} Kirchoff's law for candidate circuits or by linearizing this equation. Since the resulting model is mixed linear, the latter allows using standard combinatorial optimization methods, but, in general, solutions are infeasible. The latter allows an implicit linearization scheme that arises when applying Benders decomposition, where once a trial investment proposal is made, an LP in variables *q*, *g* and *f* results.

For each iteration, a trial expansion proposal *x* is obtained by solving a mixed integer program (the master), and the resulting network configuration is then analysed by solving a linear program (the slave), which returns to the master a Benders cut expressing the operation cost (the minimum load shedding required to respect network operating constraints) in terms of the decision variables. The slave sub-problem LP formulation is the same as the one shown in the appendix, except for the unit load shedding cost which is a large penalty (the optimal solution must respect all network constraints). The master sub-problem has an additional non-negative continuous variable (the operation cost, expressed as the total load shedding), which is represented by means of Benders cuts which are expressed as linear functions of the investment variables. After each iteration, the Benders cut generated by the slave is included in the master sub-problem. Only integrality constraints for investment variables and Benders cuts are part of the master subproblem.

The master's solution value provides a lower bound (the master is a relaxation of the model), while the master's solution investment cost plus the slave's solution operation cost provides an upper bound (the solution is feasible, but may not be optimal). Convergence is assured when these bounds meet, and if not, a new iteration is performed. Although the Benders decomposition has finite convergence, this linearization scheme may result in discarding feasible solutions, since the non-convex nature of the model does not guarantee that the Benders cuts do not cut off part of the feasible set.

This undesirable behavior was mitigated by means of a hierarchical Benders decomposition method [1,2]. In this approach, initially the 2^{nd} Kirchoff's law are relaxed and Benders decomposition is applied to the resulting mixed integer linear program. The Benders cuts thus obtained are valid cutting planes for the original formulation. Next, the original formulation is solved by the same method, but now the previous cuts guide the solution through a smooth path, as experimental results have shown.

The computational effort of this decomposition scheme is high due to the need to solve a mixed integer linear program (the master) for each iteration. In general, many Benders iterations are required until convergence. During the Benders iterations, the incorporation of cuts result in increasing ill conditioning of the master problem, and therefore slows down the solution time of the master subproblems. Although no proof can be given with respect to optimality of the resulting solution, this approach was successfully applied to medium scale problem instances.

III. THE DISJUNCTIVE MIXED INTEGER FORMULATION OF THE TRANSMISSION EXPANSION PROBLEM

In this formulation, the non-linear constraints of the non-linear formulation are avoided by using a disjunctive form to which they are equivalent. The standard disjunctive mixed integer model is formulated as follows:

$\text{Min}_{\{x,f,g,\theta\}}\ c\,x$

s.t.

(power node balance equation – first Kirchoff's law)

$$\sum_{k=(i,j)\ j\in\Omega_i} f_k + g_i = d_i \quad , \quad i=1,n$$

(2nd Kirchoff's law for existing circuits)

$f_k - \gamma_k\,(\theta_i - \theta_j) = 0 \quad , \quad k=(i,j)\ ,\ j\in\Omega_i^0\ ,\ i=1,n$

(2nd Kirchoff's law for candidate circuits, expressed in disjunctive form)

$-M_k(1-x_k) \le f_k - \gamma_k\,(\theta_i - \theta_j) \le M_k(1-x_k)$, $k=(i,j),\ j\in\Omega_i^+, i=1,n$

(existing circuit flow upper and lower bounds)

$-f_k^{max} \le f_k \le f_k^{max}$, $k=(i,j)\ ,\ j\in\Omega_i^0\ ,\ i=1,n$

(candidate circuit flow upper and lower bounds)

$-f_k^{max}\ x_k \le f_k \le f_k^{max}\ x_k$, $k=(i,j)\ ,\ j\in\Omega_i^+ , i=1,n$

(bus generation upper bounds)

$0 \le g_i \le g_i^{max}$, $i=1,n$

(integrality constraints of investment variables)

$x\in\{0,1\}^m$

(reference bus angle is fixed, other bus angles are free variables)

$\theta_{ref} = 0$

Note that the 2nd Kirchoff's law for each candidate circuit is now expressed as two linear inequalities. When a candidate circuit binary variable is set to zero, the corresponding disjunctive constraints enforce that no flow will go through the circuit, while if it is set one the flow will obey the second Kirchoff's law equation, as required.

This standard disjunctive formulation was independently proposed by [3,4]. Benders decomposition was also applied to this approach [6], using a very large penalty factor. It has been proven [5] that if a candidate circuit k is such that there is an existing circuit on the same branch, the minimum value of the penalty parameter M_k is given by f_k^{max} times the ratio of the candidate susceptance and the existing circuit susceptance. Also, if the candidate circuit is in a new right-of-way, its penalty parameter is the product of its susceptance times the solution value of a shortest path problem on the network between the branch's terminal nodes, where the "distance" between each pair of nodes is measured by the ratio of the branch's flow capacity and its reactance. By using these optimal penalties, ill conditioning can be alleviated during the solution of the linear relaxation along the B&B tree.

We now discuss an alternative disjunctive formulation with better conditioning properties. In this formulation, Kirchoff's 2nd law for each candidate circuit k=(i,j) is represented by two inequalities, each one related to the possible flow direction (from i to j and from j to i), resulting in lower and upper bounds for flow in each direction:

(2nd Kirchoff's law for candidate circuit k=(i,j), upper bound)

$f_k^+ - \gamma_k\Delta\theta_k^+ \le 0$, $k=(i,j)\ ,\ j\in\Omega_i^+\ ,\ i=1,n$

$f_k^- - \gamma_k\Delta\theta_k^- \le 0$, $k=(i,j)\ ,\ j\in\Omega_i^+\ ,\ i=1,n$

(2nd Kirchoff's law for candidate circuit k=(i,j), lower bound)

$f_k^+ - \gamma_k\Delta\theta_k^+ \ge -M_k(1-x_k)$, $k=(i,j)\ ,\ j\in\Omega_i^+\ ,\ i=1,n$

$f_k^- - \gamma_k\Delta\theta_k^- \ge -M_k(1-x_k)$, $k=(i,j)\ ,\ j\in\Omega_i^+\ ,\ i=1,n$

The flow in each candidate circuit k is now expressed as the difference of two non-negative flow variables, f_k^+ and f_k^-:

(flow in each candidate circuit k=(i,j))

$f_k = f_k^+ - f_k^-$, $k=(i,j)\ ,\ j\in\Omega_i^+,\ i=1,n$

Each branch angle difference is now expressed as the difference of two non-negative angle differences, $\Delta\theta_k^+$ and $\Delta\theta_k^-$.

(new constraints relating angle difference $\Delta\theta_k$ with its corresponding non-negative angle difference variables):

$\Delta\theta_k = \Delta\theta_k^+ - \Delta\theta_k^-$, $k=(i,j)\ ,\ j\in\Omega_i^+\ ,\ i=1,n.$

With the new candidate circuit flow variables, the flow bounds are now expressed by:

(candidate circuit flow upper and lower bounds)

$f_k^+ - f_k^{max}\ x_k \le 0$, $k=(i,j)\ ,\ j\in\Omega_i^+ , i=1,n$

$f_k^- - f_k^{max}\ x_k \le 0$, $k=(i,j)\ ,\ j\in\Omega_i^+ , i=1,n$

The objective function and other unmentioned constraints remain unaltered, as well as variables f^+, f^-, g, x and q.

Comparing this formulation with the previous, it can be seen that:

- the upper bound is tighter since is doesn't include in the RHS the positive term with the penalty;
- the lower bound is exact when $x_k=1$, and the RHS is better than the one in the previous formulation when $x_k=0$.

The resulting formulation has more continuous variables, but being tighter should be better than the previous standard disjunctive formulation. Note that, contrarily to the Benders decomposition approach, which is an iterative scheme, the mixed integer disjunctive model is solved only once. Since it has the same number of binary variables as the non-linear formulation, and also the due to the tighter formulation, the B&B solution processing effort should be much lower.

Another ingredient is necessary to accelerate the B&B tree search, being commonly used in combinatorial optimization methods. It is an upper bound (UB) to the solution value, and the better it is, more effective is the effect of pruning the tree, avoiding solving many LP relaxations during the search. A natural UB is the solution value of a heuristic solution. The computational effort of the heuristic must be much lower than the one required by the B&B solver, but also its solution quality must be such that the optimality gap (the difference between the heuristic solution value and the optimal one) is low. These requirements are satisfied by the solutions obtained using the GRASP metaheuristic (see appendix), as results have shown for several transmission planning problem instances [10]. Very small gaps are obtained after a few GRASP iterations, consuming small computing time.

In the next section, the standard and alternative disjunctive formulations will be compared using a medium scale transmission network problem instance.

IV. THE STANDARD AND ALTERNATIVE DISJUNCTIVE MODELS: A 46 BUS CASE STUDY

This network is used as a benchmark for transmission expansion solution methods (see [1],[2],[4],[5],[8],[10]). It has 46 nodes (12 with generation) and 62 existing circuits, with 79 candidate rights-of-way and 3 possible duplications in each one, resulting in a total of 237 binary variables. The network represents the 230KV and 500KV high voltage nodes of the reduced South Brazilian system. There are 11 disconnected nodes, where 2 of these have new generators, and therefore have to be connected. The total load is 6800MW and the total generation capacity is 10545MW.

For the fixed dispatch case (the generation capacity of all generators is reduced so that total capacity equals total load), the best solution obtained both by hierarchical Benders decomposition and GRASP has a cost of $154.26 millions, and 16 circuits are built: 2 x 26/29, 2 x 42/43, 2 x 24/25, 2 x 29/30, 2 x 5/6, 19/25, 46/6, 31/32, 28/30 and 20/21 (see Figure 1). Recently, this solution was proved optimal by a Benders decomposition approach applied to the standard disjunctive formulation [5].

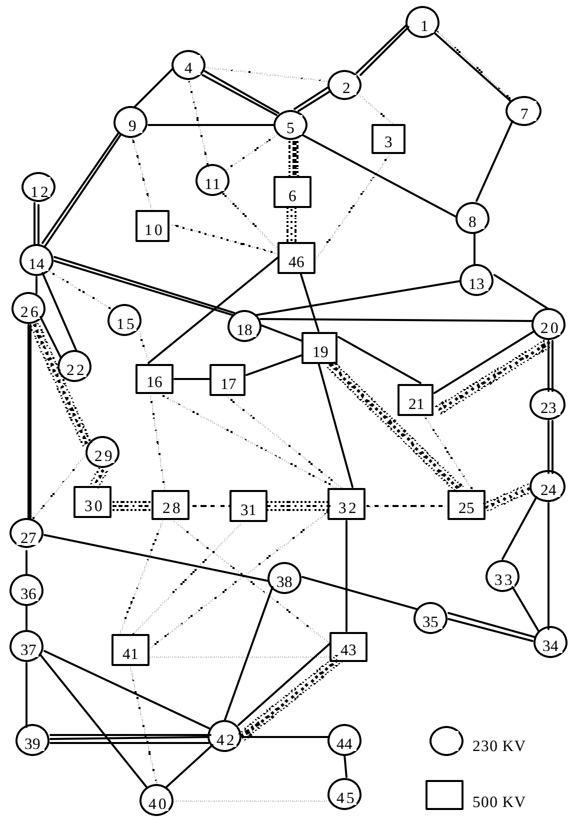

Figure 1 - 46 bus network and solution (without redispatch)

The optimality gap reduction along the GRASP iterations depends on the random generator seed. Although one cannot draw conclusive figures about the reduction, for this problem instance, using the default seed, the gap dropped from 14% at iteration 1 to 5% at iteration 38, the latter being a reasonable UB to use.

For the standard disjunctive formulation, we used a fixed penalty factor for all candidate circuits. It took 4 hours to prove optimality of this solution on a Pentium III-450MHz PC with 128Mb of memory, using the solver's default parameters, without an UB. Using the alternative formulation with "optimal" penalty factors, the CPU time required to prove optimality dropped to 2 hours and 10 minutes, indicating the superiority of the alternative formulation with respect to the standard one.

Now suppose we use the same alternative formulation, and have an UB for the solution value. In order to verify the impact of providing an UB to the B&B solver, 9 runs were made, varying the UB from 1.05 to 1.3 times the optimal solution value. Computing time results for the alternative formulation are shown in Table 1.

Table 1 – CPU minutes of B&B solver using UB

UB%	.5	1	2	5	10	15	20	25	30
Time	50	57	64	81	91	95	96	103	105

The UB information was very effective: if we use the 5% gap UB obtained after 38 GRASP iterations, the solution time required to prove optimality would be 81 minutes. It should be noted that this solution was rapidly found in the B&B tree, taking only 5 minutes. Note that he solution time increases in a linear fashion as the UB increases. These results show that it is worthwhile to have any UB, if it can be obtained in a small fraction of the time required by the B&B search. This was the case for GRASP, since several sub-optimal solutions are found among the initial iterations.

For the redispatch case (original generator limits), the best solution obtained both by decomposition and GRASP techniques has a cost of $70.21 millions, and 8 circuits are built: 13/20, 20/23, 46/6, 2 x 20/21, 42/43 and 2 x 5/6. Using the standard formulation with the same "optimal" penalty factor, it took 33 seconds to prove optimality of the solution. The CPU time using the alternative formulation was 57 seconds. The greater flexibility gained with generator redispatching allows a great deal of savings, and eases considerably the solution process, so for this case both disjunctive formulations perform well.

V. THE ALTERNATIVE DISJUNCTIVE MODEL: A 79 BUS CASE STUDY

The Southeastern Brazilian network has 79 nodes and 156 circuits. The peak load is 38000MW, and 268 candidates are provided for voltage levels from 230KV to 750KV. There are 7 disconnected nodes, two of which have new generators. The best solution was obtained using GRASP, being found at iteration number 280. This solution has a cost of $422 millions, where 24 circuits are built: 2 x 224/227, 2 x 210/41, 2 x 255/259, 2 x 220/242, 2 x 226/242, 220/250, 234/237, 221/224, 245/253, 245/239, 244/245, 226/259, 211/246, 226/227, 250/251, 207/206, 207/209, 249/250 and 216/215 .

The B&B solver was applied to the alternative formulation without an upper bound. The optimal penalty factor was calculated for each candidate right-of-way. This problem instance is badly conditioned due to the very large penalties associated to candidate rights-of-way connecting the two isolated generation nodes. The optimal solution could not be found by the B&B code due to the size of the search tree that exausted the available memory, but several sub-optimal solutions were found. As an example, a feasible solution with 8% of optimality gap was obtained within 33 CPU minutes on the same PC.

With an upper bound of $423, and using high branching priorities for all candidate circuits connecting isolated generation busses 244 and 245, the B&B solver was able to find the same solution after 50 minutes, and prove the optimality after 90 minutes, on a Pentium III 800 MHz with 128MB of memory. Specialized inequalities for this problem are being tested, and we expect to report results soon.

VI. CONCLUSIONS

The disjunctive mixed integer formulation of the transmission expansion problem allows using classical combinatorial optimization techniques and therefore guarantees obtaining the optimal solution, contrarily to the nonlinear formulation. An alternative disjunctive formulation was presented which is tighter than the standard one, resulting in significant reduction of computational effort as shown in a benchmark case study. For a real world problem instance, the alternative formulation proved optimality of the solution found by the GRASP metaheuristic. The use of an upper bound easily found by GRASP, that is input to the B&B solver applied to the alternative disjunctive mixed integer formulation is a promising strategy to solve large scale problem instances, and even to prove optimality.

The disjunctive formulation is being studied in order to incorporate valid inequalities that can be efficiently dealt with using branch-and-cut schemes within the B&B code.

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APPENDIX – THE GRASP METAHEURISTIC FOR THE TRANSMISSION EXPANSION PROBLEM

The greedy randomized adaptive search procedure [11] is a metaheuristic method that has been successfully applied to many hard combinatorial problems [12]. It is composed of two different phases: construction and local search. In the construction phase, a feasible solution is produced in a greedy manner by repeatedly randomly drawing promising candidates from a restricted candidate list (RCL) until feasibility is attained. The local search phase explores the construction phase's solution neighborhood in order to reach a local minimum. The random nature of the construction phase gives GRASP a multistart nature, therefore allows searching all the solution space, avoiding getting trapped in local minima. The RCL is built according to a greedy function, tailored for the problem at hand. The local search also depends on the problem nature, due to the need to characterize the neighborhood of a solution and define valid moves to neighboring solutions. Since the local search is typically an exponential process in terms of the neighborhood size, one must balance the effort of the search and the benefit obtained in attaining a local minimum. The procedure is repeated for a pre-specified number of iterations, and returns the best solution found. Being a heuristic method, optimality cannot be proven, therefore the procedure is stopped when the number of iterations reaches a specified number. For many problem instances, the best solution of the transmission expansion problem was found by GRASP very early along the iterations.

In this GRASP implementation for the transmission expansion problem, some enhancements have been used: among others, the size of the RCL list is dynamically self-adjusted (the so called reactive GRASP approach), and a linear distribution function is used to bias the selection of the RCL candidate variables, resulting in consistently better performance for several problem instances, while avoiding the need to calibrate the RCL size parameter.

The greedy function adopted for the transmission expansion problem is derived from the dual variables of the network performance problem which measures the total load shedding required for each network configuration in order to respect the linearized power flow equations, the bounds on circuit flows and bus generation limits. Given a network configuration resulting from a trial solution, a linear program is formulated as follows. For each power balance node equation, a slack variable (the load shedding) is introduced with unit cost in the objective function, limited by the node's load. The objective function is the sum of load shedding costs. The network performance problem formulation is presented below.

$$\text{Min}_{\{r,f,g,\theta\}} \sum_i r_i$$

s.t.

(power nodel balance equation – first Kirchoff's law)

$$\sum_{k=(i,j)\; j\in\Omega_i} f_k + g_i + r_i = d_i \;,\; i=1,n$$

(2^{nd} Kirchoff's law for existing circuits)

$$f_k - \gamma_k\,(\theta_i - \theta_j) = 0 \quad,\quad k=(i,j) \;,\; j\in\Omega_i \;,\; i=1,n$$

(existing circuit flow upper and lower bounds)

$$-f_k^{max} \le f_k \le f_k^{max} \;,\; k=(i,j) \;,\; j\in\Omega_i \;,\; i=1,n$$

(bus generation upper bounds)

$$0 \le g_i \le g^{max} \;,\; i=1,n$$

(load shedding bounds)

$$r_i \le 0 \;,\; i=1,n$$

(reference bus angle is fixed, other bus angles are free variables)

$$\theta_{ref} = 0$$

The objective function measures the total amount of unfeasibility in load supply, and is null if the trial investment proposal is feasible. This problem is also the slave problem of the Benders decomposition applied to the non-linear formulation. It has been shown [1] that the multiplier with respect to a candidate circuit's susceptance γ_k is given by

$$\pi_{\gamma k} = (\theta_j - \theta_i)\,(\pi_i - \pi_j),$$

where k=(i,j) and π_i is the Lagrange multiplier associated to the balance equation for node i (and likewise for node j).

The greedy function for any candidate circuit *k* is defined as the ratio $\pi_{\gamma k}/c_k$, and takes into account the cost of the circuit in order to penalize high voltage circuits (which are most expensive) with a low greedy value. The RCL is composed of the best candidates ranked by their greedy function values, and the RCL size is controlled by the parameter α, where $0 \le \alpha \le 1$. When randomly sampling elements from RCL, the bigger is α, more random is the construction, and the smaller is α, the greedier is the construction. In the basic GRASP, α is usually set to a low value around 10% of the number of candidates. In reactive GRASP, this RCL size parameter is self adjusted along the iterations, according to a probability distribution of possible α values within the unit interval. The distribution is estimated according to the relative frequency of the ratio of the best solution to the average solution value observed along every K iterations, for each α value. Another enhancement is, instead of sampling randomly from the RCL, to use a linear bias distribution to induce the selection of circuits with higher greedy function values.

Each time a candidate is selected and added to the network, a network performance problem must be solved, and a new RCL is obtained. When a feasible solution is completed, a check consisting in removing each added candidate in decreasing cost order is performed so as to eliminate eventually redundant additions made along the construction phase.

The local search strategy for the transmission expansion problem is based on 2-exchange movements, meaning that for each built candidate in the current solution, a possible neighbor consists in flipping off this candidate and flipping on another candidate not in the solution. Each such neighbor must have smaller cost than the current solution and be checked for feasibility by solving a network performance problem. Once a feasible movement if made, the same one-at-a-time circuit removal check is also performed. The local search stops when a local minimum is found, or a maximum number of movements is made.

Every new solution found is checked against the incumbent, and the incumbent is updated if the cost is lower. The incumbent is returned at the end. Note that one can keep other near-optimal solutions in a pool so as to compare them using detailed network models.

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