Test problems for quasi-satellite packing: Cylinders packing with behavior constraints and all the optimal solutions known

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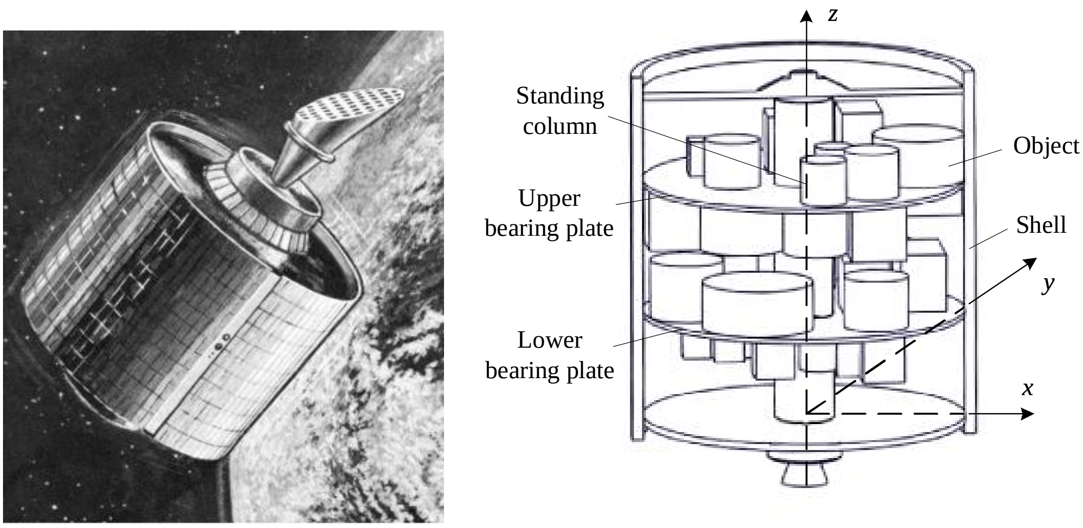
Abstract: This paper presents seven test problems with all the optimal solutions known on the background of the layout optimization problem of a simplified international communication satellite module, aiming to evaluate the algorithm performance on solving three-dimensional packing problem with behavior constraints. The test problems are constructed in the following step. First, place N (=217) cylinders of different size within a big cylindrical container such that the cylinders are tangent to each other or touch the container. Then, sort the cylinders in descending sequence and divide them into q=23 groups according to radius. At last, we choose seven groups (q=1,2,⋯,7) of cylinders to construct test problems (N_{max}=49) with all the optimal solutions known. User can select the test problem with different number of cylinders ($N_{max}\leq 49$) according to the algorithm performance. Also the cylinder number of the test problem can be increased by the proposed construction method.

Keyword: cylinder packing; test problem; behavior constraint; layout optimization; satellite module

1 Instruction

The packing problem belong to NP-hard problem that is extensively used in engineering applications such as spacecraft, car, ship, machining center, shield machine and drilling platform. New effective algorithms are always expected to solve the packing problems, and one algorithm should be evaluated after it is proposed. The evaluation of an algorithm is as hard as its construction. In general, the algorithms are often evaluated by three following methods: (1) algorithm theory analysis, (2) test problem with the optimal solutions, (3) engineering applications and practices. However, algorithm theory analysis (such as the complexity analysis and the convergence proof) is difficult for some algorithms. And assessing the algorithm by engineering application is rather complicated in some situations. Therefore, evaluation by test problem is frequently employed. For example, De Jong(1975), Schwerin and Wäscher (1997), Burke and Kendall (1999), Hopper and Turton (2002) presented different kinds of classic test problems for function optimization. Teng *et al.* (2004) constructed a test problem for two-dimensional packing with the optimal solution. Bischoff and Ratcliff (1995) and Hoare *et al.* (2001) proposed test problems for three-dimensional packing without behavior constraint (such as container loading). But there are not many test problems for three-dimensional packing with behavior constraints. In this paper, we created seven test problems for three-dimensional packing with behavior constraints (such as dynamic equilibrium or moment of inertia) based on the two-dimensional packing problems (Teng *et al.*, 2004). The test problems take the layout optimization problem in the cabin of an international communication satellite of European Space Agency (shown in Figure 1) as the background, which are called the test problems of quasi-satellite packing for short. The test problems are constructed to evaluate the performance of different algorithms on solving cylinder packing problem with behavior constraint by placing N (=49) cylinders into a cylindrical container. In this paper, the test problems without behavior constraints refer to the packing problem only subjected to non-interference constraint. The test problems with behavior constraints refer to the packing problem subjected to both non-interference constraint and behavior constraints (such as dynamic equilibrium constraints).



a. the international communication satellite b. the simplified satellite module

Figure 1 Schematic diagram of the satellite and the simplified satellite module.

2 Test problems for quasi-satellite packing

2.1 Coordinate system

Figure 2 shows the related coordinate systems used in the test problem, which are defined as follows.

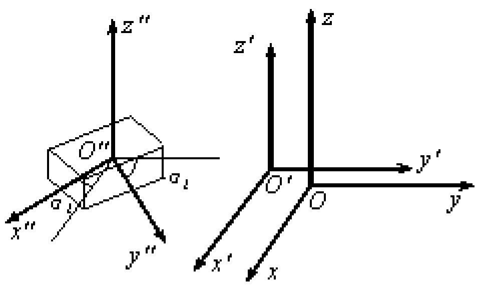


Figure 2 The related coordinate systems

(1) Reference coordinate system *Oxyz*.

O: the origin of the coordinate systems, located on the center of the container base.

z: the longitudinal symmetric axis of the satellite, the upward direction is positive.
x: the perpendicular to the longitudinal surface
y: the composition of right hand rectangular coordinate system with x, z axis.

The reference coordinate system is used to calculate the centroid and moment of inertia of the satellite and define the installation position of the objects.

(2) Planet coordinate system $O'x'y'z'$

O': origin of coordinate, it allocated on the centroid of satellite
z': longitudinal symmetric axis of the satellite, it is parallel with z axis or coincides with z axis.
x', y',: they are parallel with x and y axis, respectively

(3) Local coordinate system of object $O''x''y''z''$

O'': origin of coordinate, it is allocated on the centroid of object.

x'', y'', z'': geometric symmetry axis of the object. In general, z'' is parallel with z axis, and x'', z'' have an angle a_i with x, y axis.

The local coordinate system is used to calculate the moment of inertia of the object in relation to its self axis.

2.2 The test problem for N_{max}(≦49) cylinders packing with behavior constraints

In the coordinate system *Oxyz*, a cylinder with radius R=1 and height H=R is considered as the container, whose bottom face is on the plane *xoy*. Many small cylinders A_i (i = 1,2,…,N, N=49) called objects are placed into the container, whose radius r_i is shown in appendix table 1 and the height h_i=R. The object is supposed as rigid body and the centroid coincides with the mass center. The mass of object m_i=cV_i, where V_i denotes the volume of the cylinder, c is the scale factor (density). The container is called as satellite system after all the objects are packed. The arrangement of the objects should maximize the space utilization of the satellite system and subject to the following constraints:

(1) No overlap exists among the objects or between container and objects.
(2) Static stability constraints.
(3) Moments of inertia constraints.
(4) Equilibrium constraint.

The mathematical model of the test problem is built as follows (see Sun and Teng, 2003):

Find X={$C_i(x_i, y_i, z_i)$| i=1,2,…, N } such that

$$\text{Min}\,F(X) = V_s / V = \left(\frac{p}{4} R_s h_i\right) \Big/ \left(\frac{p}{4} RH\right) = R_s / R \tag{1}$$

Wherein: $C_i(x_i, y_i, z_i)$ is the center of the objects A_i. $F(X)$ denotes the space utilization. V_s is the volume of the envelope cylinder enclosing all the objects. V is the volume of the container. Because the value of R is given, $F(X)$ is equal to following formula.

$$\text{Min}F(X) = \text{Min}\,R_s = \text{Min}(\max(\sqrt{x_i^2 + y_i^2} + r_i)),\ i=1,2,\ldots,N \tag{2}$$

s.t.

(1) Non-interference constraint:

$$g_1(X)= \sum_{i=0}^{N-1}\sum_{j=i+1}^{N} \Delta V_{ij} = 0 \tag{3}$$

(2) Static stability constraints:

$$g_2(X) = (x_e - x_c) - \Delta x_c \le 0, i \in I \tag{4a}$$

$$g_3(X) = (y_e - y_c) - \Delta y_c \le 0 \tag{4b}$$

$$g_4(X) = (z_e - z_c) - \Delta z_c \le 0 \tag{4c}$$

Wherein: m_0 denotes the mass of the container. x_e , y_e and z_e are e the expected position of satellite system centroid with respect to reference system *Oxyz*; (Δx_c,Δy_c,Δz_c) are their allowable errors. (x_c, y_c, z_c)is the expected centroid position of the satellite system, which can be calculated as follows:

$$x_c = \sum_{i=0}^{N} m_i x_i \Big/ \sum_{i=0}^{N} m_i \tag{5a}$$

$$y_c = \sum_{i=0}^{N} m_i y_i \Big/ \sum_{i=0}^{N} m_i \tag{5b}$$

$$z_c = \sum_{i=0}^{N} m_i z_i \Big/ \sum_{i=0}^{N} m_i \tag{5c}$$

(3) Moments of inertia constraints:

$$g_5(X) = |J_x(X)| - \vartriangle J_x \le 0 \tag{6a}$$

$$g_6(X) = |J_y(X)| - \vartriangle J_y \le 0 \tag{6b}$$

$$g_7(X) = |J_z(X)| - \vartriangle J_z \le 0 \tag{6c}$$

Wherein: $(J_x(X), J_y(X), J_z(X))$ are moments of inertia of the satellite system with respect to reference coordinate system *Oxyz*, which can be calculated as the formulas (7). $(\vartriangle J_x, \vartriangle J_y, \vartriangle J_z)$ are their allowable errors.

$$J_x(X) = \sum_{i=0}^{N} (J_{x'i} \cdot \cos a_i^2 + J_{y'} \cdot \sin a_i^2) + \sum_{i=0}^{N} m_i (y_i^2 + z_i^2) - (y_c^2 + z_c^2) \sum_{i=0}^{N} m_i \tag{7a}$$

$$J_y(X) = \sum_{i=0}^{N} (J_{y'i} \cdot \cos a_i^2 + J_{x'i} \cdot \sin a_i^2) + \sum_{i=0}^{N} m_i (x_i^2 + z_i^2) - (x_c^2 + z_c^2) \sum_{i=0}^{N} m_i \tag{7b}$$

$$J_z(X) = \sum_{i=0}^{N} J_{z'i} + \sum_{i=0}^{N} m_i (x_i^2 + y_i^2) - (x_c^2 + y_c^2) \sum_{i=0}^{N} m_i \tag{7c}$$

Wherein: $(J_{x'i}, J_{y'i}, J_{z'i})$ denote moments of inertia of the *i*th object with respect to local coordinate system $O''x''y''z''$, which can be calculated as the formulas (8). a_i is the included angle between the positive axis *x*, *y* of reference coordinate system *Oxyz* with positive axis x'', y'' of local coordinate system $O''x''y''z''$.

$$J_{x''i} = J_{y''i} = m_i(3r_i^2 + h_i^2)/12 \tag{8a}$$

$$J_{z''i} = m_i r_i^2 / 2 \tag{8b}$$

(4) Equilibrium constraint:

$$g_8(X) = |j_x(X)| - \Delta j_x \le 0 \tag{9a}$$

$$g_9(X) = |j_y(X)| - \Delta j_y \le 0 \tag{9b}$$

$$g_{10}(X) = |j_z(X)| - \Delta j_z \le 0 \tag{9c}$$

where $(j_x(X), j_y(X), j_z(X))$ indicate the angles between the principal axis of inertia of the module and the reference axis *ox*, *oy*, *oz* along each direction, calculated as the formulas (10). ($(\Delta j_x, \Delta j_y, \Delta j_z)$ are their allowable error.

$$j_x(X) = \arctan\left(\frac{2J_{xy}(X)}{J_x(X) - J_y(X)}\right) \Big/ 2 \tag{10a}$$

$$j_y(X) = \arctan\left(\frac{2J_{xz}(X)}{J_z(X) - J_x(X)}\right) \Big/ 2 \tag{10b}$$

$$j_z(X) = \arctan\left(\frac{2J_{yz}(X)}{J_z(X) - J_y(X)}\right) \Big/ 2 \tag{10c}$$

Wherein: $(J_{xy}(X), J_{xz}(X), J_{yz}(X))$ are the product of inertia of the system with respect to the reference coordinate system *Oxyz*, and they can be calculated as follows:

$$J_{xy}(X) = \sum_{i=0}^{N} \frac{J_{x'i} + m_i(y_i^2 + z_i^2) - J_{y'i} - m_i(x_i^2 + z_i^2)}{2} \cdot \sin 2a_i \quad + \sum_{i=0}^{N} m_i x_i y_i - x_c y_c \sum_{i=0}^{N} m_i \tag{11a}$$

$$J_{xz}(X) = \sum_{i=0}^{N} m_i x_i z_i - x_c z_c \sum_{i=0}^{N} m_i \tag{11b}$$

$$J_{yz}(X) = \sum_{i=0}^{N} m_i y_i z_i - y_c z_c \sum_{i=0}^{N} m_i \tag{11c}$$

2.3 The optimal solutions of the test problem

2.3.1 The optimal solutions for test problem in coordinate system *Oxyz*

Assume the errors of static stability, moments of inertia and equilibrium all equal zero, we can obtain the optimal solution. The centroid coordinate (x_i, y_i, z_i) of object A_i ($0 < i \le 49$) in the optimal solution is shown in appendix table 1. The optimal layout is seen in Figure 3 and the top view of the layout is seen in Figure 4.

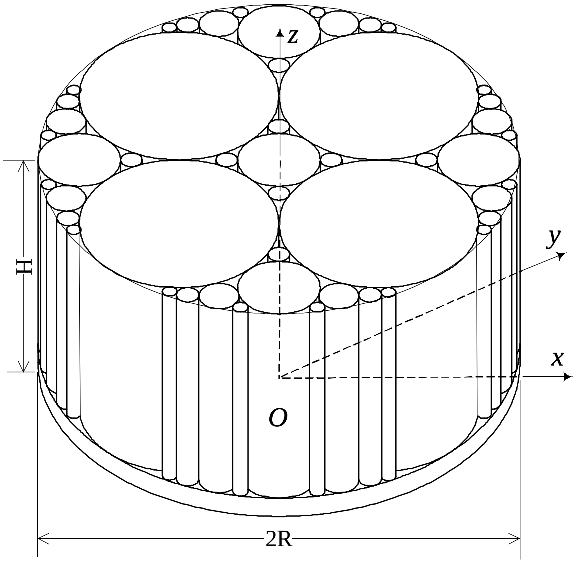


Figure 3 The optimal layout of the test problem

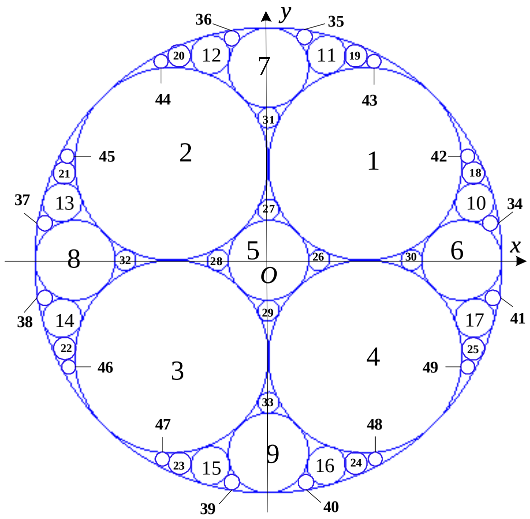


Figure 4 The top view of the optimal layout

2.3.1 Different expression of the solutions

The solutions of the test problem are the relative positions of a set of cylinders in essence. If all the cylinders in the optimal layout rotate an angle θ with respect to longitudinal symmetry axis of the container in counter-clockwise direction, the layout after rotation is still the optimal solutions of the test problem. And $C_i(x_i,y_i,z_i)$, the centre coordinate of A_i , would transformed to C_i (x_i', y_i', z_i'):

$$\begin{cases} x_i' = x_i \cos q - y_i \sin q \\ y_i' = x_i \sin q + y_i \cos q \\ z_i' = z_i \end{cases} \tag{12}$$

So we can get another expression of the optimal solutions according to the formula (12). But all the optimal solutions have the same relative positions of the cylinders.

2.4 Some explanations about the test problems

(1) The proof of the optimal solutions for the test problems can be got from the proof in Teng *et al.*(2004) by analogy. So it is omitted.

(2) Generally speaking, the test problem of q=7(N_{max}=49) is difficult for the current algorithms to solve. User can choose the test problem of q<7(N_{max}<49) according to the algorithm performance to do evaluation. However, when the algorithms with more powerful performance appears, the test problem containing more objects (N_{max}>49) can be created according to the proposed construction method.

(3) Butting two layouts in Figure 3, we can obtain the layout containing N=49 objects on both sides of the bearing plate, which is shown in Figure 5. The layout is more similar to the satellite layout in Figure 1.We can construct another test problem with N=49×2 objects according to above layout, whose mathematic model can be seen in section 3.2. Assume all the allowable errors of the

constraints equal zero, $C_i(x_i, y_i, z_i)$ in the optimal solutions is seen in table 1, and $C_i^u(x_i^u, y_i^u, z_i^u)$ can be obtained from formula (13). Wherein $C_i(x_i, y_i, z_i)$ indicates the center of object on upside of the bearing plate, and $C_i^u(x_i^u, y_i^u, z_i^u)$ indicates the center of underside one.

$$\begin{cases} x_i^u = x_i \\ y_i^u = y_i \\ z_i^u = -z_i \end{cases} \tag{13}$$

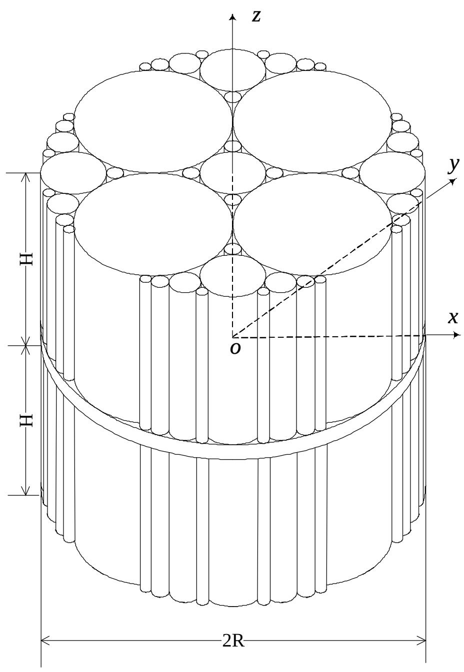


Figure 5 The layout containing 49 objects up and down bearing board (removing container)

3 Conclusions

This paper proposes seven test problems for quasi-satellite packing with all the optimal solutions known, consisting of N=4, 9, 17, 33, 25, 41, 49 cylinders, aiming to evaluate the performance of the algorithm on solving three-dimensional layout optimization problem with behavior constraint. The test problems have following characteristics:

(1) The optimal layout of the test problem have symmetry, and all the parameters of the test problems such as the radii and the centers of the objects can be denoted by the radius of container.

(2) As the optimal solution is known, the offset between the experimental result and the optimal result can be quantified. It is much easier to compare the performance of different algorithms.

(3) Test problems consisting of different groups (q<7) of objects can be chosen to do test according to the performance of the algorithm. Also the test problems containing more objects (N > 49) can be obtained by the proposed construction method.

(4) Butting the satellite layout in Figure 2, we can obtain the test problem with N=49 objects on both sides of satellite sustain board, whose optimal layout has the mathematical beauty of symmetry.

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Appendix A

Table 1 The radius and centroid coordinate of the object in the optimal solutions

No. i	Centroid(x_i,y_i,z_i)	*Radius* r_i
1	($\sqrt{2}$ -1, $\sqrt{2}$ -1, 0.5)	$\sqrt{2}$ -1
2	(-$\sqrt{2}$ +1, $\sqrt{2}$ -1, 0.5)	$\sqrt{2}$ -1
3	(-$\sqrt{2}$ +1, -$\sqrt{2}$ +1, 0.5)	$\sqrt{2}$ -1
4	($\sqrt{2}$ -1, -$\sqrt{2}$ +1, 0.5)	$\sqrt{2}$ -1
5	(0, 0, 0.5)	3-2$\sqrt{2}$
6	(2$\sqrt{2}$ -2,0, 0.5)	3-2$\sqrt{2}$
7	(0, 2$\sqrt{2}$ -2, 0.5)	3-2$\sqrt{2}$
8	(2$\sqrt{2}$ -2,0, 0.5)	3-2$\sqrt{2}$
9	(0, 2$\sqrt{2}$ -2, 0.5)	3-2$\sqrt{2}$
10	((3+$\sqrt{2}$)/5,3/5($\sqrt{2}$ -1, 0.5))	($\sqrt{2}$ -1)/5
11	(3/5($\sqrt{2}$ -1),(3+$\sqrt{2}$)/5, 0.5)	($\sqrt{2}$ -1)/5
12	(-3/5($\sqrt{2}$ -1),(3+$\sqrt{2}$)/5, 0.5)	($\sqrt{2}$ -1)/5
13	(-(3+$\sqrt{2}$)/5,3/5($\sqrt{2}$ -1) , 0.5)	($\sqrt{2}$ -1)/5
14	(-(3+$\sqrt{2}$)/5,-3/5($\sqrt{2}$ -1) , 0.5)	($\sqrt{2}$ -1)/5
15	(-3/5($\sqrt{2}$ -1),-(3+$\sqrt{2}$)/5, 0.5)	($\sqrt{2}$ -1)/5
16	(3/5($\sqrt{2}$ -1),-(3+$\sqrt{2}$)/5, 0.5)	($\sqrt{2}$ -1)/5
17	(-(3+$\sqrt{2}$)/5, 3/5($\sqrt{2}$ -1), 0.5)	($\sqrt{2}$ -1)/5
18	(2(25+29$\sqrt{2}$)/151,8(10$\sqrt{2}$ -7)/151, 0.5)	(10$\sqrt{2}$ -7)/151
19	(8(10$\sqrt{2}$ -7)/151,2(25+29$\sqrt{2}$)/151, 0.5)	(10$\sqrt{2}$ -7)/151
20	(-8(10$\sqrt{2}$ -7)/151,2(25+29$\sqrt{2}$)/151, 0.5)	(10$\sqrt{2}$ -7)/151
21	(-2(25+29$\sqrt{2}$)/151,8(10$\sqrt{2}$ -7)/151, 0.5)	(10$\sqrt{2}$ -7)/151
22	(-2(25+29$\sqrt{2}$)/151,-8(10$\sqrt{2}$ -7)/151, 0.5)	(10$\sqrt{2}$ -7)/151
23	(-8(10$\sqrt{2}$ -7)/151,-2(25+29$\sqrt{2}$)/151, 0.5)	(10$\sqrt{2}$ -7)/151
24	(8(10$\sqrt{2}$ -7)/151,-2(25+29$\sqrt{2}$)/151, 0.5)	(10$\sqrt{2}$ -7)/151
25	(2(25+29$\sqrt{2}$)/151,-8(10$\sqrt{2}$ -7)/151, 0.5)	(10$\sqrt{2}$ -7)/151
26	((10-6$\sqrt{2}$)/7,0, 0.5)	(8$\sqrt{2}$ -11)/7
27	(0, (10-6$\sqrt{2}$)/7, 0.5)	(8$\sqrt{2}$ -11)/7
28	(-(10-6$\sqrt{2}$)/7,0, 0.5)	(8$\sqrt{2}$ -11)/7
29	(0, -(10-6$\sqrt{2}$)/7, 0.5)	(8$\sqrt{2}$ -11)/7
30	((20$\sqrt{2}$ -24)/7,0, 0.5)	(8$\sqrt{2}$ -11)/7
31	(0, (20$\sqrt{2}$ -24)/7, 0.5)	(8$\sqrt{2}$ -11)/7
32	(-(20$\sqrt{2}$ -24)/7,0, 0.5)	(8$\sqrt{2}$ -11)/7
33	(0,-(20$\sqrt{2}$ -24)/7, 0.5)	(8$\sqrt{2}$ -11)/7
34	(($\sqrt{2}$ +11)/13,5($\sqrt{2}$ -1)/13, 0.5)	($\sqrt{2}$ -1)/13
35	(5($\sqrt{2}$ -1)/13,($\sqrt{2}$ +11)/13, 0.5)	($\sqrt{2}$ -1)/13

36	(-5($\sqrt{2}$ -1)/13,($\sqrt{2}$ +11)/13, 0.5)	**($\sqrt{2}$ -1)/13**
37	(-($\sqrt{2}$ +11)/13,5($\sqrt{2}$ -1)/13, 0.5)	**($\sqrt{2}$ -1)/13**
38	(-($\sqrt{2}$ +11)/13,-5($\sqrt{2}$ -1)/13, 0.5)	**($\sqrt{2}$ -1)/13**
39	(-5($\sqrt{2}$ -1)/13,-($\sqrt{2}$ +11)/13, 0.5)	**($\sqrt{2}$ -1)/13**
40	(5($\sqrt{2}$ -1)/13,(- $\sqrt{2}$ +11)/13, 0.5)	**($\sqrt{2}$ -1)/13**
41	(($\sqrt{2}$ +11)/13,-5($\sqrt{2}$ -1)/13, 0.5)	**($\sqrt{2}$ -1)/13**
42	((17+31 $\sqrt{2}$)/71,15(17 $\sqrt{2}$ -9)/497, 0.5)	**(17 $\sqrt{2}$ -9)/497**
43	(15(17 $\sqrt{2}$ -9)/497,(17+31 $\sqrt{2}$)/71, 0.5)	**(17 $\sqrt{2}$ -9)/497**
44	(-15(17 $\sqrt{2}$ -9)/497,(17+31 $\sqrt{2}$)/71, 0.5)	**(17 $\sqrt{2}$ -9)/497**
45	(-(17+31 $\sqrt{2}$)/71,15(17 $\sqrt{2}$ -9)/497, 0.5)	**(17 $\sqrt{2}$ -9)/497**
46	(-(17+31 $\sqrt{2}$)/71,-15(17 $\sqrt{2}$ -9)/497, 0.5)	**(17 $\sqrt{2}$ -9)/497**
47	(-15(17 $\sqrt{2}$ -9)/497,-(17+31 $\sqrt{2}$)/71, 0.5)	**(17 $\sqrt{2}$ -9)/497**
48	(15(17 $\sqrt{2}$ -9)/497,-(17+31 $\sqrt{2}$)/71, 0.5)	**(17 $\sqrt{2}$ -9)/497**
49	((17+31 $\sqrt{2}$)/71,-15(17 $\sqrt{2}$ -9)/497, 0.5)	**(17 $\sqrt{2}$ -9)/497**

Appendix B:

We employ the following nine evolutionary algorithms to solve test problems: GALib (Genetic Algorithms), NicheGA (Niche Genetic Algorithms), aiNet (Artificial Immune Net Algorithms), ClonAlg (Clonal selection Algorithm), DE (Differential Evolution algorithm), Tabu Search, SA(Simulated Annealing), *w*PSO (Particle Swarm Optimization Algorithm with inertial factor) and *x*PSO (Particle Swarm Optimization Algorithm with compression factor).

The number of circles contained in the test problems increases with the group number q, that is N_{max}=4, 9, 17, 25, 33, ···. Each algorithm performs 50 times, respectively. The frequency of fitness evaluation is l_{max}=500000 (max generations). The results indicate that the test problem with the size (N) less than 9 ($N\leq 9$), just can be solved within 500000 generations. If the result satisfies the criteria that R_s is less than 1.01, the value of overlapping is less than 0.01, and the value of static equilibrium is less than 0.01 for each running, then the computation is regarded as success. Table 2 shows the calculations of nine approaches for 50 random running times.

***Success rate**: Success rate η_c is defined as the number of times n_c that the algorithm successfully finds the optimum over the total number N_c of times that the algorithm is applied, which is computed as $\eta_c=n_c/N_c\times 100\%$.*

Table 2. The results of nine evolutionary algorithms solving the constrained packing test problem (N=9) (each algorithm run for 50 times)

No.	Algorithms	Time t(s)		enveloping Radius R_s		Overlapping (g_1+g_2)		Static equilibrium g_3		SR η_r (%)	Population size
		Mean	Optimal	Mean	Optimal	Mean	Optimal	Mean	Optimal		
1	DE	10.7222	10.5259	1.0029	1.0016	0.0035	0.0011	0.0056	0.0024	90	60
2	TS	1.8597	0.0940	1.0073	1.0076	0.0077	0.0062	0.0104	0.0084	88	50
3	aiNet	44.0301	45.2316	1.0089	1.0082	0.0062	0.0038	0.0246	0.0070	58	100
4	Clonalg	32.5311	31.8959	1.0092	0.9999	0.0156	0.0001	0.1343	0.0002	22	100
5	SA	12.6850	12.7763	1.0083	0.9997	0.0401	0.0001	0.0138	0.0001	14	1
6	GAlib	40.9113	41.3441	1.0844	1.0036	0.0014	0.0005	0.0001	0.0001	10	40
7	*w*PSO	13.9175	13.1538	1.0555	0.9999	0.0041	0.0001	0.0143	0.0001	4	25
8	NicheGA	51.8713	52.3516	1.0258	1.0039	0.0365	0.0024	0.0208	0.0021	4	100
9	*x*PSO	16.2753	16.2364	1.0572	1.0228	0.0064	0.0089	0.0150	0.0198	0	25