Solving Bin Packing Related Problems Using an Arc Flow Formulation

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Solving Bin Packing Related Problems Using an Arc Flow Formulation

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Abstract

We present a new method for solving bin packing problems, including two-constraint variants, based on an arc flow formulation with side constraints. Conventional formulations for bin packing problems are usually highly symmetric and provide very weak lower bounds. The arc flow formulation proposed provides a very strong lower bound, and is able to break symmetry completely.

The proposed formulation is usable with various variants of this problem, such as bin packing, cutting stock, cardinality constrained bin packing, and 2D-vector bin packing. We report computational results obtained with standard benchmarks, all of them showing a large advantage of this formulation with respect to the traditional ones.

Keywords: bin packing, cutting stock, integer programming, arc flow formulation, cardinality constrained bin packing, 2D-vector bin packing

1. Introduction

The bin packing problem is a combinatorial NP-hard problem (see, e.g., Garey et al. 1979) in which objects of different volumes must be packed into a finite number of bins, each with capacity W, in a way that minimizes the number of bins used. Besides being NP-hard, bin packing is also hard to approximate within $3/2 - \varepsilon$. If such approximation exists, one could partition n non-negative numbers into two sets with the same sum in polynomial time. However, this problem is also known to be NP-hard. Simchi-Levi (1994) showed that first-fit decreasing and best-fit decreasing heuristics have an absolute performance ratio of 1.5, which is the best possible absolute performance ratio for the bin packing problem, unless $P = NP$.

There are many variants of this problem and they have many applications, such as filling up containers, loading trucks with volume or weight capacity limits, creating file backups in removable media, technology mapping in field-programmable gate array semiconductor chip design, among others.

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The bin packing problem can be seen as a special case of the cutting stock problem. In this problem we have a number of rolls of paper of fixed width waiting to be cut, for satisfying demand of different customers, who want different numbers of rolls of various widths. We have to cut the rolls in such a way that waste is minimized. Note that in the paper industry solving this problem to optimality can be economically significant; a small improvement in reducing waste can have a huge impact in yearly savings.

There are many similarities between the bin packing problem and the one-dimensional cutting stock problem. However, in the cutting stock problem, the items of equal size (which are usually ordered in large quantities) are grouped into orders with a required level of demand, while in the bin packing problem the demand for a given size is usually close to one. According to Wäscher et al. (2007), cutting stock problems are characterized by a weakly heterogeneous assortment of small items, in contrast with the bin packing problems.

One of the bin packing variants is the cardinality constrained bin packing, in which in addition to the capacity constraint, the number of items per bin is also limited. This problem can be seen as a special case of two-constraint bin packing (also called 2D-vector bin packing by some authors), in which each item has a weight and a length. On 2D-vector bin packing, on each dimension there is a difficult problem, whereas for the cardinality constrained bin packing in one of the dimensions the problem is very easy: we just need to count the number of items.

In this article we present a method based on an arc flow formulation with side constraints by Valério de Carvalho (1999). This model has a set of flow conservation constraints and a set of constraints to ensure that the demand is satisfied. The corresponding path flow formulation is equivalent to the classical formulation for the cutting stock problem. We extent this idea to some variants of the bin packing problem.

The remainder of this article is organized as follows. Section 2 gives account of previous approaches to these problems. Section 2.2 presents Valério de Carvalho's method. Section 3 describes our new method. Our results are presented in Section 4. Finally, Section 5 gives the conclusions.

2. Previous work

In this section we will give account of previous approaches to bin packing and related problems. We will introduce Martello and Toth's formulation for bin packing, Kantorovich's formulation for cutting stock, Caprara and Toth's formulation for two-constraint bin packing and it's adaptation to cardinality constrained bin packing.

We will also describe the classical formulation of Gilmore and Gomory (1961) for the cutting stock problem, which is equivalent to the model described in Valério de Carvalho (1999). Gilmore and Gomory's model provides a very strong LP relaxation, but it has an exponential size; while Valério de Carvalho's model is also exponential, but it is much smaller. While Gilmore and Gomory's model is exponential in the number of decision variables with respect with the input size, Valério de Carvalhos's model is pseudo-polynomial in terms of the decision variables and of the number of constraints. In both models we consider every valid packing pattern. However, in the

Valério de Carvalho's model, patterns are derived from paths in a graph, whereby the model is usually much smaller.

Valério de Carvalho (2002) provides an excellent survey on integer programming models for bin packing and cutting stock problems. Here we will just look at the more common and straightforward approaches.

2.1. Common approaches

2.1.1. Martello and Toth's formulation

Martello and Toth (1990) developed a branch-and-bound algorithm for the bin packing problem based on the following mathematical programming formulation:

$$\text{minimize} \quad \sum_{k=1}^{K} y_k \tag{1}$$

$$\text{subject to} \quad \sum_{k=1}^{K} x_{ik} = 1, \qquad i = 1, \ldots, n, \tag{2}$$

$$\sum_{i=1}^{n} w_i x_{ik} \leq W y_k, \qquad k = 1, \ldots, K, \tag{3}$$

$$y_k \in \{0, 1\}, \qquad k = 1, \ldots, K, \tag{4}$$

$$x_{ik} \in \{0, 1\}, \qquad i = 1, \ldots, n, \ k = 1, \ldots, K. \tag{5}$$

where K is a known upper bound on the number of bins needed (it may be obtained using, for example, the first-fit decreasing heuristics), n is the number of items, w_i is the weight of item i, W is the bin capacity, and the variables are:

$$y_k = \begin{cases} 1 & \text{if bin } k \text{ is used,} \\ 0 & \text{otherwise;} \end{cases}$$

$$x_{ik} = \begin{cases} 1 & \text{if item } i \text{ is assigned to bin } k, \\ 0 & \text{otherwise.} \end{cases}$$

Martello and Toth (1990) proved that the lower bound for the linear relaxation of this model, which is equal to the minimum amount of space that is necessary to accommodate all the items if they could be divided, can be very weak for instances with large waste.

Property 1 (Martello and Toth's formulation). *The lower bound obtained by model (1)-(5) is equal to* $\lceil \sum_{i=1}^{n} w_i / W \rceil$.

Property 2 (Martello and Toth's formulation). *In the worst case, as* W *increases, when all the items have a size* $w_i = \lfloor W/2 + 1 \rfloor$*, the lower bound approaches* $1/2$ *of the optimal solution.*

PROOF. If $w_i = \lfloor W/2+1 \rfloor$ then $\sum_{i=1}^{n} w_i = n\lfloor W/2+1 \rfloor \leq nW/2+n$. Therefore $\lceil \sum_{i=1}^{n} w_i/W \rceil \leq \lceil (nW/2+n)/W \rceil = \lceil n/2+n/W \rceil$. As W increases, this lower bound approaches $\lceil n/2 \rceil$ while the optimal solution is n.

This is a drawback of this model, as good quality lower bounds are vital in branch-and-bound procedures. Another drawback is due to the symmetry of the problem, which makes this model very inefficient in practice.

2.1.2. Kantorovich's formulation

In principle, the bin packing problem and the cutting stock problem are equivalent. However, the bin packing problem takes a list of items as input, while the cutting stock problem takes a list of different item sizes and the corresponding demands. The size of the input for the bin packing problem can be exponentially larger than the input for the cutting stock problem. Therefore, a polynomial-size formulation for the bin packing problem is not necessarily polynomial-size for the cutting stock problem.

Kantorovich (1960) introduced the following mathematical programming formulation for the cutting stock problem, where the objective is to minimize the number of rolls used to cut all the items demanded:

$$\text{minimize} \quad \sum_{k=1}^{K} y_k \tag{6}$$

$$\text{subject to} \quad \sum_{k=1}^{K} x_{ik} \geq b_i, \qquad i = 1, \ldots, m, \tag{7}$$

$$\sum_{i=1}^{m} w_i x_{ik} \leq W y_k, \qquad k = 1, \ldots, K, \tag{8}$$

$$y_k \in \{0,1\}, \qquad k = 1, \ldots, K, \tag{9}$$

$$x_{ik} \geq 0, \text{ integer}, \qquad i = 1, \ldots, m, \; k = 1, \ldots, K, \tag{10}$$

where K is a known upper bound on the number of rolls needed, m is the number of different item sizes, w_i and b_i are the weight and demand of item i, and W is the roll capacity. The variables are y_k, which is 1 if roll k is used and 0 otherwise, and x_{ik}, the number of times item i is cut in the roll k.

Property 3 (Kantorovich's formulation). *The lower bound given by the linear relaxation of model (6)-(10) is equal to* $\lceil \sum_{i=1}^{m} w_i b_i/W \rceil$.

PROOF. Defining $\sigma = \sum_{i=1}^{m} w_i b_i/W$, an optimal solution of the LP relaxation of (6)-(10) can be computed in $O(n)$ time as $y_i^* = \sigma/K$ for $i = 1, \ldots, K$ and $x_{ik}^* = b_i/K$. Therefore, $\sum_{k=1}^{K} y_k^* = \sum_{k=1}^{K} \sigma/K = \sigma$. It is easy to check that this solution is feasible. By summing up inequalities (8) and considering $\sum_{k=1}^{K} x_{ik} = b_i$, we have $\sum_{k=1}^{K} \sum_{i=1}^{m} w_i x_{ik} \leq \sum_{k=1}^{K} W y_k \Leftrightarrow \sum_{i=1}^{m} \sum_{k=1}^{K} w_i x_{ik} \leq \sum_{k=1}^{K} W y_k \Leftrightarrow \sum_{i=1}^{m} w_i \sum_{k=1}^{K} x_{ik} \leq \sum_{k=1}^{K} W y_k \Leftrightarrow \sum_{i=1}^{m} w_i b_i \leq \sum_{k=1}^{K} W y_k \Leftrightarrow \sum_{i=1}^{m} w_i b_i/W \leq \sum_{k=1}^{K} y_k \Leftrightarrow \sigma \leq \sum_{k=1}^{K} y_k$. Therefore, the solution is optimal once $\sum_{k=0}^{K} y_k^* = \sigma$. By rounding up the lower bound obtained, we have $\lceil \sigma \rceil = \lceil \sum_{i=1}^{m} w_i b_i/W \rceil$.

In the worst case the lower bound provided by this model approaches 1/2 of the optimal solution, since this lower bound is equivalent to the one provided by Martello and Toth's formulation.

Vance (1998) applied a Dantzig-Wolfe decomposition (see, e.g., Dantzig and Wolfe 1960) to model (6)-(10), keeping constraints (6), (7) in the master problem, and the subproblem being defined by the integer solutions to the knapsack constraints (8). Vance also showed that when all the rolls have the same width, the reformulated model is equivalent to the classical Gilmore-Gomory model.

Gilmore and Gomory (1961) proposed the following model for the cutting stock problem. A combination of orders in the width of the roll is called a cutting pattern. Let column vectors $a^j = (a_1^j, \ldots, a_m^j)$ represent all possible cutting patterns j. The element a_d^j represents the number of rolls of width w_d obtained in cutting pattern j. Let x_j be a decision variable that designates the number of rolls to be cut according to cutting pattern j. The cutting stock problem that can be modeled in terms of these variables as follows:

$$\text{minimize} \quad \sum_{j \in J} x_j \tag{11}$$

$$\text{subject to} \quad \sum_{j \in J} a_i^j x_j \geq b_i, \qquad i = 1, \ldots, m, \tag{12}$$

$$x_j \geq 0, \ \text{integer}, \qquad \forall j \in J, \tag{13}$$

where J is the set of valid cutting patterns that satisfy:

$$\sum_{i=1}^{m} a_i^j w_i \leq W \text{ and } a_i^j \geq 0, \ \text{integer}. \tag{14}$$

Since constraints (14) just accept integer linear combinations of items, the search space of the continuous relaxation is reduced and the lower bound provided is stronger.

It may be impractical to enumerate all the columns in this formulation, as their number may be very large, even for moderately sized problems. To tackle this problem, Gilmore and Gomory (1963) introduced column generation.

2.1.3. Caprara and Toth's formulation

Two-constraint bin packing can be formulated in a way similar to the standard version. Caprara and Toth (2001) introduced the following mathematical programming formulation for this problem:

$$
\begin{aligned}
&\text{minimize} && \sum_{k=1}^{K} y_k && && (15)\\
&\text{subject to} && \sum_{k=1}^{K} x_{ik} = 1, && i = 1, \dots, n, && (16)\\
& && \sum_{i=1}^{n} w_i x_{ik} \le A y_k, && k = 1, \dots, K, && (17)\\
& && \sum_{i=1}^{n} v_i x_{ik} \le B y_k, && k = 1, \dots, K, && (18)\\
& && y_k \in \{0, 1\}, && k = 1, \dots, K, && (19)\\
& && x_{ik} \in \{0, 1\}, && i = 1, \dots, n,\ k = 1, \dots, K, && (20)
\end{aligned}
$$

where K is a known upper bound on the number of bins needed, n is the number of items, w_i is the weight of item i in the first dimension, v_i is the weight of item i in the second dimension, and A and B are bin capacities in the first and second dimensions, respectively. Each variable y_k is 1 if bin k is used and 0 otherwise, and x_{ik} is 1 if item i is assigned to bin k, 0 otherwise.

Property 4 (Caprara and Toth's formulation). *The lower bound obtained by model (15)-(20) is equal to* $\max(\lceil \sum_{i=1}^{n} w_i/A \rceil, \lceil \sum_{i=1}^{n} v_i/B \rceil)$.

Proof. Defining $\sigma = \max(\sum_{i=1}^{n} w_i/A, \sum_{i=1}^{n} v_i/B)$, Caprara (1998) proved that an optimal solution (x^*, y^*) of the LP relaxation of (15)-(20) can be computed in $O(n)$ time as $y_i^* = \sigma/n$ for $i = 1, \dots, n$ and $x_{ik}^* = 1/n$. Therefore, $\sum_{k=1}^{n} y_k^* \sum_{k=1}^{n} \max(\sum_{i=1}^{n} w_i/A, \sum_{i=1}^{n} v_i/B)/n = \max(\sum_{i=1}^{n} w_i/A, \sum_{i=1}^{n} v_i/B)$. It is easy to check that this result is also valid if we limit the number of bins by a known upper bound K. In this case, an optimal solution can be $y_i^* = \sigma/K$ for $i = 1, \dots, K$ and $x_{ik}^* = 1/K$. By rounding up the lower bound obtained, we have $\max(\lceil \sum_{i=1}^{n} w_i/A \rceil, \lceil \sum_{i=1}^{n} v_i/B \rceil)$.

Property 5 (Caprara and Toth's formulation). *The worst case performance ratio of the lower bound provided by the Caprara and Toth's formulation is* 1/3.

Proof. Caprara (1998) proved for the p-dimensional vector packing that the worst-case performance of the lower bound provided by the linear relaxation of the model (21)-

(25)

$$
\begin{aligned}
\text{minimize} \quad & \sum_{k=1}^{n} y_k && && (21)\\
\text{subject to} \quad & \sum_{k=1}^{n} x_{ik} = 1, && i = 1, \ldots, n, && (22)\\
& \sum_{i=1}^{n} a_{li} x_{ik} \leq y_k, && k = 1, \ldots, n, l = 1, \ldots, p, && (23)\\
& y_k \in \{0, 1\}, && k = 1, \ldots, n, && (24)\\
& x_{ik} \in \{0, 1\}, && i = 1, \ldots, n, k = 1, \ldots, n && (25)
\end{aligned}
$$

is equal to $1/(p+1)$. For $p = 2$, this model is equivalent to Caprara and Toth's formulation for 2-Dimensional Vector Packing. Therefore, in this problem the worst case performance ratio is equal to $1/3$.

In this model, the lower bound weakness is a huge drawback. The worst case performance may be asymptotically achieved, for example, with $A = B = W$, $n = 3W$, $2W$ items of size $(\lfloor W/2 + 1 \rfloor, \lfloor W/3 + 1 \rfloor)$ and W items of size $(0, \lfloor 2W/3 + 1 \rfloor)$. As W increases, the lower bound approaches $\lceil n/3 \rceil$, while the optimal solution is n.

Cardinality constrained bin packing can be seen as a special case of two-constraint bin packing; we can formulate this problem based on Caprara and Toth's formulation in the following way:

$$
\begin{aligned}
\text{minimize} \quad & \sum_{k=1}^{K} y_k && && (26)\\
\text{subject to} \quad & \sum_{k=1}^{K} x_{ik} = 1, && i = 1, \ldots, n, && (27)\\
& \sum_{i=1}^{n} x_{ik} w_i \leq W y_k, && k = 1, \ldots, K, && (28)\\
& \sum_{i=1}^{n} x_{ik} \leq C y_k, && k = 1, \ldots, K, && (29)\\
& y_k \in \{0, 1\}, && k = 1, \ldots, K, && (30)\\
& x_{ik} \in \{0, 1\}, && i = 1, \ldots, n, k = 1, \ldots, K, && (31)
\end{aligned}
$$

where K is a known upper bound on the number of bins needed, n is the number of items, w_i is the weight of item i, W is the capacity and C is the bin maximum cardinality.

Property 6 (Cardinality constrained version). *The lower bound obtained by the linear relaxation of model (26)-(31) is equal to* $\max(\lceil \sum_{i=1}^{n} w_i/W \rceil, \lceil n/C \rceil)$.

Proof. Fixing $A = W$, $v_i = 1$, and $B = C$ in Caprara and Toth's formulation we have the model for the cardinality constrained version. From Property 4 we know that the lower bound is equal to $\max(\lceil \sum_{i=1}^{n} w_i/A \rceil, \lceil (\sum_{i=1}^{n} v_i/B \rceil)$. Once, in this case $\sum_{i=1}^{n} v_i = n$, the lower bound is equal to $\max(\lceil \sum_{i=1}^{n} w_i/W \rceil, \lceil n/C \rceil)$.

In practice, these models (except Gilmore-Gomory's model) have at least the same limitations as Martello and Toth's formulation for standard bin packing.

2.2. Valério de Carvalho's method

Valério de Carvalho (1999) proposed an arc flow formulation with side constraints for the bin packing problem. In this section we describe his method; in Section 3 we will describe how we improved and generalized his approach.

2.2.1. Arc flow formulation

Given bins of integer capacity W and a set of different item sizes $w_1, w_2, \ldots, w_m$, the problem of determining a valid solution to a single bin can be modeled as the problem of finding a path in an acyclic directed graph with $W + 1$ vertices.

Consider a graph $G = (V, A)$ with $V = \{0, 1, 2, \ldots, W\}$ and $A = \{(i, j) : j - i = w_d$, for $1 \leq d \leq m$ and $0 \leq i < j \leq W\}$, meaning that there exists an arc between two vertices i and $j > i$ if there is an item of size $w_d = j - i$. The number of arcs is bounded by $\mathcal{O}(mW)$. Additional arcs between $(k, k + 1)$, for $k = 0, \ldots, W - 1$, are included for representing unoccupied portions of the bin.

Property 7 (Flow formulation for a packing). *There is a packing of the m items in a single bin if and only if there is a path between vertices 0 and W. The length of arcs that constitute the path (excepting loss) define the item sizes to be packed.*

Proof.

($\Rightarrow$) If there is a packing of the m items in a single bin then, by construction, the graph will contain a path $(0, a_1)(a_1, a_2) \ldots (a_{m-1}, a_m) \ldots (*, W)$ corresponding to the packing $(a_1, a_2 - a_1, \ldots, a_m - a_{m-1})$; $(*, W)$ represent loss, unitary arcs from a_m to W.

($\Leftarrow$) If there is a path $(0, a_1)(a_1, a_2) \ldots (a_n, W)$ between vertices 0 and W with n arcs (with $m \leq n$ arcs corresponding to items and possibly some loss arcs), it is always possible to obtain the corresponding packing pattern from the differences $j - i$ from each arc (i, j) in the path (excepting loss).

Example 1. *Figure 1 shows the graph associated with an instance with bins of capacity $W = 7$ and items of sizes 5, 3, 2 with demands 3, 1, 2, respectively; the path shown corresponds to a bin with two items of size $w_2 = 3$ and one unit of loss.*

This kind of formulation has been used initially by Shapiro (1968) to model knapsack problems as the problem of determining the longest path in a directed graph. The same idea can be used to model bin packing problems: a solution to a single bin corresponds to the flow of one unit between vertices 0 and W, and a path carrying a larger flow will correspond to using the same packing solution in multiple bins. Different paths correspond to different packing patterns.

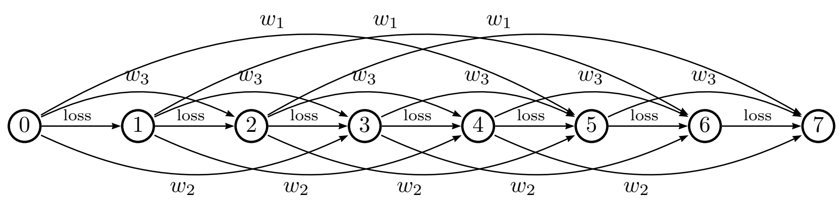

Figure 1: Graph associated with an instance with bins of capacity $W = 7$ and items of sizes 5, 3 and 2 (top) and a path between vertices 0 and W that corresponds to a possible packing (bottom).

The bin packing problem is now equivalently formulated as that of determining the minimum flow between vertex 0 and vertex W, with additional constraints enforcing the sum of the flows in the arcs of each order to be greater than or equal to the corresponding demand. Consider decision variables x_{ij} associated with the arcs defined above, which correspond to the number of items of size $j - i$ placed in any bin at a distance of i units from the beginning of the bin. A variable z, representing the number of bins required, aggregates the flow in the graph, and can be seen as a feedback arc from vertex W to vertex 0. The model is as follows:

$$\text{minimize} \quad z \tag{32}$$

$$\text{subject to} \quad \sum_{(i,j)\in A} x_{ij} - \sum_{(j,k)\in A} x_{jk} = \begin{cases} -z & \text{if } j = 0, \\ z & \text{if } j = W, \\ 0 & \text{for } j = 1, \dots, W-1, \end{cases} \tag{33}$$

$$\sum_{(k,k+w_i)\in A} x_{k,k+w_i} \geq b_i, \quad i = 1, \dots, m, \tag{34}$$

$$x_{ij} \geq 0, \text{ integer}, \qquad \forall (i,j) \in A. \tag{35}$$

Property 8 (equivalence to the classical Gilmore-Gomory). *This model is equivalent to the classical Gilmore-Gomory model (11)-(14) for cutting stock problem and hence the linear relaxation bounds are identical.*

Proof. Valério de Carvalho (1999) proved that this formulation is equivalent to the classical Gilmore-Gomory model (11)-(14) by applying Dantzig-Wolfe decomposition to model (32)-(35) keeping (32) and (34) in the master problem, and (33) and (35) in the subproblem. Once the subproblem is a flow problem that will only generate valid patterns, we can substitute (33),(35) by patterns and obtain the classical model. From this equivalence follows that lower bounds provided by both models are the same.

Any feasible solution to model (32)-(35) can be transformed into a feasible solution to the bin packing problem using z bins. By the flow decomposition properties, non-negative flows can be represented by paths and cycles. Once the graph G is acyclic, any flow can be decomposed into directed paths connecting the only excess node (node 0) to the only deficit node (node W); see Section 3 for more details. The graph in Figure 2 shows an optimal solution for Example 1. The optimal solution for the bin packing problem can be obtained by decomposing the flow into four paths: three paths $(0,5)(5,7)$ corresponding to three bins with one item of size 5 and one of size 2; and one path $(0,3)(3,4)(4,5)(5,6)(6,7)$ corresponding to one bin with one item of size 3 and loss.

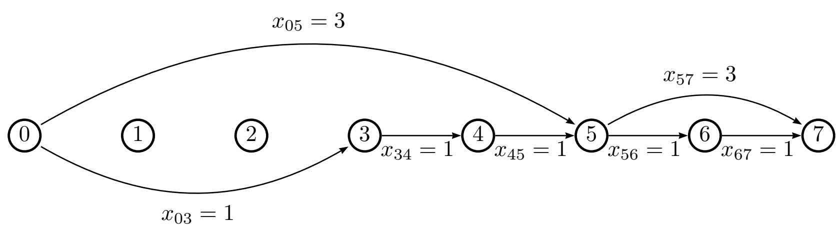

Figure 2: Optimal solution for Example 1.

2.2.2. Symmetry reduction criteria

In order to reduce the symmetry of the solution space and the size of the model, Valério de Carvalho introduced some symmetry reduction criteria. The idea is to consider only a subset of arcs from A. If we search a solution in which the items are ordered by decreasing values of width, the following criteria may be used to reduce the number of arcs that are taken into account.

Criterion 1. *An arc $(k, k + w_e)$ of size w_e can only leave a node $k > 0$ if there is another arc $(k - w_d, k)$ of size $w_d \geq w_e$ entering k; any node can leave node $k = 0$.*

Criterion 2. *All the loss arcs $(k, k + 1)$ can be removed for $k < w_m$ (recall that w_m is the smallest item).*

Criterion 3. *Given any node k that is the head of an arc of size w_d or $k = 0$, the only valid arcs for size w_e ($w_e < w_d$) are those that start at nodes $k + sw_e$, for $s = 0, 1, 2, \ldots, b_e - 1$, with $k + (s + 1)w_e \leq W$, where b_e is the demand of items of size w_e.*

Criterion 1 tries to impose a order on the size and placement of a bin's items; Criterion 2 imposes that a bin never starts with loss; and Criterion 3 tries to limit the number of consecutive arcs of a certain size by the demand.

The graph in Figure 3 (with 11 arcs) results from applying these criteria to the graph in Figure 1 (with 21 arcs). Notice that this set of criteria may not completely break the symmetry of the solution space. For example, in the graph of Figure 3 the paths $(0,3)(3,5)(5,6)(6,7)$ and $(0,3)(3,4)(4,5)(5,7)$ correspond to the same pattern (one item of size w_2 and one of size w_3).

Valério de Carvalho (1999) developed a branch-and-price procedure that combines deferred variable generation and branch-and-bound. At each iteration, the subproblem generates a set of columns, which altogether correspond to an attractive valid packing for a single bin.

Thanks to symmetry reduction, the memory required to hold the entire graph, even for reasonably large instances, is less than a few gigabytes. This eliminates the requirement of using column generation and, more importantly, opens the possibility

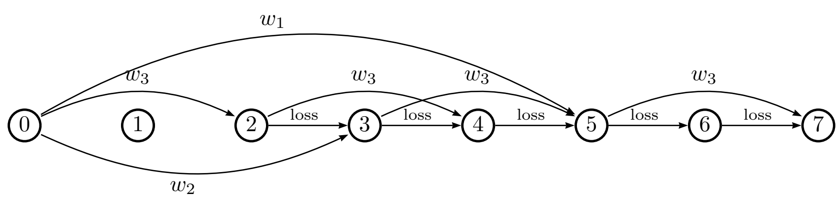

Figure 3: Graph corresponding to Example 1 after applying symmetry reduction.

of using general-purpose mixed-integer programming solvers to tackle this problem directly. We used `gurobi` (Gu et al. (2011)) to solve every instance from OR-Library (2012) using this model. The average run time was less than 2 seconds. Using the same solver and Martello and Toth's model, we were able to solve only 7 out of the 160 instances within a 10 minutes time limit (even allowing `gurobi` to use problem-specific heuristics).

Valério de Carvalho's model proved to be very efficient on bin packing and cutting stock. The main ideas behind his formulation described in this section are the starting point for our methods.

3. A new arc flow formulation

It would be interesting to use Valério de Carvalho's model to cardinality constrained bin packing, or, more generally, on two-constraint bin packing. However, this approach does not allow us to control the number of items we assign to each bin.

In this section we propose a generalization of his model, in the following way.

$$\text{minimize} \quad z \tag{36}$$

$$\text{subject to} \quad \sum_{(i,j)\in A} x_{ij} - \sum_{(j,k)\in A} x_{jk} = \begin{cases} -z & \text{if } j = s, \\ z & \text{if } j = t, \\ 0 & \text{otherwise,} \end{cases} \tag{37}$$

$$\sum_{(i,j)\in A_l} x_{ij} \geq b_l, \qquad l = 1, \ldots, m, \tag{38}$$

$$x_{ij} \geq 0, \text{ integer}, \qquad \forall (i,j) \in A, \tag{39}$$

where $A_l \subseteq A$ represents the set of arcs associated with item l (i.e., the set of arcs that can contribute to the demand of the item l) , s represents the source and t the target. In Valério de Carvalho's model, a variable x_{ij} contributes to an item with weight $j-i$, in our case it contributes to the item with label l iff $(i,j) \in A_l$. This new model is more general; Valério de Carvalho's model is a subcase, where l always corresponds to the size of an item. As in Valério de Carvalho's model, each arc can only contribute to an item, but the new model has several differences from the original formulation:

- nodes are more general (e.g., they can encompass two-dimensions);
- there may be some arcs with no label;
- arcs may be unrelated to distance (i.e., $(i,j) \in A_l$ even if $j - i \neq l$).

Using this model it is possible to use more general graphs, but we always need to ensure that it is a directed acyclic graph whose flow will only generate every valid packing, and that every arc is associated with at most one item (i.e., $\forall i \neq j \; A_i \cap A_j = \varnothing$). Since `gurobi` works very well with Valério de Carvalho's model, it is expectable that it also works very well on this model with a similar graph.

Property 9 (equivalence to the classical Gilmore-Gomory). *The model (36)-(39) is equivalent to the classical Gilmore-Gomory model (11)-(14) with the same patterns as the ones obtained from paths in the graph.*

Proof. Extending Valério de Carvalho's proof, we apply Dantzig-Wolfe decomposition to model (36)-(39) keeping (36),(38) in the master problem and (37),(39) in the subproblem. Since the subproblem is a flow model that will only generate valid patterns, we can substitute (37),(39) by patterns and obtain the classical model. From this equivalence follows that lower bounds provided by both models are the same.

Using this new formulation it is possible to model the bin packing problem for the instance of Example 1, using the graph in Figure 4.

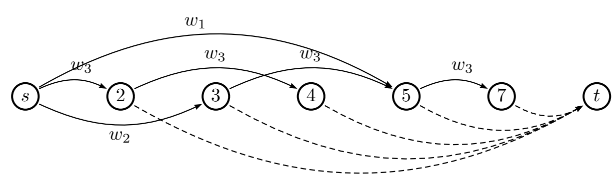

Figure 4: Another possible graph for bin packing. In this graph, $A_1 = \{(s,5)\}$, $A_2 = \{(s,3)\}$ and $A_3 = \{(s,2),(2,4),(3,5),(5,7)\}$ are the sets of arcs associated with each item. Since the loss arcs connect the nodes directly to the target (instead of connecting consecutive nodes) we do not always need to have a node for each integer value less than, or equal to the capacity.

This new type of graph allows us to have less nodes in instances with very large bin dimensions. We used `gurobi` to solve every instance from OR-Library (2012) using this type of graph, and the average run time was less than 2 seconds. This formulation also has the advantage of reducing symmetry.

3.1. Cardinality constrained bin packing

If there is a constraint limiting the number of items that can be placed in a bin, not every path on the previous model is feasible. For example, if we have a 2-item cardinality limit, we need to exclude paths containing three or more items. We could add constraints for excluding paths that violate cardinality, but this would make the problem much harder to solve. An alternative idea is to extend the graph in order to include cardinality information in the nodes, as in Figure 5.

In the graph presented in Figure 5, if we want to limit the cardinality to 2, we just need to remove node $(7,3)$.

Property 10 (Cardinality constrained bin packing bounds). *The lower bound provided by the linear relaxation on the cardinality constrained bin packing is at least* $\max(z^\star_{bp}, n/C)$ *where* $z^\star_{bp}$ *is the lower bound obtained from the linear relaxation for the standard bin packing problem using the arc flow formulation.*

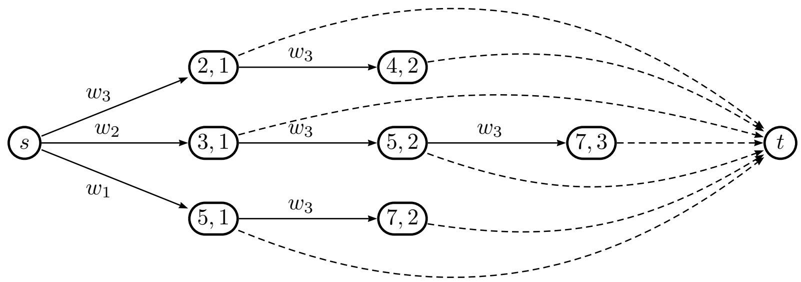

Figure 5: Graph associated with Example 1, but now with cardinality limit 3. A node (w, c) means that any path from the source will reach the node with at most c items whose sum of weights is at most w. We connect nodes (w, c) with nodes $(w + w_i, c + 1)$ using an arc belonging to A_i associated with an item of weight w_i.

Proof. Let $z^\star$ be the lower bound. Suppose that $z^\star < \max(z^\star_{bp}, n/C)$. We have to consider two cases. If $z^\star_{bp} > n/C$ then we have a contradiction, since we can use the same solution for standard bin packing (i.e., with no cardinality constraint), and by assumption $z^\star < \max(z^\star_{bp}, n/C) = z^\star_{bp}$; but $z^\star_{bp}$ is optimal. If $n/C > z^\star_{bp}$ we also have a contradiction, since by assumption $z^\star < \max(z^\star_{bp}, n/C) = n/C$; but we cannot have more than C items in each pattern.

For testing this model, we have done the following experiment. For each instance i of the OR-Library, we determined the maximum number of items C_i that fit into a single bin, in order to study the effect of cardinality even when it does not exclude any packing pattern. We then created instances with cardinality limits $C = C_i, C_i-1, C_i-2, \ldots, 1$. As shown in Section 4, this model works very well on cardinality constrained bin packing, for every cardinality. Run time increases just slightly when both cardinality and capacity constraints are likely to be binding; otherwise, the average run time per instance remains below two seconds.

3.2. Two-constraint bin packing

We can extend the idea we used for cardinality constrained bin packing for the more general case of two-constraint bin packing. For a given item i, instead of having an arc connecting node (a, b) to a node $(a + w_i, b + 1)$, there will be an arc connecting node (a, b) to a node $(a + w_i, b + v_i)$, where w_i is the weight and v_i is the length of item i.

Consider an instance with bins of capacity $(W_1, W_2) = (4, 4)$ and items of sizes $(w_1, v_1) = (1, 3)$, $(w_2, v_2) = (3, 1)$ and $(w_3, v_3) = (2, 2)$, with demands 1, 1 and 2, respectively. Figure 6 shows the corresponding graph.

Property 11 (Two-constraint bin packing bounds). *The lower bound provided by the linear relaxation on the two-constraint constrained bin packing is at least* $\max(z^\star_1, z^\star_2)$ *where* $z^\star_1$ *and* $z^\star_2$ *are the lower bounds obtained from the linear relaxation for standard bin packing problems using the arc flow formulation on the first and second dimensions, respectively.*

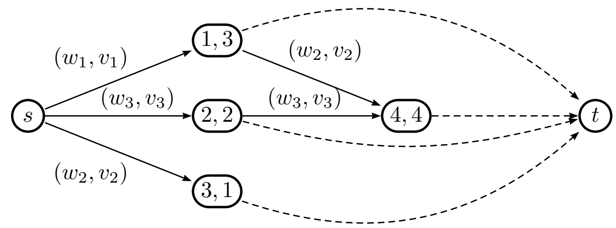

Figure 6: Two-constraint bin packing example We connect nodes (a, b) with nodes $(a + w_i, b + v_i)$ using an arc belonging to A_i, associated with an item of weight (w_i, v_i).

Proof. Let $z^\star$ be the lower bound. Suppose that $z^\star < \max(z_1^\star, z_2^\star)$. We have to consider two cases. Case 1: If $z_1^\star > z_2^\star$ we have a contradiction, since we can use the same solution for a standard bin packing problem with the first constraint, and by assumption $z^\star < \max(z_1^\star, z_2^\star) = z_1^\star$; but $z_1^\star$ is optimal. Case 2: Analogous Proof.

Note that the lower bound $z^\star$ on this problem can be better than $\max(z_1^\star, z_2^\star)$. For example: Consider an instance with bins of capacity $(W_1, W_2) = (3, 3)$ and items of sizes (1,3), (3,1) and (2,2), with demands 1, 1 and 2, respectively. The lower bounds provided by $z_1^\star$ and $z_2^\star$ are equal to 3, while $z^\star$ is equal to 4.

On cardinality constrained bin packing, the number of nodes in the arc flow formulation is bounded by $\mathcal{O}(WC)$, and as C is usually very small the bound is fair. However, on two-constraint bin packing the number of nodes is bounded by $\mathcal{O}(W_1W_2)$ and both W_1, W_2 may be very large.

3.3. Algorithms

In this section we describe an algorithm to construct a graph respecting symmetry reduction criteria, which is applicable to both standard bin packing and to the two-constraint variants; we also propose an algorithm to reconstruct the bin packing solution based on that of the arc flow formulation.

Except for the source and target, all nodes are labeled with pairs (a, b). On bin packing/cutting stock problems, we have arcs between nodes $(a, 0)$ and $(a+w_i, 0)$ (i.e., the second dimension is always 0). On cardinality constrained bin packing, we have arcs between nodes (a, b) and $(a + w_i, b + 1)$. Finally, on two-constraint bin packing, we have arcs between nodes (a, b) and $(a + w_i, b + v_i)$.

Using Algorithm 1, the graph can be constructed in pseudo-polynomial time $\mathcal{O}(|V|n^2)$; notice that $|V|$ is $\mathcal{O}(W_1W_2+W_1+W_2)$, but usually is much smaller than the maximum possible value.

The graph construction process is summarized in Algorithms 1 and 2. Algorithms 1 receives as input a list of labels (e.g., for the bin packing problem, $labels = \{(w_i, 0)|w_i \in items\}$), the demands of the items associated with each label, and the capacity limits on each dimension.

Initializations are performed in lines 3-15 of Algorithm 1. We start by sorting labels by decreasing values of the sum of the weights in each dimension; notice that different orders lead to different graphs, and hence to different numbers of nodes and arcs. In lines 4-10, we create a list *lst* of labels that will be used to create paths. Algorithm 2 creates the nodes and arcs (excepting loss arcs). Finally, in lines 17-21, we add the loss arcs and complete the graph. Algorithm 2 creates the nodes and arcs by enumerating paths corresponding to every possibly packing pattern respecting the decreasing order of weights. The parameter p indicates the current item, and s_1 and s_2 indicate the remaining space available in each dimension. We use a hash-table $visited[p, s_1, s_2]$ to keep track of the computations done, in order to avoid repeating work. Whenever it is not possible to include any other item or $visited[p, s_1, s_2]$ is true, we add the path to the graph, in lines 16-23. In line 12, whenever we have enough space for all the remaining items we do not generate paths that do not include some of these items, because these paths are unnecessary.

Algorithms 1 and 2 create a list of vertices V, a list of arcs A, and the list of arcs associated with each label A_l. This graph along with the demands of each label are enough to model the problem.

Algorithm 1 Graph Construction algorithm

Input: $labels$, $demand$, W_1, W_2
Output: $G = (V, A)$, A_l **for all** $l \in labels$

1: **procedure** construct-graph:
2: **global** lst, q, $visited$, V, A, A_l **for all** $l \in labels$
3: sort($labels$, key$(w_1, w_2) = w_1 + w_2$, reversed)
4: $lst \leftarrow [\ ]$
5: **for all** $(l_1, l_2) \in labels$ **do**
6:     **for** $i = 1 \rightarrow demand[l_1, l_2]$ **do**
7:         **if** $i \times l_1 > W1$ **or** $i \times l_2 > W2$ **then break**
8:         $lst \leftarrow lst + [(l_1, l_2)]$
9:     **end for**
10: **end for**
11: $q \leftarrow$ len(lst)
12: $V \leftarrow \{\}$
13: $A \leftarrow \{\}$
14: $A_l \leftarrow \{\}$, **for all** $l \in labels$
15: $visited[p, s_1, s_2] \leftarrow$ **false**, **for all** p, s_1, s_2
16: cons(1, W_1, W_2, [ ]) // described in Algorithm 2
17: **for all** $v \in V$ **do**
18:     $A \leftarrow A \cup \{(v, t)\}$
19: **end for**
20: $s \leftarrow (0, 0)$
21: $V \leftarrow V \cup \{s, t\}$ // s is the source node and t is the target node
22: **end procedure**

Using the algorithm just described to construct a graph for the instance of Example 1 adding cardinality limit 3, we obtain the graph represented in Figure 7. Then, we add loss arcs and obtain the graph in the Figure 8; this is the graph of the flow problem to be solved by a general-purpose mixed-integer optimization solver.

Algorithm 2 Graph Construction recursive procedure

```
procedure cons(p, s1, s2, path):
global lst, q, visited, V, A, A_l for all l ∈ labels
if p ≤ q and visited[p, s1, s2] = false then
    visited[p, s1, s2] ← true
    f ← false
    for i = p → n do
        w ← lst[i]
        if w1 ≤ s1 and w2 ≤ s2 and (i = p or lst[i] ≠ lst[i − 1]) then
            f ← true
            cons(i + 1, s1 − w1, s2 − w2, path + [w])  // recursive call
        end if
        if Σ_{k=i}^{q} lst[k] ≤ (s1, s2) then break
    end for
    if f = true then return
end if
u ← (0, 0)
for all w ∈ path do
    v ← (u1 + w1, u2 + w2)
    V ← V ∪ {v}
    A ← A ∪ {(u, v)}
    A_w ← A_w ∪ {(u, v)}
    u ← v
end for
end procedure
```

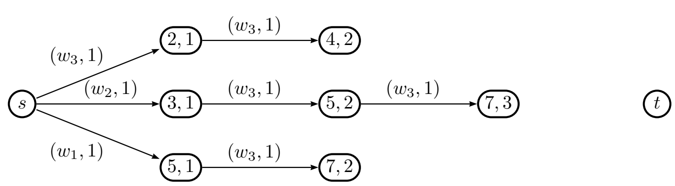

Figure 7: Graph produced by the algorithm with Example 1: arcs corresponding to items.

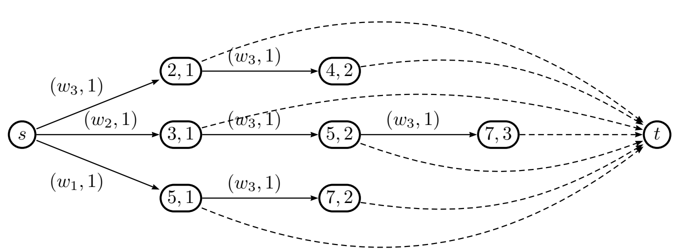

Figure 8: Graph produced by the algorithm with Example 1, after adding loss arcs.

After having the solution of the arc flow integer optimization model, we use a flow decomposition algorithm to obtain the bin packing solution.

Property 12. *Any integer solution to the arc flow model can be transformed into an integer solution to the bin packing problem.*

Proof. By the flow decomposition properties, non-negative flows can be represented by paths and cycles. Since we require an acyclic graph, any flow can be decomposed into directed paths connecting the only excess node (node s) to the only deficit node (node t).

We may use Algorithm 3 for recovering the solution; the number of steps required for obtaining the bin packing is $\mathcal{O}(|E|n)$. There are better algorithms, but this one was chosen for its simplicity and readability. Simple improvement is to try to fill more than one bin at a time.

Algorithm 3 Solution Decomposition algorithm

Input: $G = (V, A)$, A_*, f, demand

procedure decompose:
while there is a path p from s to t such that $f(u, v) > 0\ \forall (u, v) \in p$ **do**
    add a new bin b
    for all $(u, v) \in p$ **do**
        $f(u, v) \leftarrow f(u, v) - 1$
        if $(u, v) \in A_l$ **and** $demand[l] > 0$ **then**
            $demand[l] \leftarrow demand[l] - 1$
            assign an item associated with label l to b.
        end if
    end for
end while
end procedure

3.4. Breaking symmetry

It is possible to remove symmetry completely, for example by having a dimension where we introduce the label of the smallest incoming arc. In this case, we just need to avoid creating arcs with length greater than the label in order to break symmetry completely. However, this usually leads to much larger graphs and the problem becomes harder to solve. Therefore, in practice it is preferable to leave some symmetry and let the branch-and-bound algorithm handle it.

The introduction of cardinality may reduce symmetry, but it does not always break it completely, as shown in Figure 11.

Property 13. *The lower bound provided by the linear relaxation when we have no symmetry is not better than the lower bound obtained with a graph containing symmetry.*

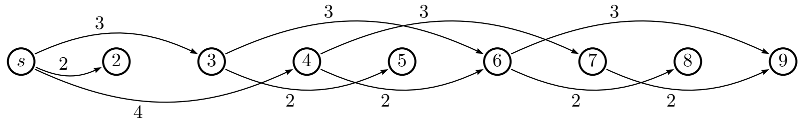

Figure 9: Graph associated with a bin packing instance with bins of capacity $W = 9$ and items of sizes 4, 3 and 2 with demands 1, 3 and 1, respectively. The paths $\{(s,4),(4,7),(7,9)\}$ and $\{(s,4),(4,6),(6,9)\}$ correspond to the same pattern (4,3,2) but the seccond one does not respect the order.

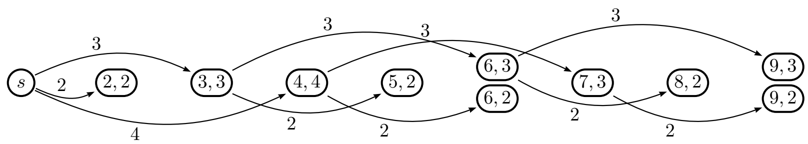

Figure 10: Breaking symmetry completely using the smallest incoming arc as label. The only path corresponding to the pattern (4,3,2) is $\{(s,(4,4)),((4,4),(7,3)),((7,3),(9,2))\}$.

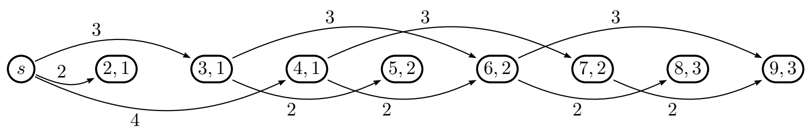

Figure 11: A graph where cardinality does not break symmetry completely. The paths $\{(s,(4,1)),((4,1),(7,2)),((7,2),(9,3))\}$ and $\{(s,(4,1)),((4,1),(6,2)),((6,2),(9,3))\}$ correspond to the same pattern (4,3,2) but the second one does not respect the order.

Proof. By Property 8 we have the same lower bound as the Gilmore-Gomory model with the same patterns as the ones generated by paths in the graph. Removing symmetry does not exclude any valid pattern, as we just exclude paths for which there is another path corresponding to the same pattern. Therefore, the lower bound is the same.

3.5. Integrality gap for the bin packing problem

There have been many studies (see, e.g., Scheithauer and Terno 1995, Scheithauer and Terno 1997) about the integrality gap for the bin packing problem, many of them about the integrality gap using the Gilmore and Gomory (1961) model. The arc flow formulation presented in this paper is equivalent to Gilmore-Gomory's model and hence the lower bounds are equal. Therefore, the results found on these studies are also valid for the arc flow formulation.

Rietz et al. (2002) describe families of instances without the integer round-up property (see Definition 2 below) of the one-dimensional cutting stock problem. One of the families is the so-called divisible case, where every item size w_i is a factor of the bin capacity W, and was firstly proposed by Nica (1994). As the method presented in this paper usually solves bin packing problems quickly, it was used to solve millions of instances from this family; the largest gap found was $1.0378\ldots$, the same gap as the one found by Scheithauer and Terno (1997).

Definition 1 (Integer Property). *A linear integer optimization problem P has the integer property (IP) if*

$$z^*_{ip}(E) = z^*_{lp}(E) \text{ for every instance } E \in P$$

Definition 2 (Integer Round-Up Property). *A linear integer optimization problem P has the integer round-up property (IRUP) if*

$$z^*_{ip}(E) = \lceil z^*_{lp}(E) \rceil \text{ for every instance } E \in P$$

Definition 3 (Modified Integer Round-Up Property). *A linear integer optimization problem P has the modified integer round-up property (MIRUP) if*

$$z^*_{ip}(E) = \lceil z^*_{lp}(E) \rceil + 1 \text{ for every instance } E \in P$$

Gau (1994) presents an instance with a gap of 1.0666, which is the largest gap known so far. Scheithauer and Terno (1997) conjecture that the general one-dimensional cutting stock problem has the modified integer round-up property (MIRUP). Moreover, many instances for the one dimensional bin packing problem present the integer round-up property (IRUP). Concerning the results obtained using the arc flow formulation on cardinality constrained bin packing and two-constraint bin packing, most of the instances of these problems also have the IRUP, and no instance violated the MIRUP. The lower bound provided by the linear relaxation of the arc flow formulation is usually very tight on every problem described in this paper; hence, the branch-and-bound process usually finds the optimal solution quickly.

4. Results

CPU times were obtained using a single thread in a Quad-Core Intel Xeon at 2.66GHz, running Mac OS X 10.6.6, with 16 GBytes of memory. The algorithm that generates the graph was implemented in `Python` 2.6.1, and `gurobi` 4.6.1, a state-of-the-art mixed integer programming solver, was used to solve the model. The parameters used on `gurobi` were Threads = 1, Method = 2 (Interior point methods), MIPFocus = 1, Heuristics = 1, MIPGap = 0, MIPGapAbs = 1-1E-5 and the remaining parameters were `gurobi`'s default values. The branch-and-cut solver used in `gurobi` uses a series of cuts; in our models the most frequently used were Gomory, Zero half and MIR.

On the following tables we present average values over 20 instances for each class u120, u250,...,t501 of OR-Library's data set. DEIS's data set has several sizes for each class, each pair (class, size) having 10 instances; we report average results on these 10 instances. For more detailed results please consult the appendix. Table 1 presents the meaning of each column in subsequent tables.

4.1. Standard bin packing and cutting stock

We used the arc flow formulation to solve every instance from the OR-Library (2012) bin packing test data set. The test data set has two classes of instances, further divided into subclasses of varying sizes: uniform instances, where items have

Table 1: Meaning of the data displayed in subsequent tables.

Label	Description
W	bin capacity
W_1	bin capacity on the first dimension
W_2	bin capacity on the second dimension
C	maximum bin cardinality
n	number of items
m	number of different item sizes
w	average weight
$w^{\min}$	weight of the smallest item
$w^{\max}$	weight of the largest item
z^*	optimum objective value
lb^{lp}	arc flow linear relaxation lower bound
lb^{lp1}	arc flow linear relaxation lower bound on the first dimension
lb^{lp2}	arc flow linear relaxation lower bound on the second dimension
lb^{sp}	space lower bound
lb^{crd}	cardinality lower bound
lb^{d1}	space lower bound on the first dimension
lb^{d2}	space lower bound on the second dimension
n^{bb}	number of nodes explored on the branch-and-bound procedure
t^{pp}	time spent on preprocessing in seconds
t^{lp}	time spent on the linear relaxation of the root node in seconds
t^{ip}	time spent on the branch and bound procedure in seconds
t^{tot}	total time in seconds (without solution decomposition)
$\#v$	number of vertices of the graph
$\#a$	number of arcs of the graph
$\%v$	ratio between the number of vertices and the maximum number of vertices (approx.: W_1W_2)
$\#op$	number of open instances solved

randomly generated weights, and the harder triplets instances, where the optimal solution for each bin is completely filled with three items. Each class contains 20 instances. In Table 2 we have the data set characteristics. In Table 3 we have the results of our method on this data set, and in Table 4 we report the results using Valério de Carvalho's formulation. The average run time is less than two seconds using either Valério de Carvalho's formulation or our alternative approach.

Table 2: Bin packing test data set characteristics.

class	W	n	m	$w^{\min}$	$w^{\max}$	w
u120	150	120	63.20	20	100	60.57
u250	150	250	77.25	20	100	60.64
u500	150	500	80.80	20	100	60.19
u1000	150	1,000	81.00	20	100	60.00
t60	1,000	60	49.95	250	499	333.33
t120	1,000	120	86.15	250	499	333.33
t249	1,000	249	140.10	250	499	333.33
t501	1,000	501	194.25	250	499	333.33

We generated cutting stock instances from the OR-Library (2012) bin packing test data set by multiplying the demand of each item by one million. Table 5 reports the results obtained. Even with extremely large demands, the graph remains very small. Therefore the problem does not become significantly harder to solve.

4.2. Cardinality constrained bin packing

As explained in Section 3.1, we added cardinality constraints to the instances of OR-Library. We tested every instance with every cardinality limit between 2 and the maximum number of items that fit into a single bin. Even though, for cardinality

Table 3: Bin packing results.

class	z^*	lb^{lp}	lb^{sp}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
u120	49.05	48.50	48.46	0.00	0.07	0.03	0.15	0.24
u250	101.60	101.09	101.07	0.00	0.09	0.04	0.35	0.49
u500	201.20	200.64	200.64	0.00	0.10	0.05	0.26	0.42
u1000	400.55	400.01	400.01	6.60	0.10	0.05	0.57	0.72
t60	20.00	20.00	20.00	0.00	0.16	0.05	0.13	0.34
t120	40.00	40.00	40.00	0.55	0.33	0.12	0.86	1.31
t249	83.00	83.00	83.00	0.00	0.64	0.28	2.04	2.95
t501	167.00	167.00	167.00	0.00	1.07	0.52	6.77	8.36

Note that the average run time ($t^{\text{lp}} + t^{\text{ip}}$) is less than 2 seconds.

Table 4: Bin packing results using Valério de Carvalho's graph.

class	z^*	lb^{lp}	lb^{sp}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
u120	49.05	48.50	48.46	0.00	0.07	0.03	0.20	0.29
u250	101.60	101.09	101.07	0.00	0.09	0.04	0.35	0.49
u500	201.20	200.64	200.64	0.00	0.10	0.05	0.33	0.49
u1000	400.55	400.01	400.01	0.00	0.10	0.05	0.31	0.46
t60	20.00	20.00	20.00	0.00	0.17	0.06	0.38	0.61
t120	40.00	40.00	40.00	0.00	0.34	0.13	1.44	1.91
t249	83.00	83.00	83.00	0.00	0.65	0.28	3.11	4.04
t501	167.00	167.00	167.00	0.00	1.09	0.54	4.39	6.02

Note that the average run time ($t^{\text{lp}} + t^{\text{ip}}$) is less than 2 seconds.

Table 5: Cutting stock results.

class	z^*	lb^{lp}	n	$\#v$	$\#a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
u120	4.8E+07	4.8E+07	1.2E+08	128.00	2335.70	11.20	0.07	0.04	0.40	0.51
u250	1E+08	1E+08	2.5E+08	131.80	3111.55	5.50	0.10	0.05	0.37	0.52
u500	2E+08	2E+08	5E+08	132.95	3335.65	0.00	0.10	0.05	0.16	0.32
u1000	4E+08	4E+08	1E+09	133.00	3348.00	0.00	0.10	0.05	0.14	0.30
t60	2E+07	2E+07	6E+07	548.30	5530.90	0.00	0.17	0.07	0.13	0.37
t120	4E+07	4E+07	1.2E+08	589.15	10826.25	0.00	0.35	0.16	0.52	1.03
t249	8.3E+07	8.3E+07	2.5E+08	643.10	19446.20	0.00	0.66	0.39	1.93	2.98
t501	1.7E+08	1.7E+08	5E+08	697.25	30003.80	0.00	1.09	0.72	5.56	7.37

Note that, on the class u1000, we have one billion items in each instance and the average run time ($t^{\text{lp}} + t^{\text{ip}}$) is less than 2 seconds.

2, the problem can be solved in polynomial time as a maximum non-bipartite matching problem in a graph where each item is represented by a node and every compatible pair of items is connect by an edge, we present the results to highlight Property 11. We have the results for this problem, in Table 6 for uniform classes, and in Table 7 for triplets classes.

Table 6: Cardinality constrained bin packing results on uniform classes.

class	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
u120	7	49.05	48.50	48.46	17.14	0.00	0.11	0.06	0.18	0.35
	6	49.05	48.50	48.46	20.00	0.00	0.11	0.05	0.16	0.32
	5	49.05	48.50	48.46	24.00	0.00	0.11	0.05	0.20	0.36
	4	49.05	48.50	48.46	30.00	0.00	0.10	0.04	0.18	0.33
	3	**49.05**	**48.50**	**48.46**	**40.00**	**0.00**	**0.09**	**0.03**	**0.11**	**0.23**
	2	60.00	60.00	48.46	60.00	0.00	0.05	0.01	0.02	0.09
u250	7	101.60	101.09	101.07	35.71	0.35	0.16	0.09	0.38	0.63
	6	101.60	101.09	101.07	41.67	0.35	0.16	0.09	0.41	0.65
	5	101.60	101.09	101.07	50.00	0.00	0.15	0.08	0.30	0.54
	4	101.60	101.09	101.07	62.50	0.00	0.14	0.07	0.29	0.50
	3	**101.60**	**101.09**	**101.07**	**83.33**	**0.00**	**0.12**	**0.04**	**0.18**	**0.34**
	2	125.00	125.00	101.07	125.00	0.00	0.08	0.02	0.03	0.13
u500	7	201.20	200.64	200.64	71.43	0.00	0.18	0.10	0.56	0.85
	6	201.20	200.64	200.64	83.33	0.00	0.18	0.10	0.45	0.73
	5	201.20	200.64	200.64	100.00	0.00	0.17	0.09	0.43	0.70
	4	201.20	200.64	200.64	125.00	0.00	0.16	0.08	0.38	0.61
	3	**201.20**	**200.64**	**200.64**	**166.67**	**0.00**	**0.13**	**0.05**	**0.21**	**0.40**
	2	250.00	250.00	200.64	250.00	0.00	0.09	0.02	0.05	0.16
u1000	7	400.55	400.01	400.01	142.86	0.00	0.18	0.10	0.79	1.07
	6	400.55	400.01	400.01	166.67	2.00	0.18	0.10	1.15	1.44
	5	400.55	400.01	400.01	200.00	0.00	0.18	0.09	0.71	0.98
	4	400.55	400.01	400.01	250.00	0.00	0.16	0.07	0.39	0.63
	3	**400.55**	**400.01**	**400.01**	**333.33**	**0.00**	**0.13**	**0.05**	**0.21**	**0.40**
	2	500.00	500.00	400.01	500.00	0.00	0.09	0.02	0.05	0.16

Table 7: Cardinality constrained bin packing results on triplets classes.

class	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
t60	4	20.00	20.00	20.00	15.00	0.00	0.17	0.05	0.16	0.38
	3	**20.00**	**20.00**	**20.00**	**20.00**	**0.00**	**0.17**	**0.05**	**0.16**	**0.38**
	2	30.00	30.00	20.00	30.00	0.00	0.06	0.01	0.02	0.09
t120	4	40.00	40.00	40.00	30.00	0.00	0.35	0.11	1.27	1.73
	3	**40.00**	**40.00**	**40.00**	**40.00**	**0.00**	**0.35**	**0.12**	**1.80**	**2.27**
	2	60.00	60.00	40.00	60.00	0.00	0.15	0.03	0.06	0.24
t249	4	83.00	83.00	83.00	62.25	0.00	0.66	0.25	2.92	3.83
	3	**83.00**	**83.00**	**83.00**	**83.00**	**0.80**	**0.67**	**0.28**	**6.31**	**7.26**
	2	125.00	124.50	83.00	124.50	0.00	0.35	0.10	0.25	0.70
t501	4	167.00	167.00	167.00	125.25	0.00	1.12	0.45	8.15	9.71
	3	**167.00**	**167.00**	**167.00**	**167.00**	**4.65**	**1.13**	**0.52**	**20.57**	**22.23**
	2	251.00	250.50	167.00	250.50	0.00	0.67	0.23	0.61	1.50

The proposed formulation works very well on cardinality constrained bin packing for every cardinality. We used this approach to solve every instance from OR-Library. Run time increases just slightly when both cardinality and capacity constraints are likely to be binding; otherwise, the average run time per instance remains below two seconds. When just one of the constraints is binding, the average run time remains below two seconds; in the other cases, taking into account that many of these instances were previously unsolved, run time is still very low. For some of the instances we knew, by construction, that there would be a solution with at most three items in each bin, but we were not aware of any good method to solve the cardinality constrained bin packing problem in general.

4.3. Two-constraint bin packing

We used the arc flow formulation to solve 260 instances from the DEIS - Operations Research Group Library of Instances (2011) two-constraint bin packing test data set. Table 8 summarizes the characteristics of these two-constraint bin packing test data sets, and the average run times in seconds for the extended arc flow formulation.

On this problem, the use of interior point methods methods to solve the linear relaxation at the root node is very important, as the graphs (and the corresponding linear programs) become very large in this case.

Table 8: Two-constraint bin packing test data set characteristics and average run times in seconds.

class	n	W_1	W_2	$\#v$	$\%v$	n^{bb}	t^{tot}	$\#op$
1	25	1000.0	1000.0	10915.00	0.01	0.00	2.95	0
	50	1000.0	1000.0	81556.20	0.08	0.00	121.25	0
2	**25**	**1000.0**	**1000.0**	**633.20**	**0.00**	**0.00**	**0.07**	**10**
	50	**1000.0**	**1000.0**	**2796.80**	**0.00**	**0.00**	**0.43**	**10**
	100	**1000.0**	**1000.0**	**18686.60**	**0.02**	**0.00**	**6.87**	**10**
3	**25**	**1000.0**	**1000.0**	**194.40**	**0.00**	**0.00**	**0.02**	**10**
	50	**1000.0**	**1000.0**	**538.70**	**0.00**	**0.00**	**0.06**	**10**
	100	**1000.0**	**1000.0**	**2002.30**	**0.00**	**0.00**	**0.28**	**10**
6	25	150.0	150.0	714.30	0.03	0.00	0.08	0
	50	150.0	150.0	2232.20	0.10	0.00	0.37	1
	100	150.0	150.0	5168.90	0.23	0.00	2.32	5
	200	150.0	150.0	8107.70	0.36	0.00	35.03	8
7	25	150.0	150.0	1243.10	0.06	0.00	0.21	0
	50	150.0	150.0	2670.70	0.12	0.00	1.82	1
	100	150.0	150.0	3981.80	0.18	0.00	18.77	7
	200	150.0	150.0	5282.10	0.23	0.00	198.30	3
8	**25**	**150.0**	**150.0**	**218.90**	**0.01**	**0.00**	**0.03**	**10**
	50	**150.0**	**150.0**	**632.30**	**0.03**	**0.00**	**0.08**	**10**
	100	**150.0**	**150.0**	**1206.00**	**0.05**	**0.00**	**0.28**	**10**
	200	**150.0**	**150.0**	**1514.20**	**0.07**	**0.00**	**0.88**	**10**
9	25	915.8	923.9	7351.80	0.01	0.00	1.77	0
	50	890.6	893.2	42167.60	0.05	0.00	33.82	1
10	24	100.0	100.0	788.50	0.08	0.00	0.09	0
	51	100.0	100.0	2149.00	0.21	0.00	0.52	0
	99	100.0	100.0	3643.90	0.36	0.00	3.55	0
	201	100.0	100.0	5214.00	0.52	88.10	120.03	0

Note that among these instances there were 126 instances with no known optimum. We solved 260 instances out of the 400 instances and there are still 52 open instances.

The average run time is less than 20 seconds. Notice that among the instances presented in Table 8 there were 126 instances with no known optimum; all of the instances in this data set could be solved to optimality with the arc flow formulation that we proposed.

One may ask why so many of these instances were not solved before. A reasonable explanation may be the fact that, for example on instances from classes 2, 3 and 8, the lower bound provided by previous formulations is rather loose. None of the instances from these classes have been solved before.

Table 9: Two-constraint bin packing results for $n = 25$.

class	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{d1}	lb^{d2}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
1	6.90	6.07	5.48	5.42	5.76	5.67	0.00	0.92	0.19	1.83	2.95
2	**14.20**	**14.00**	**11.93**	**12.18**	**10.80**	**10.99**	**0.00**	**0.05**	**0.01**	**0.01**	**0.07**
3	**14.20**	**14.00**	**12.47**	**12.80**	**11.47**	**11.59**	**0.00**	**0.02**	**0.00**	**0.00**	**0.02**
6	10.10	9.70	7.37	7.47	9.07	9.17	0.00	0.05	0.01	0.02	0.08
7	9.60	9.18	7.37	8.34	9.07	8.86	0.00	0.11	0.02	0.08	0.21
8	**13.00**	**12.50**	**7.37**	**8.91**	**9.07**	**9.92**	**0.00**	**0.02**	**0.00**	**0.00**	**0.03**
9	7.30	6.38	6.13	6.09	6.30	6.30	0.00	0.59	0.11	1.07	1.77

Table 10: Two-constraint bin packing results for $n = 50$.

class	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{d1}	lb^{d2}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
1	13.50	12.94	11.50	11.14	12.22	11.86	0.00	13.83	5.83	101.59	121.25
2	**31.50**	**31.30**	**25.00**	**25.25**	**23.35**	**23.51**	**0.00**	**0.21**	**0.03**	**0.19**	**0.43**
3	**31.50**	**31.30**	**26.30**	**26.55**	**23.99**	**24.09**	**0.00**	**0.04**	**0.01**	**0.01**	**0.06**
6	21.50	20.86	13.65	13.68	19.09	19.18	0.00	0.19	0.04	0.14	0.37
7	19.70	19.31	13.65	15.78	19.09	18.88	0.00	0.41	0.37	1.04	1.82
8	**25.00**	**25.00**	**14.00**	**14.04**	**19.09**	**18.98**	**0.00**	**0.06**	**0.01**	**0.01**	**0.08**
9	14.50	13.56	12.65	12.67	13.49	13.49	0.00	5.32	1.53	26.97	33.82

Table 11: Two-constraint bin packing results for $n = 100$.

class	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{d1}	lb^{d2}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
2	**57.40**	**57.40**	**50.48**	**51.38**	**48.94**	**49.80**	**0.00**	**1.72**	**0.32**	**4.83**	**6.87**
3	**56.90**	**56.85**	**49.35**	**49.40**	**49.35**	**49.38**	**0.00**	**0.17**	**0.02**	**0.09**	**0.28**
6	41.00	40.47	22.14	22.14	39.40	39.43	0.00	0.76	0.29	1.27	2.32
7	40.20	39.64	22.92	24.40	39.40	39.25	0.00	1.35	2.30	15.11	18.77
8	**50.00**	**50.00**	**23.51**	**21.80**	**39.40**	**36.89**	**0.00**	**0.19**	**0.03**	**0.05**	**0.28**

Table 12: Two-constraint bin packing results for $n = 200$.

class	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{d1}	lb^{d2}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
6	81.10	80.61	29.21	29.24	79.31	79.42	0.00	3.44	4.52	27.08	35.03
7	80.10	79.53	30.37	32.43	79.31	79.17	0.00	4.65	11.57	182.08	198.30
8	**100.00**	**100.00**	**32.06**	**30.77**	**79.31**	**73.30**	**0.00**	**0.53**	**0.14**	**0.21**	**0.88**

Table 13: Two-constraint bin packing results on the class 10.

n	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{d1}	lb^{d2}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
24	8.00	8.00	6.25	6.23	8.00	8.00	0.00	0.06	0.01	0.02	0.09
51	17.00	17.00	9.06	9.11	17.00	17.00	0.00	0.28	0.10	0.14	0.52
99	33.00	33.00	11.12	11.31	33.00	33.00	0.00	1.02	1.55	0.98	3.55
201	67.00	67.00	13.56	14.55	66.84	67.00	88.10	3.98	15.47	100.58	120.03

5. Conclusions

We proposed a very powerful method to solve bin packing problems, including cardinality constrained and two-constraint variants. The formulation presented is simple, but provides a very strong lower bound on every problem described in this paper. This method allowed us to solve many previously open instances. Actually, using the proposed formulation and a state-of-the-art mixed-integer programming solver, current benchmark instances for standard bin packing, cutting stock and cardinality constrained bin packing are not challenging anymore.

For the two-constraint bin packing problem we could solve many open instances, but there are still instances that could not be solved within a reasonable time. The main problem now is to compute the linear relaxation at the root node, since model sometimes has millions of variables and constraints. Even for these large problems, the branch-and-bound process usually finds the optimal solution quickly, mainly because of the tight lower bound.

Note that it is possible to solve p-dimensional vector packing problems using an arc flow formulation by generalizing the idea used to solve two-constraint bin packing problems. However, depending on the number of dimensions and capacities, the size of the graph may be a problem.

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Appendix A. Detailed results

Table A.14: Bin packing results.

				New graph					VC's graph				
instance	z^*	lb^{lp}	lb^{sp}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
u120_00	48	47.27	47.19	0	0.06	0.03	0.31	0.40	0	0.06	0.02	0.21	0.29
u120_01	49	48.05	48.03	0	0.06	0.03	0.06	0.15	0	0.06	0.02	0.05	0.14
u120_02	46	45.29	45.29	0	0.07	0.04	0.31	0.41	0	0.07	0.04	0.10	0.21
u120_03	49	48.63	48.57	0	0.07	0.03	0.12	0.22	0	0.08	0.03	0.25	0.35
u120_04	50	49.09	49.03	0	0.06	0.03	0.13	0.22	0	0.06	0.02	0.06	0.14
u120_05	48	47.49	47.48	0	0.07	0.03	0.06	0.15	0	0.06	0.02	0.05	0.14
u120_06	48	47.58	47.58	0	0.07	0.03	0.22	0.32	0	0.07	0.04	0.10	0.21
u120_07	49	48.66	48.63	0	0.06	0.03	0.29	0.38	0	0.06	0.02	0.70	0.79
u120_08	50	49.91	49.85	0	0.07	0.03	0.09	0.20	0	0.07	0.03	0.20	0.30
u120_09	46	45.80	45.80	0	0.07	0.04	0.47	0.58	0	0.07	0.04	0.29	0.40
u120_10	52	51.28	51.20	0	0.06	0.03	0.10	0.19	0	0.06	0.02	0.41	0.50
u120_11	49	48.39	48.31	0	0.06	0.02	0.04	0.12	0	0.06	0.02	0.03	0.12
u120_12	48	47.87	47.87	0	0.07	0.04	0.15	0.26	0	0.07	0.04	0.69	0.80
u120_13	49	48.01	48.01	0	0.06	0.03	0.07	0.17	0	0.07	0.03	0.09	0.19
u120_14	50	49.17	49.15	0	0.06	0.03	0.05	0.13	0	0.06	0.02	0.20	0.28
u120_15	48	47.38	47.35	0	0.07	0.03	0.15	0.25	0	0.07	0.02	0.04	0.14
u120_16	52	51.33	51.25	0	0.06	0.03	0.10	0.19	0	0.06	0.02	0.27	0.35
u120_17	52	51.50	51.35	0	0.06	0.02	0.09	0.18	0	0.06	0.02	0.03	0.12
u120_18	49	48.38	48.37	0	0.07	0.03	0.04	0.14	0	0.07	0.03	0.04	0.14
u120_19	49	48.86	48.81	0	0.08	0.04	0.08	0.20	0	0.08	0.03	0.10	0.20
u250_00	99	98.55	98.55	0	0.08	0.04	0.20	0.32	0	0.08	0.04	0.69	0.81
u250_01	100	99.03	99.03	0	0.10	0.06	0.30	0.46	0	0.10	0.05	0.23	0.38
u250_02	102	101.42	101.42	0	0.09	0.04	0.09	0.22	0	0.09	0.03	0.11	0.23
u250_03	100	99.43	99.43	0	0.09	0.04	0.33	0.47	0	0.10	0.04	0.34	0.49
u250_04	101	100.61	100.61	0	0.09	0.05	0.60	0.74	0	0.09	0.04	0.47	0.61
u250_05	101	100.83	100.83	0	0.09	0.05	0.58	0.72	0	0.10	0.04	0.40	0.54
u250_06	102	101.03	101.03	0	0.09	0.05	0.12	0.27	0	0.10	0.05	0.13	0.28
u250_07	103	102.89	102.79	0	0.09	0.04	0.09	0.22	0	0.09	0.03	0.54	0.66
u250_08	105	104.92	104.91	0	0.09	0.04	1.45	1.58	0	0.09	0.03	0.38	0.50
u250_09	101	100.20	100.20	0	0.10	0.05	0.08	0.22	0	0.10	0.04	0.26	0.39
u250_10	105	104.39	104.37	0	0.09	0.04	0.27	0.40	0	0.09	0.03	0.48	0.60
u250_11	101	100.71	100.71	0	0.10	0.05	0.31	0.47	0	0.10	0.05	0.23	0.38
u250_12	105	104.98	104.93	0	0.09	0.04	0.65	0.77	0	0.09	0.03	0.59	0.71
u250_13	103	102.04	101.96	0	0.09	0.03	0.08	0.20	0	0.09	0.03	0.09	0.21
u250_14	100	99.17	99.17	0	0.10	0.05	0.14	0.29	0	0.10	0.04	0.35	0.49
u250_15	105	104.86	104.81	0	0.09	0.03	0.20	0.33	0	0.09	0.03	0.27	0.40
u250_16	97	96.51	96.51	0	0.09	0.05	0.29	0.43	0	0.10	0.05	0.40	0.54
u250_17	100	99.17	99.17	0	0.09	0.05	0.24	0.38	0	0.09	0.04	0.40	0.54
u250_18	100	99.70	99.70	0	0.10	0.06	0.70	0.85	0	0.09	0.05	0.49	0.63
u250_19	102	101.36	101.36	0	0.10	0.05	0.33	0.48	0	0.10	0.05	0.20	0.34
u500_00	198	197.58	197.58	0	0.11	0.06	0.27	0.43	0	0.11	0.05	0.39	0.55
u500_01	201	200.85	200.85	0	0.10	0.05	0.33	0.48	0	0.11	0.05	0.48	0.63
u500_02	202	201.44	201.44	0	0.10	0.05	0.15	0.30	0	0.10	0.05	0.45	0.60
u500_03	204	203.81	203.81	0	0.11	0.06	0.29	0.45	0	0.11	0.05	0.88	1.03
u500_04	206	205.11	205.11	0	0.10	0.05	0.10	0.24	0	0.10	0.04	0.11	0.25
u500_05	206	205.09	205.09	0	0.10	0.06	0.30	0.46	0	0.11	0.05	0.43	0.59

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				New graph					VC's graph				
instance	z^*	lb^{lp}	lb^{sp}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
u500_06	207	206.91	206.89	0	0.10	0.04	0.58	0.72	0	0.10	0.04	0.30	0.44
u500_07	204	203.98	203.98	0	0.10	0.06	0.43	0.59	0	0.10	0.05	0.26	0.41
u500_08	196	195.68	195.68	0	0.11	0.05	0.32	0.47	0	0.11	0.04	0.62	0.77
u500_09	202	201.06	201.06	0	0.11	0.05	0.14	0.30	0	0.11	0.05	0.46	0.61
u500_10	200	199.07	199.07	0	0.11	0.05	0.14	0.29	0	0.11	0.05	0.16	0.31
u500_11	200	199.43	199.43	0	0.10	0.05	0.31	0.46	0	0.11	0.05	0.19	0.35
u500_12	199	198.62	198.62	0	0.11	0.05	0.30	0.46	0	0.10	0.05	0.56	0.71
u500_13	196	195.59	195.59	0	0.11	0.05	0.40	0.56	0	0.11	0.05	0.42	0.57
u500_14	204	203.03	203.03	0	0.10	0.06	0.29	0.45	0	0.10	0.05	0.11	0.27
u500_15	201	200.13	200.13	0	0.10	0.05	0.14	0.29	0	0.10	0.04	0.33	0.48
u500_16	202	201.01	201.01	0	0.10	0.05	0.15	0.31	0	0.10	0.05	0.12	0.27
u500_17	198	197.43	197.43	0	0.10	0.05	0.16	0.32	0	0.10	0.05	0.14	0.29
u500_18	202	201.29	201.29	0	0.11	0.05	0.14	0.30	0	0.11	0.05	0.13	0.28
u500_19	196	195.63	195.63	0	0.11	0.06	0.36	0.52	0	0.11	0.05	0.14	0.31
u1000_00	399	398.43	398.43	0	0.10	0.05	0.15	0.31	0	0.11	0.05	0.35	0.51
u1000_01	406	405.25	405.25	0	0.10	0.05	0.13	0.28	0	0.11	0.05	0.41	0.56
u1000_02	411	410.20	410.20	0	0.10	0.05	0.16	0.31	0	0.10	0.04	0.16	0.31
u1000_03	411	410.87	410.87	0	0.10	0.06	0.17	0.33	0	0.11	0.05	0.14	0.30
u1000_04	397	396.74	396.74	0	0.11	0.05	0.35	0.51	0	0.11	0.05	0.41	0.56
u1000_05	399	398.49	398.49	0	0.10	0.05	0.24	0.39	0	0.10	0.05	0.31	0.46
u1000_06	395	394.21	394.21	0	0.10	0.05	0.17	0.32	0	0.10	0.04	0.10	0.25
u1000_07	404	403.16	403.16	0	0.10	0.05	0.32	0.48	0	0.10	0.04	0.36	0.51
u1000_08	399	398.43	398.43	0	0.10	0.05	0.33	0.48	0	0.10	0.04	0.44	0.58
u1000_09	397	396.93	396.93	0	0.11	0.05	0.20	0.35	0	0.11	0.05	0.38	0.53
u1000_10	400	399.34	399.34	0	0.11	0.05	0.31	0.47	0	0.10	0.04	0.43	0.57
u1000_11	401	400.52	400.52	0	0.11	0.05	0.34	0.49	0	0.10	0.04	0.41	0.55
u1000_12	393	392.24	392.24	0	0.10	0.05	0.15	0.30	0	0.10	0.05	0.14	0.29
u1000_13	396	395.27	395.27	0	0.10	0.05	0.32	0.47	0	0.11	0.05	0.33	0.48
u1000_14	394	393.89	393.89	132	0.11	0.05	6.28	6.44	0	0.11	0.05	0.43	0.59
u1000_15	402	401.81	401.81	0	0.11	0.05	0.87	1.02	0	0.10	0.04	0.35	0.50
u1000_16	404	403.03	403.03	0	0.10	0.05	0.15	0.30	0	0.10	0.04	0.42	0.57
u1000_17	404	403.80	403.80	0	0.10	0.05	0.27	0.42	0	0.10	0.04	0.30	0.45
u1000_18	399	398.19	398.19	0	0.10	0.05	0.11	0.26	0	0.11	0.04	0.16	0.32
u1000_19	400	399.33	399.33	0	0.11	0.05	0.30	0.46	0	0.11	0.04	0.13	0.28
t60_00	20	20.00	20.00	0	0.16	0.05	0.09	0.31	0	0.17	0.06	0.14	0.37
t60_01	20	20.00	20.00	0	0.19	0.06	0.09	0.34	0	0.20	0.07	1.11	1.38
t60_02	20	20.00	20.00	0	0.15	0.05	0.06	0.26	0	0.17	0.05	0.47	0.70
t60_03	20	20.00	20.00	0	0.15	0.05	0.12	0.31	0	0.16	0.05	0.50	0.70
t60_04	20	20.00	20.00	0	0.14	0.04	0.12	0.31	0	0.15	0.05	0.07	0.27
t60_05	20	20.00	20.00	0	0.16	0.06	0.06	0.28	0	0.17	0.07	0.07	0.30
t60_06	20	20.00	20.00	0	0.16	0.04	0.41	0.61	0	0.17	0.06	0.81	1.04
t60_07	20	20.00	20.00	0	0.15	0.05	0.09	0.30	0	0.17	0.06	0.48	0.71
t60_08	20	20.00	20.00	0	0.15	0.05	0.06	0.26	0	0.16	0.05	0.44	0.66
t60_09	20	20.00	20.00	0	0.14	0.05	0.07	0.26	0	0.16	0.05	0.44	0.65
t60_10	20	20.00	20.00	0	0.17	0.05	0.37	0.58	0	0.17	0.06	0.56	0.79
t60_11	20	20.00	20.00	0	0.14	0.04	0.05	0.23	0	0.14	0.04	0.38	0.56
t60_12	20	20.00	20.00	0	0.18	0.06	0.11	0.34	0	0.19	0.06	0.29	0.54
t60_13	20	20.00	20.00	0	0.17	0.06	0.07	0.29	0	0.18	0.06	0.12	0.36
t60_14	20	20.00	20.00	0	0.16	0.05	0.07	0.29	0	0.18	0.06	0.08	0.32

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				New graph					VC's graph				
instance	z^*	lb^{lp}	lb^{sp}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
t60_15	20	20.00	20.00	0	0.18	0.06	0.35	0.59	0	0.19	0.07	0.62	0.88
t60_16	20	20.00	20.00	0	0.15	0.04	0.06	0.25	0	0.16	0.05	0.43	0.63
t60_17	20	20.00	20.00	0	0.15	0.04	0.11	0.30	0	0.16	0.05	0.45	0.67
t60_18	20	20.00	20.00	0	0.15	0.05	0.10	0.30	0	0.16	0.05	0.08	0.29
t60_19	20	20.00	20.00	0	0.17	0.06	0.06	0.29	0	0.18	0.06	0.09	0.33
t120_00	40	40.00	40.00	0	0.34	0.12	1.46	1.93	0	0.36	0.13	0.33	0.83
t120_01	40	40.00	40.00	0	0.30	0.11	1.76	2.17	0	0.32	0.12	0.97	1.40
t120_02	40	40.00	40.00	0	0.37	0.12	0.27	0.76	0	0.38	0.14	0.19	0.71
t120_03	40	40.00	40.00	0	0.34	0.13	1.15	1.62	0	0.36	0.14	1.08	1.58
t120_04	40	40.00	40.00	0	0.38	0.13	2.22	2.72	0	0.40	0.15	0.22	0.77
t120_05	40	40.00	40.00	0	0.35	0.14	0.21	0.69	0	0.36	0.13	1.14	1.62
t120_06	40	40.00	40.00	0	0.33	0.11	0.29	0.73	0	0.32	0.12	1.01	1.45
t120_07	40	40.00	40.00	0	0.32	0.11	0.40	0.83	0	0.34	0.12	1.07	1.53
t120_08	40	40.00	40.00	0	0.33	0.10	0.23	0.66	0	0.33	0.12	2.64	3.09
t120_09	40	40.00	40.00	0	0.30	0.11	0.60	1.02	0	0.31	0.12	2.15	2.59
t120_10	40	40.00	40.00	0	0.31	0.09	0.68	1.08	0	0.33	0.12	2.02	2.47
t120_11	40	40.00	40.00	0	0.34	0.14	0.25	0.73	0	0.36	0.13	1.18	1.67
t120_12	40	40.00	40.00	0	0.31	0.10	1.03	1.44	0	0.32	0.12	2.36	2.79
t120_13	40	40.00	40.00	0	0.33	0.12	1.92	2.37	0	0.34	0.13	2.07	2.54
t120_14	40	40.00	40.00	11	0.32	0.12	1.74	2.18	0	0.33	0.13	1.19	1.65
t120_15	40	40.00	40.00	0	0.32	0.10	0.26	0.68	0	0.34	0.13	2.29	2.75
t120_16	40	40.00	40.00	0	0.33	0.12	0.46	0.90	0	0.34	0.12	1.15	1.60
t120_17	40	40.00	40.00	0	0.37	0.13	1.14	1.65	0	0.38	0.14	2.48	3.01
t120_18	40	40.00	40.00	0	0.32	0.11	1.03	1.46	0	0.34	0.12	2.36	2.82
t120_19	40	40.00	40.00	0	0.34	0.10	0.18	0.62	0	0.34	0.13	0.85	1.32
t249_00	83	83.00	83.00	0	0.61	0.26	0.47	1.34	0	0.61	0.26	2.05	2.93
t249_01	83	83.00	83.00	0	0.62	0.28	1.93	2.84	0	0.64	0.29	3.30	4.24
t249_02	83	83.00	83.00	0	0.63	0.28	4.99	5.90	0	0.65	0.29	6.53	7.47
t249_03	83	83.00	83.00	0	0.65	0.27	0.43	1.35	0	0.65	0.29	4.40	5.33
t249_04	83	83.00	83.00	0	0.60	0.24	1.38	2.22	0	0.62	0.28	0.81	1.71
t249_05	83	83.00	83.00	0	0.67	0.31	1.27	2.25	0	0.66	0.30	6.36	7.31
t249_06	83	83.00	83.00	0	0.61	0.24	1.15	1.99	0	0.62	0.27	1.90	2.79
t249_07	83	83.00	83.00	0	0.60	0.28	4.43	5.31	0	0.60	0.26	0.78	1.64
t249_08	83	83.00	83.00	0	0.63	0.31	6.02	6.96	0	0.64	0.29	4.22	5.15
t249_09	83	83.00	83.00	0	0.62	0.28	0.82	1.72	0	0.65	0.29	0.37	1.32
t249_10	83	83.00	83.00	0	0.65	0.26	0.86	1.77	0	0.64	0.28	3.15	4.07
t249_11	83	83.00	83.00	0	0.65	0.27	1.06	1.98	0	0.66	0.28	2.18	3.12
t249_12	83	83.00	83.00	0	0.64	0.27	1.26	2.17	0	0.65	0.29	4.81	5.76
t249_13	83	83.00	83.00	0	0.62	0.29	1.45	2.36	0	0.65	0.30	0.56	1.51
t249_14	83	83.00	83.00	0	0.69	0.33	2.04	3.06	0	0.68	0.30	4.59	5.56
t249_15	83	83.00	83.00	0	0.65	0.30	1.66	2.62	0	0.67	0.30	5.27	6.24
t249_16	83	83.00	83.00	0	0.68	0.30	1.24	2.22	0	0.69	0.30	2.49	3.47
t249_17	83	83.00	83.00	0	0.67	0.30	2.73	3.70	0	0.66	0.30	2.34	3.29
t249_18	83	83.00	83.00	0	0.61	0.25	3.79	4.66	0	0.63	0.27	5.21	6.10
t249_19	83	83.00	83.00	0	0.61	0.24	1.76	2.62	0	0.64	0.26	0.81	1.70
t501_00	167	167.00	167.00	0	1.00	0.46	2.41	3.87	0	1.01	0.51	1.37	2.89
t501_01	167	167.00	167.00	0	1.03	0.50	3.15	4.67	0	1.05	0.50	2.96	4.51
t501_02	167	167.00	167.00	0	1.04	0.54	4.67	6.25	0	1.02	0.53	2.03	3.58
t501_03	167	167.00	167.00	0	1.14	0.63	6.82	8.59	0	1.14	0.61	5.82	7.57

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				New graph					VC's graph				
instance	z^*	lb^{lp}	lb^{sp}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
t501_04	167	167.00	167.00	0	1.03	0.52	8.13	9.68	0	1.11	0.59	3.02	4.72
t501_05	167	167.00	167.00	0	1.11	0.55	4.24	5.90	0	1.14	0.60	7.17	8.92
t501_06	167	167.00	167.00	0	1.12	0.52	2.43	4.07	0	1.14	0.56	1.45	3.15
t501_07	167	167.00	167.00	0	1.02	0.48	6.79	8.29	0	1.04	0.51	5.53	7.08
t501_08	167	167.00	167.00	0	1.13	0.52	8.27	9.93	0	1.12	0.55	5.72	7.39
t501_09	167	167.00	167.00	0	1.01	0.48	9.47	10.95	0	1.03	0.54	5.58	7.15
t501_10	167	167.00	167.00	0	1.02	0.45	11.02	12.50	0	1.01	0.51	6.76	8.28
t501_11	167	167.00	167.00	0	1.03	0.49	10.22	11.74	0	1.06	0.54	6.44	8.03
t501_12	167	167.00	167.00	0	1.01	0.44	10.04	11.49	0	1.03	0.48	5.57	7.08
t501_13	167	167.00	167.00	0	1.14	0.57	8.28	9.98	0	1.15	0.57	6.40	8.12
t501_14	167	167.00	167.00	0	1.20	0.60	11.34	13.13	0	1.23	0.58	4.62	6.43
t501_15	167	167.00	167.00	0	1.12	0.56	2.19	3.86	0	1.14	0.55	5.06	6.75
t501_16	167	167.00	167.00	0	1.14	0.53	3.89	5.56	0	1.13	0.55	3.43	5.11
t501_17	167	167.00	167.00	0	1.10	0.54	8.05	9.69	0	1.14	0.52	1.30	2.97
t501_18	167	167.00	167.00	0	1.05	0.54	2.61	4.20	0	1.03	0.53	3.88	5.45
t501_19	167	167.00	167.00	0	1.02	0.50	11.39	12.91	0	1.03	0.52	3.75	5.29

Table A.15: Cutting stock results.

instance	z^*	lb^{lp}	lb^{sp}	n	$\#v$	$\#a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
u120_00	47265958	47265957.45	47186666.67	1.2E+08	129	2167	0	0.07	0.03	0.06	0.16
u120_01	48048612	48048611.11	48033333.33	1.2E+08	128	2193	0	0.07	0.03	0.06	0.16
u120_02	45293334	45293333.33	45293333.33	1.2E+08	132	2530	0	0.08	0.04	0.30	0.42
u120_03	48623077	48623076.92	48566666.67	1.2E+08	130	2588	4	0.08	0.04	1.23	1.35
u120_04	49085035	49085034.01	49026666.67	1.2E+08	128	2218	0	0.07	0.03	0.05	0.16
u120_05	47486395	47486394.56	47480000.00	1.2E+08	128	2237	0	0.07	0.04	0.28	0.39
u120_06	47580000	47580000.00	47580000.00	1.2E+08	130	2514	0	0.08	0.04	0.02	0.14
u120_07	48656463	48656462.59	48633333.33	1.2E+08	124	2199	220	0.07	0.03	1.75	1.85
u120_08	49911565	49911564.63	49853333.33	1.2E+08	130	2578	0	0.08	0.04	0.22	0.33
u120_09	45800000	45800000.00	45800000.00	1.2E+08	130	2536	0	0.08	0.04	0.02	0.14
u120_10	51280317	51280316.34	51200000.00	1.2E+08	127	2226	0	0.07	0.03	0.87	0.98
u120_11	48392858	48392857.14	48313333.33	1.2E+08	127	2066	0	0.07	0.03	0.04	0.14
u120_12	47866667	47866666.67	47866666.67	1.2E+08	129	2511	0	0.08	0.04	0.46	0.58
u120_13	48013334	48013333.33	48013333.33	1.2E+08	128	2283	0	0.07	0.03	0.13	0.24
u120_14	49166667	49166666.67	49153333.33	1.2E+08	125	2146	0	0.07	0.04	0.48	0.59
u120_15	47384058	47384057.97	47346666.67	1.2E+08	129	2317	0	0.07	0.04	0.27	0.38
u120_16	51333334	51333333.33	51253333.33	1.2E+08	124	2138	0	0.06	0.03	1.31	1.40
u120_17	51500000	51500000.00	51353333.33	1.2E+08	124	2137	0	0.06	0.02	0.02	0.11
u120_18	48381503	48381502.89	48366666.67	1.2E+08	127	2437	0	0.07	0.04	0.10	0.22
u120_19	48860545	48860544.22	48813333.33	1.2E+08	131	2693	0	0.08	0.04	0.35	0.48
u250_00	98553334	98553333.33	98553333.33	2.5E+08	130	2652	0	0.08	0.04	0.51	0.63
u250_01	99026667	99026666.67	99026666.67	2.5E+08	133	3226	0	0.10	0.06	0.14	0.30
u250_02	101421769	101421768.71	101420000.00	2.5E+08	131	3063	0	0.10	0.05	0.13	0.27
u250_03	99426667	99426666.67	99426666.67	2.5E+08	132	3213	0	0.10	0.05	0.29	0.44
u250_04	100613334	100613333.33	100613333.33	2.5E+08	132	3013	0	0.10	0.05	0.07	0.22
u250_05	100826667	100826666.67	100826666.67	2.5E+08	132	3129	0	0.10	0.05	0.12	0.26
u250_06	101026667	101026666.67	101026666.67	2.5E+08	133	3249	0	0.10	0.05	0.09	0.25
u250_07	102885186	102885185.19	102786666.67	2.5E+08	131	3054	0	0.09	0.05	0.21	0.35
u250_08	104918368	104918367.35	104913333.33	2.5E+08	131	3121	0	0.10	0.05	0.30	0.45

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instance	z^*	lb^{lp}	lb^{sp}	n	$\#v$	$\#a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
u250_09	100201389	100201388.89	100200000.00	2.5E+08	132	3235	0	0.10	0.05	1.73	1.88
u250_10	104391157	104391156.46	104373333.33	2.5E+08	131	3069	0	0.09	0.05	0.65	0.79
u250_11	100713334	100713333.33	100713333.33	2.5E+08	132	3243	0	0.10	0.06	0.09	0.26
u250_12	104977163	104977162.29	104926666.67	2.5E+08	129	2915	0	0.09	0.05	0.05	0.20
u250_13	102036586	102036585.37	101960000.00	2.5E+08	132	3030	0	0.10	0.04	0.06	0.20
u250_14	99166667	99166666.67	99166666.67	2.5E+08	133	3231	110	0.10	0.06	2.42	2.58
u250_15	104861112	104861111.11	104813333.33	2.5E+08	131	3092	0	0.09	0.05	0.09	0.23
u250_16	96513334	96513333.33	96513333.33	2.5E+08	133	3155	0	0.10	0.05	0.26	0.41
u250_17	99166667	99166666.67	99166666.67	2.5E+08	132	3107	0	0.10	0.05	0.09	0.24
u250_18	99700000	99700000.00	99700000.00	2.5E+08	133	3086	0	0.10	0.05	0.05	0.20
u250_19	101360000	101360000.00	101360000.00	2.5E+08	133	3348	0	0.11	0.05	0.04	0.20
u500_00	197580000	197580000.00	197580000.00	5E+08	133	3348	0	0.11	0.06	0.24	0.41
u500_01	200846667	200846666.67	200846666.67	5E+08	133	3348	0	0.11	0.05	0.23	0.39
u500_02	201440000	201440000.00	201440000.00	5E+08	133	3276	0	0.10	0.05	0.01	0.17
u500_03	203813334	203813333.33	203813333.33	5E+08	133	3348	0	0.10	0.06	0.08	0.24
u500_04	205113334	205113333.33	205113333.33	5E+08	132	3235	0	0.10	0.05	0.22	0.37
u500_05	205086667	205086666.67	205086666.67	5E+08	133	3348	0	0.10	0.06	0.09	0.25
u500_06	206905798	206905797.10	206886666.67	5E+08	133	3348	0	0.11	0.05	0.10	0.26
u500_07	203980000	203980000.00	203980000.00	5E+08	133	3296	0	0.10	0.06	0.17	0.33
u500_08	195680000	195680000.00	195680000.00	5E+08	133	3348	0	0.10	0.05	0.04	0.19
u500_09	201060000	201060000.00	201060000.00	5E+08	133	3348	0	0.10	0.06	0.14	0.29
u500_10	199066667	199066666.67	199066666.67	5E+08	133	3348	0	0.10	0.05	0.21	0.37
u500_11	199426667	199426666.67	199426666.67	5E+08	133	3348	0	0.10	0.05	0.30	0.46
u500_12	198620000	198620000.00	198620000.00	5E+08	133	3348	0	0.11	0.05	0.04	0.20
u500_13	195586667	195586666.67	195586666.67	5E+08	133	3348	0	0.11	0.05	0.38	0.54
u500_14	203026667	203026666.67	203026666.67	5E+08	133	3348	0	0.10	0.06	0.08	0.24
u500_15	200133334	200133333.33	200133333.33	5E+08	133	3338	0	0.10	0.06	0.13	0.29
u500_16	201006667	201006666.67	201006666.67	5E+08	133	3348	0	0.10	0.05	0.05	0.20
u500_17	197426667	197426666.67	197426666.67	5E+08	133	3348	0	0.10	0.05	0.21	0.37
u500_18	201293334	201293333.33	201293333.33	5E+08	133	3348	0	0.11	0.05	0.23	0.39
u500_19	195633334	195633333.33	195633333.33	5E+08	133	3348	0	0.11	0.06	0.27	0.43
u1000_00	398426667	398426666.67	398426666.67	1E+09	133	3348	0	0.11	0.06	0.06	0.23
u1000_01	405253334	405253333.33	405253333.33	1E+09	133	3348	0	0.10	0.06	0.20	0.36
u1000_02	410200000	410200000.00	410200000.00	1E+09	133	3348	0	0.10	0.06	0.04	0.20
u1000_03	410866667	410866666.67	410866666.67	1E+09	133	3348	0	0.10	0.06	0.07	0.23
u1000_04	396740000	396740000.00	396740000.00	1E+09	133	3348	0	0.10	0.05	0.11	0.27
u1000_05	398493334	398493333.33	398493333.33	1E+09	133	3348	0	0.11	0.05	0.14	0.30
u1000_06	394206667	394206666.67	394206666.67	1E+09	133	3348	0	0.11	0.05	0.31	0.47
u1000_07	403160000	403160000.00	403160000.00	1E+09	133	3348	0	0.11	0.06	0.08	0.24
u1000_08	398433334	398433333.33	398433333.33	1E+09	133	3348	0	0.10	0.05	0.15	0.30
u1000_09	396926667	396926666.67	396926666.67	1E+09	133	3348	0	0.10	0.05	0.30	0.46
u1000_10	399340000	399340000.00	399340000.00	1E+09	133	3348	0	0.10	0.05	0.02	0.17
u1000_11	400520000	400520000.00	400520000.00	1E+09	133	3348	0	0.11	0.05	0.07	0.23
u1000_12	392240000	392240000.00	392240000.00	1E+09	133	3348	0	0.11	0.05	0.14	0.30
u1000_13	395273334	395273333.33	395273333.33	1E+09	133	3348	0	0.10	0.06	0.21	0.37
u1000_14	393886667	393886666.67	393886666.67	1E+09	133	3348	0	0.10	0.05	0.05	0.20
u1000_15	401806667	401806666.67	401806666.67	1E+09	133	3348	0	0.10	0.05	0.26	0.41
u1000_16	403026667	403026666.67	403026666.67	1E+09	133	3348	0	0.10	0.05	0.13	0.29
u1000_17	403800000	403800000.00	403800000.00	1E+09	133	3348	0	0.10	0.05	0.02	0.17
u1000_18	398193334	398193333.33	398193333.33	1E+09	133	3348	0	0.11	0.05	0.24	0.40
u1000_19	399333334	399333333.33	399333333.33	1E+09	133	3348	0	0.11	0.05	0.24	0.40

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instance	z^*	lb^{lp}	lb^{sp}	n	$\#v$	$\#a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
t60_00	20000000	20000000.00	20000000.00	6E+07	549	5582	0	0.18	0.05	0.11	0.34
t60_01	20000000	20000000.00	20000000.00	6E+07	556	6687	0	0.21	0.05	0.13	0.39
t60_02	20000000	20000000.00	20000000.00	6E+07	549	5498	0	0.17	0.10	0.06	0.33
t60_03	20000000	20000000.00	20000000.00	6E+07	541	5145	0	0.16	0.05	0.05	0.26
t60_04	20000000	20000000.00	20000000.00	6E+07	541	4961	0	0.16	0.08	0.03	0.27
t60_05	20000000	20000000.00	20000000.00	6E+07	548	5485	0	0.17	0.07	0.10	0.33
t60_06	20000000	20000000.00	20000000.00	6E+07	548	5410	0	0.16	0.05	0.18	0.39
t60_07	20000000	20000000.00	20000000.00	6E+07	549	5491	0	0.17	0.06	0.99	1.22
t60_08	20000000	20000000.00	20000000.00	6E+07	546	5179	0	0.16	0.10	0.07	0.33
t60_09	20000000	20000000.00	20000000.00	6E+07	548	5111	0	0.16	0.09	0.07	0.32
t60_10	20000000	20000000.00	20000000.00	6E+07	554	5806	0	0.18	0.07	0.06	0.31
t60_11	20000000	20000000.00	20000000.00	6E+07	541	4627	0	0.14	0.04	0.03	0.22
t60_12	20000000	20000000.00	20000000.00	6E+07	556	5980	0	0.19	0.08	0.10	0.36
t60_13	20000000	20000000.00	20000000.00	6E+07	553	5950	0	0.19	0.08	0.17	0.44
t60_14	20000000	20000000.00	20000000.00	6E+07	553	5805	0	0.18	0.06	0.09	0.33
t60_15	20000000	20000000.00	20000000.00	6E+07	552	6255	0	0.19	0.09	0.06	0.34
t60_16	20000000	20000000.00	20000000.00	6E+07	540	5005	0	0.15	0.07	0.06	0.28
t60_17	20000000	20000000.00	20000000.00	6E+07	537	5330	0	0.17	0.07	0.07	0.30
t60_18	20000000	20000000.00	20000000.00	6E+07	551	5343	0	0.17	0.07	0.04	0.27
t60_19	20000000	20000000.00	20000000.00	6E+07	554	5968	0	0.19	0.09	0.08	0.36
t120_00	40000000	40000000.00	40000000.00	1.2E+08	589	11017	0	0.36	0.18	1.06	1.60
t120_01	40000000	40000000.00	40000000.00	1.2E+08	588	9974	0	0.32	0.14	1.34	1.80
t120_02	40000000	40000000.00	40000000.00	1.2E+08	594	11965	0	0.40	0.21	0.75	1.36
t120_03	40000000	40000000.00	40000000.00	1.2E+08	589	11012	0	0.36	0.19	0.16	0.71
t120_04	40000000	40000000.00	40000000.00	1.2E+08	595	12315	0	0.41	0.23	0.20	0.84
t120_05	40000000	40000000.00	40000000.00	1.2E+08	591	11431	0	0.37	0.18	0.22	0.77
t120_06	40000000	40000000.00	40000000.00	1.2E+08	589	10525	0	0.35	0.16	0.19	0.70
t120_07	40000000	40000000.00	40000000.00	1.2E+08	590	10649	0	0.35	0.19	0.24	0.78
t120_08	40000000	40000000.00	40000000.00	1.2E+08	589	10538	0	0.34	0.11	0.99	1.44
t120_09	40000000	40000000.00	40000000.00	1.2E+08	588	10203	0	0.32	0.12	1.18	1.63
t120_10	40000000	40000000.00	40000000.00	1.2E+08	586	10412	0	0.34	0.10	0.77	1.21
t120_11	40000000	40000000.00	40000000.00	1.2E+08	589	10985	0	0.37	0.20	0.14	0.71
t120_12	40000000	40000000.00	40000000.00	1.2E+08	585	9902	0	0.32	0.15	0.16	0.63
t120_13	40000000	40000000.00	40000000.00	1.2E+08	590	10710	0	0.34	0.15	0.21	0.70
t120_14	40000000	40000000.00	40000000.00	1.2E+08	586	10607	0	0.35	0.13	0.18	0.66
t120_15	40000000	40000000.00	40000000.00	1.2E+08	584	10425	0	0.34	0.15	0.43	0.92
t120_16	40000000	40000000.00	40000000.00	1.2E+08	590	10709	0	0.34	0.12	0.27	0.74
t120_17	40000000	40000000.00	40000000.00	1.2E+08	593	11782	0	0.39	0.15	1.29	1.83
t120_18	40000000	40000000.00	40000000.00	1.2E+08	588	10561	0	0.34	0.18	0.48	1.00
t120_19	40000000	40000000.00	40000000.00	1.2E+08	590	10803	0	0.35	0.14	0.16	0.65
t249_00	83000000	83000000.00	83000000.00	2.5E+08	637	18565	0	0.64	0.28	3.32	4.23
t249_01	83000000	83000000.00	83000000.00	2.5E+08	643	19253	0	0.64	0.28	1.66	2.58
t249_02	83000000	83000000.00	83000000.00	2.5E+08	642	19311	0	0.66	0.41	0.51	1.59
t249_03	83000000	83000000.00	83000000.00	2.5E+08	645	19474	0	0.65	0.40	0.58	1.64
t249_04	83000000	83000000.00	83000000.00	2.5E+08	637	18572	0	0.63	0.32	2.84	3.79
t249_05	83000000	83000000.00	83000000.00	2.5E+08	648	20160	0	0.70	0.36	0.63	1.69
t249_06	83000000	83000000.00	83000000.00	2.5E+08	641	18621	0	0.62	0.40	0.58	1.61
t249_07	83000000	83000000.00	83000000.00	2.5E+08	640	18539	0	0.63	0.39	0.58	1.60
t249_08	83000000	83000000.00	83000000.00	2.5E+08	642	19125	0	0.66	0.44	1.28	2.37
t249_09	83000000	83000000.00	83000000.00	2.5E+08	644	19288	0	0.66	0.39	1.40	2.46
t249_10	83000000	83000000.00	83000000.00	2.5E+08	643	19739	0	0.68	0.45	0.46	1.59

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instance	z^*	lb^{lp}	lb^{sp}	n	$\#v$	$\#a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
t249_11	83000000	83000000.00	83000000.00	2.5E+08	644	19607	0	0.68	0.30	7.52	8.50
t249_12	83000000	83000000.00	83000000.00	2.5E+08	644	19377	0	0.65	0.43	1.73	2.80
t249_13	83000000	83000000.00	83000000.00	2.5E+08	644	19343	0	0.66	0.43	1.77	2.86
t249_14	83000000	83000000.00	83000000.00	2.5E+08	648	20577	0	0.72	0.49	8.32	9.53
t249_15	83000000	83000000.00	83000000.00	2.5E+08	645	20370	0	0.68	0.34	1.68	2.70
t249_16	83000000	83000000.00	83000000.00	2.5E+08	647	20531	0	0.68	0.43	0.63	1.74
t249_17	83000000	83000000.00	83000000.00	2.5E+08	648	20178	0	0.68	0.33	2.29	3.29
t249_18	83000000	83000000.00	83000000.00	2.5E+08	641	19100	0	0.65	0.44	0.37	1.46
t249_19	83000000	83000000.00	83000000.00	2.5E+08	639	19194	0	0.65	0.39	0.47	1.51
t501_00	167000000	167000000.00	167000000.00	5E+08	693	28825	0	1.02	0.67	3.13	4.83
t501_01	167000000	167000000.00	167000000.00	5E+08	695	29730	0	1.05	0.81	5.88	7.73
t501_02	167000000	167000000.00	167000000.00	5E+08	693	29357	0	1.05	0.86	2.62	4.53
t501_03	167000000	167000000.00	167000000.00	5E+08	702	31263	0	1.16	0.93	3.12	5.22
t501_04	167000000	167000000.00	167000000.00	5E+08	698	30159	0	1.13	0.56	4.21	5.90
t501_05	167000000	167000000.00	167000000.00	5E+08	698	30189	0	1.10	0.59	0.51	2.20
t501_06	167000000	167000000.00	167000000.00	5E+08	699	30209	0	1.14	0.77	8.44	10.35
t501_07	167000000	167000000.00	167000000.00	5E+08	695	29504	0	1.06	0.73	8.63	10.42
t501_08	167000000	167000000.00	167000000.00	5E+08	699	30497	0	1.14	0.73	5.76	7.63
t501_09	167000000	167000000.00	167000000.00	5E+08	692	28936	0	1.04	0.50	23.62	25.16
t501_10	167000000	167000000.00	167000000.00	5E+08	693	29008	0	1.03	0.65	0.20	1.88
t501_11	167000000	167000000.00	167000000.00	5E+08	698	29943	0	1.07	0.61	5.81	7.49
t501_12	167000000	167000000.00	167000000.00	5E+08	692	28819	0	1.02	0.78	0.49	2.29
t501_13	167000000	167000000.00	167000000.00	5E+08	701	30738	0	1.12	0.84	16.03	17.99
t501_14	167000000	167000000.00	167000000.00	5E+08	706	32064	0	1.23	0.90	3.45	5.58
t501_15	167000000	167000000.00	167000000.00	5E+08	700	30686	0	1.13	0.59	3.42	5.15
t501_16	167000000	167000000.00	167000000.00	5E+08	701	30838	0	1.15	0.79	4.18	6.12
t501_17	167000000	167000000.00	167000000.00	5E+08	699	30453	0	1.12	0.75	7.68	9.55
t501_18	167000000	167000000.00	167000000.00	5E+08	696	29735	0	1.06	0.78	2.91	4.75
t501_19	167000000	167000000.00	167000000.00	5E+08	695	29123	0	1.00	0.53	1.10	2.63

Table A.16: Cardinality constrained bin packing results on uniform classes.

instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
u120_00	7	48	47.27	47.19	17.14	0	0.10	0.05	0.20	0.34
	6	48	47.27	47.19	20.00	0	0.10	0.05	0.20	0.34
	5	48	47.27	47.19	24.00	0	0.10	0.05	0.17	0.32
	4	48	47.27	47.19	30.00	0	0.10	0.04	0.18	0.32
	3	48	47.27	47.19	40.00	0	0.08	0.03	0.11	0.22
	2	60	60.00	47.19	60.00	0	0.05	0.01	0.02	0.08
u120_01	7	49	48.05	48.03	17.14	0	0.10	0.05	0.10	0.25
	6	49	48.05	48.03	20.00	0	0.10	0.05	0.10	0.25
	5	49	48.05	48.03	24.00	0	0.10	0.04	0.10	0.24
	4	49	48.05	48.03	30.00	0	0.09	0.04	0.08	0.21
	3	49	48.05	48.03	40.00	0	0.08	0.03	0.08	0.18
	2	60	60.00	48.03	60.00	0	0.05	0.01	0.01	0.07
u120_02	7	46	45.29	45.29	17.14	0	0.13	0.08	0.26	0.47
	6	46	45.29	45.29	20.00	0	0.13	0.08	0.26	0.47
	5	46	45.29	45.29	24.00	0	0.13	0.07	0.22	0.42
	4	46	45.29	45.29	30.00	0	0.11	0.05	0.32	0.48

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instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	3	46	45.29	45.29	40.00	0	0.09	0.03	0.14	0.26
	2	60	60.00	45.29	60.00	0	0.05	0.01	0.02	0.09
u120_03	7	49	48.63	48.57	17.14	0	0.13	0.05	0.18	0.36
	6	49	48.63	48.57	20.00	0	0.13	0.05	0.18	0.35
	5	49	48.63	48.57	24.00	0	0.12	0.05	0.18	0.35
	4	49	48.63	48.57	30.00	0	0.11	0.04	0.10	0.26
	3	49	48.63	48.57	40.00	0	0.10	0.03	0.08	0.20
	2	60	60.00	48.57	60.00	0	0.06	0.02	0.02	0.09
u120_04	7	50	49.09	49.03	17.14	0	0.11	0.05	0.11	0.28
	6	50	49.09	49.03	20.00	0	0.11	0.06	0.11	0.28
	5	50	49.09	49.03	24.00	0	0.11	0.05	0.09	0.25
	4	50	49.09	49.03	30.00	0	0.10	0.04	0.15	0.29
	3	50	49.09	49.03	40.00	0	0.08	0.03	0.06	0.17
	2	60	60.00	49.03	60.00	0	0.05	0.01	0.02	0.09
u120_05	7	48	47.49	47.48	17.14	0	0.11	0.05	0.16	0.32
	6	48	47.49	47.48	20.00	0	0.11	0.05	0.15	0.31
	5	48	47.49	47.48	24.00	0	0.11	0.05	0.15	0.30
	4	48	47.49	47.48	30.00	0	0.10	0.04	0.10	0.25
	3	48	47.49	47.48	40.00	0	0.09	0.03	0.09	0.21
	2	60	60.00	47.48	60.00	0	0.05	0.01	0.01	0.08
u120_06	7	48	47.58	47.58	17.14	0	0.11	0.07	0.22	0.40
	6	48	47.58	47.58	20.00	0	0.11	0.07	0.22	0.40
	5	48	47.58	47.58	24.00	0	0.11	0.07	0.92	1.10
	4	48	47.58	47.58	30.00	0	0.11	0.06	0.13	0.29
	3	48	47.58	47.58	40.00	0	0.09	0.03	0.09	0.22
	2	60	60.00	47.58	60.00	0	0.06	0.02	0.02	0.10
u120_07	7	49	48.66	48.63	17.14	0	0.10	0.04	0.16	0.30
	6	49	48.66	48.63	20.00	0	0.10	0.04	0.15	0.30
	5	49	48.66	48.63	24.00	0	0.10	0.04	0.13	0.27
	4	49	48.66	48.63	30.00	0	0.09	0.04	0.07	0.20
	3	49	48.66	48.63	40.00	0	0.08	0.03	0.08	0.19
	2	60	60.00	48.63	60.00	0	0.06	0.01	0.02	0.09
u120_08	7	50	49.91	49.85	17.14	0	0.12	0.06	0.20	0.38
	6	50	49.91	49.85	20.00	0	0.12	0.06	0.19	0.36
	5	50	49.91	49.85	24.00	0	0.12	0.06	0.66	0.83
	4	50	49.91	49.85	30.00	0	0.11	0.05	0.85	1.01
	3	50	49.91	49.85	40.00	0	0.10	0.03	0.11	0.23
	2	60	60.00	49.85	60.00	0	0.06	0.02	0.02	0.09
u120_09	7	46	45.80	45.80	17.14	0	0.13	0.09	0.66	0.87
	6	46	45.80	45.80	20.00	0	0.12	0.08	0.26	0.47
	5	46	45.80	45.80	24.00	0	0.12	0.07	0.19	0.39
	4	46	45.80	45.80	30.00	0	0.11	0.06	0.49	0.67
	3	46	45.80	45.80	40.00	0	0.09	0.04	0.07	0.20
	2	60	60.00	45.80	60.00	0	0.06	0.01	0.02	0.09
u120_10	7	52	51.28	51.20	17.14	0	0.10	0.05	0.14	0.29
	6	52	51.28	51.20	20.00	0	0.10	0.05	0.14	0.29
	5	52	51.28	51.20	24.00	0	0.10	0.04	0.16	0.30
	4	52	51.28	51.20	30.00	0	0.10	0.04	0.13	0.27
	3	52	51.28	51.20	40.00	0	0.08	0.03	0.09	0.20
	2	60	60.00	51.20	60.00	0	0.06	0.01	0.01	0.08

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instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
u120_11	7	49	48.39	48.31	17.14	0	0.10	0.05	0.10	0.25
	6	49	48.39	48.31	20.00	0	0.10	0.05	0.10	0.25
	5	49	48.39	48.31	24.00	0	0.10	0.04	0.09	0.23
	4	49	48.39	48.31	30.00	0	0.09	0.04	0.10	0.22
	3	49	48.39	48.31	40.00	0	0.08	0.02	0.07	0.17
	2	60	60.00	48.31	60.00	0	0.05	0.01	0.02	0.08
u120_12	7	48	47.87	47.87	17.14	0	0.12	0.07	0.23	0.41
	6	48	47.87	47.87	20.00	0	0.12	0.07	0.22	0.41
	5	48	47.87	47.87	24.00	0	0.11	0.06	0.23	0.40
	4	48	47.87	47.87	30.00	0	0.10	0.05	0.17	0.33
	3	48	47.87	47.87	40.00	0	0.09	0.03	0.11	0.23
	2	60	60.00	47.87	60.00	0	0.06	0.01	0.02	0.09
u120_13	7	49	48.01	48.01	17.14	0	0.11	0.06	0.18	0.35
	6	49	48.01	48.01	20.00	0	0.11	0.06	0.16	0.34
	5	49	48.01	48.01	24.00	0	0.11	0.06	0.12	0.30
	4	49	48.01	48.01	30.00	0	0.10	0.05	0.14	0.29
	3	49	48.01	48.01	40.00	0	0.09	0.03	0.09	0.21
	2	60	60.00	48.01	60.00	0	0.05	0.01	0.02	0.08
u120_14	7	50	49.17	49.15	17.14	0	0.10	0.05	0.10	0.26
	6	50	49.17	49.15	20.00	0	0.10	0.05	0.10	0.25
	5	50	49.17	49.15	24.00	0	0.10	0.05	0.09	0.24
	4	50	49.17	49.15	30.00	0	0.10	0.04	0.10	0.23
	3	50	49.17	49.15	40.00	0	0.08	0.03	0.10	0.21
	2	60	60.00	49.15	60.00	0	0.05	0.01	0.01	0.08
u120_15	7	48	47.38	47.35	17.14	0	0.12	0.05	0.13	0.30
	6	48	47.38	47.35	20.00	0	0.12	0.05	0.13	0.30
	5	48	47.38	47.35	24.00	0	0.11	0.05	0.14	0.31
	4	48	47.38	47.35	30.00	0	0.11	0.04	0.11	0.27
	3	48	47.38	47.35	40.00	0	0.09	0.03	0.09	0.21
	2	60	60.00	47.35	60.00	0	0.06	0.01	0.02	0.08
u120_16	7	52	51.33	51.25	17.14	0	0.10	0.04	0.06	0.20
	6	52	51.33	51.25	20.00	0	0.10	0.04	0.06	0.20
	5	52	51.33	51.25	24.00	0	0.10	0.04	0.06	0.20
	4	52	51.33	51.25	30.00	0	0.10	0.04	0.06	0.19
	3	52	51.33	51.25	40.00	0	0.08	0.03	0.06	0.17
	2	60	60.00	51.25	60.00	0	0.05	0.01	0.02	0.08
u120_17	7	52	51.50	51.35	17.14	0	0.09	0.03	0.08	0.21
	6	52	51.50	51.35	20.00	0	0.09	0.03	0.08	0.20
	5	52	51.50	51.35	24.00	0	0.09	0.03	0.09	0.21
	4	52	51.50	51.35	30.00	0	0.09	0.03	0.07	0.19
	3	52	51.50	51.35	40.00	0	0.08	0.02	0.09	0.20
	2	60	60.00	51.35	60.00	0	0.05	0.01	0.01	0.08
u120_18	7	49	48.38	48.37	17.14	0	0.11	0.05	0.08	0.25
	6	49	48.38	48.37	20.00	0	0.11	0.05	0.08	0.24
	5	49	48.38	48.37	24.00	0	0.11	0.05	0.09	0.24
	4	49	48.38	48.37	30.00	0	0.10	0.04	0.08	0.22
	3	49	48.38	48.37	40.00	0	0.09	0.03	0.06	0.18
	2	60	60.00	48.37	60.00	0	0.06	0.02	0.02	0.09
u120_19	7	49	48.86	48.81	17.14	0	0.13	0.06	0.23	0.42
	6	49	48.86	48.81	20.00	0	0.13	0.06	0.23	0.42

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instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	5	49	48.86	48.81	24.00	0	0.12	0.05	0.22	0.39
	4	49	48.86	48.81	30.00	0	0.11	0.04	0.17	0.33
	3	49	48.86	48.81	40.00	0	0.10	0.03	0.53	0.66
	2	60	60.00	48.81	60.00	0	0.06	0.02	0.02	0.10
u250_00	7	99	98.55	98.55	35.71	0	0.14	0.09	0.35	0.57
	6	99	98.55	98.55	41.67	0	0.14	0.08	0.34	0.57
	5	99	98.55	98.55	50.00	0	0.13	0.07	0.26	0.47
	4	99	98.55	98.55	62.50	0	0.12	0.06	0.23	0.41
	3	99	98.55	98.55	83.33	0	0.11	0.04	0.21	0.35
	2	125	125.00	98.55	125.00	0	0.07	0.02	0.05	0.13
u250_01	7	100	99.03	99.03	35.71	0	0.17	0.11	0.29	0.57
	6	100	99.03	99.03	41.67	0	0.17	0.10	0.28	0.55
	5	100	99.03	99.03	50.00	0	0.16	0.10	0.15	0.41
	4	100	99.03	99.03	62.50	0	0.15	0.08	0.32	0.55
	3	100	99.03	99.03	83.33	0	0.12	0.05	0.20	0.37
	2	125	125.00	99.03	125.00	0	0.08	0.02	0.05	0.15
u250_02	7	102	101.42	101.42	35.71	0	0.15	0.07	0.19	0.41
	6	102	101.42	101.42	41.67	0	0.15	0.07	0.32	0.54
	5	102	101.42	101.42	50.00	0	0.15	0.07	0.19	0.40
	4	102	101.42	101.42	62.50	0	0.14	0.06	0.16	0.36
	3	102	101.42	101.42	83.33	0	0.12	0.04	0.10	0.26
	2	125	125.00	101.42	125.00	0	0.08	0.02	0.03	0.13
u250_03	7	100	99.43	99.43	35.71	0	0.17	0.10	0.38	0.65
	6	100	99.43	99.43	41.67	0	0.17	0.10	0.27	0.53
	5	100	99.43	99.43	50.00	0	0.16	0.09	0.50	0.75
	4	100	99.43	99.43	62.50	0	0.15	0.07	0.20	0.41
	3	100	99.43	99.43	83.33	0	0.12	0.04	0.15	0.31
	2	125	125.00	99.43	125.00	0	0.08	0.02	0.05	0.15
u250_04	7	101	100.61	100.61	35.71	0	0.16	0.09	0.40	0.65
	6	101	100.61	100.61	41.67	0	0.16	0.10	0.44	0.70
	5	101	100.61	100.61	50.00	0	0.15	0.08	0.39	0.63
	4	101	100.61	100.61	62.50	0	0.14	0.07	0.16	0.37
	3	101	100.61	100.61	83.33	0	0.12	0.04	0.24	0.41
	2	125	125.00	100.61	125.00	0	0.08	0.02	0.02	0.12
u250_05	7	101	100.83	100.83	35.71	0	0.16	0.10	0.30	0.56
	6	101	100.83	100.83	41.67	0	0.16	0.10	0.46	0.71
	5	101	100.83	100.83	50.00	0	0.16	0.09	0.35	0.60
	4	101	100.83	100.83	62.50	0	0.15	0.07	0.31	0.53
	3	101	100.83	100.83	83.33	0	0.12	0.05	0.27	0.44
	2	125	125.00	100.83	125.00	0	0.08	0.02	0.03	0.13
u250_06	7	102	101.03	101.03	35.71	0	0.16	0.10	0.45	0.72
	6	102	101.03	101.03	41.67	0	0.17	0.10	0.36	0.63
	5	102	101.03	101.03	50.00	0	0.16	0.09	0.32	0.57
	4	102	101.03	101.03	62.50	0	0.15	0.07	0.18	0.39
	3	102	101.03	101.03	83.33	0	0.12	0.05	0.13	0.30
	2	125	125.00	101.03	125.00	0	0.08	0.02	0.03	0.13
u250_07	7	103	102.89	102.79	35.71	0	0.16	0.07	0.27	0.50
	6	103	102.89	102.79	41.67	0	0.16	0.08	0.32	0.55
	5	103	102.89	102.79	50.00	0	0.15	0.07	0.26	0.48
	4	103	102.89	102.79	62.50	0	0.14	0.06	0.32	0.52

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instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	3	103	102.89	102.79	83.33	0	0.12	0.04	0.20	0.37
	2	125	125.00	102.79	125.00	0	0.08	0.02	0.03	0.13
u250_08	7	105	104.92	104.91	35.71	0	0.15	0.06	0.81	1.02
	6	105	104.92	104.91	41.67	0	0.15	0.06	0.82	1.03
	5	105	104.92	104.91	50.00	0	0.15	0.06	0.20	0.40
	4	105	104.92	104.91	62.50	0	0.14	0.06	0.98	1.18
	3	105	104.92	104.91	83.33	0	0.12	0.04	0.18	0.33
	2	125	125.00	104.91	125.00	0	0.08	0.02	0.04	0.14
u250_09	7	101	100.20	100.20	35.71	0	0.16	0.08	0.22	0.47
	6	101	100.20	100.20	41.67	0	0.16	0.08	0.20	0.44
	5	101	100.20	100.20	50.00	0	0.16	0.08	0.38	0.61
	4	101	100.20	100.20	62.50	0	0.14	0.07	0.16	0.37
	3	101	100.20	100.20	83.33	0	0.13	0.05	0.11	0.29
	2	125	125.00	100.20	125.00	0	0.08	0.02	0.03	0.14
u250_10	7	105	104.39	104.37	35.71	0	0.16	0.07	0.33	0.56
	6	105	104.39	104.37	41.67	0	0.15	0.07	0.21	0.44
	5	105	104.39	104.37	50.00	0	0.15	0.06	0.20	0.41
	4	105	104.39	104.37	62.50	0	0.14	0.05	0.21	0.40
	3	105	104.39	104.37	83.33	0	0.12	0.04	0.18	0.34
	2	125	125.00	104.37	125.00	0	0.08	0.02	0.03	0.13
u250_11	7	101	100.71	100.71	35.71	0	0.16	0.11	0.26	0.53
	6	101	100.71	100.71	41.67	0	0.16	0.11	0.46	0.73
	5	101	100.71	100.71	50.00	0	0.16	0.10	0.21	0.47
	4	101	100.71	100.71	62.50	0	0.15	0.08	0.29	0.52
	3	101	100.71	100.71	83.33	0	0.13	0.06	0.11	0.30
	2	125	125.00	100.71	125.00	0	0.08	0.02	0.03	0.13
u250_12	7	105	104.98	104.93	35.71	7	0.14	0.07	1.20	1.40
	6	105	104.98	104.93	41.67	7	0.15	0.07	1.18	1.39
	5	105	104.98	104.93	50.00	0	0.14	0.06	0.65	0.86
	4	105	104.98	104.93	62.50	0	0.14	0.06	0.59	0.78
	3	105	104.98	104.93	83.33	0	0.12	0.04	0.47	0.63
	2	125	125.00	104.93	125.00	0	0.08	0.02	0.03	0.12
u250_13	7	103	102.04	101.96	35.71	0	0.16	0.06	0.22	0.44
	6	103	102.04	101.96	41.67	0	0.16	0.06	0.21	0.42
	5	103	102.04	101.96	50.00	0	0.15	0.05	0.21	0.42
	4	103	102.04	101.96	62.50	0	0.14	0.05	0.27	0.45
	3	103	102.04	101.96	83.33	0	0.12	0.03	0.12	0.27
	2	125	125.00	101.96	125.00	0	0.07	0.02	0.04	0.13
u250_14	7	100	99.17	99.17	35.71	0	0.17	0.11	0.32	0.60
	6	100	99.17	99.17	41.67	0	0.17	0.10	0.39	0.67
	5	100	99.17	99.17	50.00	0	0.17	0.10	0.25	0.52
	4	100	99.17	99.17	62.50	0	0.15	0.08	0.19	0.43
	3	100	99.17	99.17	83.33	0	0.13	0.05	0.13	0.31
	2	125	125.00	99.17	125.00	0	0.08	0.02	0.03	0.14
u250_15	7	105	104.86	104.81	35.71	0	0.16	0.07	0.26	0.49
	6	105	104.86	104.81	41.67	0	0.15	0.07	0.25	0.47
	5	105	104.86	104.81	50.00	0	0.15	0.07	0.28	0.49
	4	105	104.86	104.81	62.50	0	0.14	0.05	0.22	0.41
	3	105	104.86	104.81	83.33	0	0.12	0.04	0.13	0.29
	2	125	125.00	104.81	125.00	0	0.08	0.02	0.03	0.14

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instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
u250_16	7	97	96.51	96.51	35.71	0	0.17	0.11	0.30	0.58
	6	97	96.51	96.51	41.67	0	0.16	0.11	0.35	0.62
	5	97	96.51	96.51	50.00	0	0.16	0.09	0.22	0.47
	4	97	96.51	96.51	62.50	0	0.15	0.08	0.21	0.44
	3	97	96.51	96.51	83.33	0	0.12	0.05	0.19	0.36
	2	125	125.00	96.51	125.00	0	0.08	0.02	0.04	0.13
u250_17	7	100	99.17	99.17	35.71	0	0.15	0.11	0.29	0.55
	6	100	99.17	99.17	41.67	0	0.15	0.09	0.27	0.51
	5	100	99.17	99.17	50.00	0	0.15	0.09	0.33	0.57
	4	100	99.17	99.17	62.50	0	0.14	0.07	0.29	0.50
	3	100	99.17	99.17	83.33	0	0.12	0.05	0.08	0.24
	2	125	125.00	99.17	125.00	0	0.08	0.02	0.03	0.13
u250_18	7	100	99.70	99.70	35.71	0	0.16	0.11	0.44	0.70
	6	100	99.70	99.70	41.67	0	0.17	0.11	0.60	0.87
	5	100	99.70	99.70	50.00	0	0.16	0.10	0.29	0.56
	4	100	99.70	99.70	62.50	0	0.15	0.08	0.31	0.54
	3	100	99.70	99.70	83.33	0	0.12	0.04	0.15	0.32
	2	125	125.00	99.70	125.00	0	0.08	0.02	0.03	0.13
u250_19	7	102	101.36	101.36	35.71	0	0.17	0.11	0.37	0.64
	6	102	101.36	101.36	41.67	0	0.17	0.10	0.37	0.64
	5	102	101.36	101.36	50.00	0	0.16	0.10	0.42	0.68
	4	102	101.36	101.36	62.50	0	0.16	0.08	0.20	0.44
	3	102	101.36	101.36	83.33	0	0.13	0.05	0.15	0.33
	2	125	125.00	101.36	125.00	0	0.09	0.02	0.04	0.15
u500_00	7	198	197.58	197.58	71.43	0	0.18	0.10	0.47	0.75
	6	198	197.58	197.58	83.33	0	0.18	0.11	0.35	0.65
	5	198	197.58	197.58	100.00	0	0.18	0.09	0.46	0.74
	4	198	197.58	197.58	125.00	0	0.16	0.08	0.27	0.51
	3	198	197.58	197.58	166.67	0	0.14	0.05	0.26	0.45
	2	250	250.00	197.58	250.00	0	0.09	0.02	0.07	0.18
u500_01	7	201	200.85	200.85	71.43	0	0.18	0.10	0.48	0.76
	6	201	200.85	200.85	83.33	0	0.18	0.10	0.55	0.83
	5	201	200.85	200.85	100.00	0	0.18	0.09	0.47	0.74
	4	201	200.85	200.85	125.00	0	0.16	0.08	0.46	0.69
	3	201	200.85	200.85	166.67	0	0.13	0.05	0.12	0.30
	2	250	250.00	200.85	250.00	0	0.08	0.02	0.04	0.15
u500_02	7	202	201.44	201.44	71.43	0	0.17	0.11	0.36	0.64
	6	202	201.44	201.44	83.33	0	0.18	0.11	0.42	0.71
	5	202	201.44	201.44	100.00	0	0.17	0.09	0.35	0.62
	4	202	201.44	201.44	125.00	0	0.15	0.07	0.35	0.57
	3	202	201.44	201.44	166.67	0	0.13	0.05	0.12	0.30
	2	250	250.00	201.44	250.00	0	0.08	0.02	0.04	0.15
u500_03	7	204	203.81	203.81	71.43	0	0.18	0.11	0.51	0.80
	6	204	203.81	203.81	83.33	0	0.18	0.11	0.53	0.82
	5	204	203.81	203.81	100.00	0	0.18	0.10	0.49	0.76
	4	204	203.81	203.81	125.00	0	0.16	0.08	0.18	0.43
	3	204	203.81	203.81	166.67	0	0.14	0.05	0.16	0.35
	2	250	250.00	203.81	250.00	0	0.09	0.02	0.04	0.15
u500_04	7	206	205.11	205.11	71.43	0	0.17	0.09	0.29	0.55
	6	206	205.11	205.11	83.33	0	0.17	0.09	0.29	0.55

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instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	5	206	205.11	205.11	100.00	0	0.16	0.08	0.19	0.43
	4	206	205.11	205.11	125.00	0	0.15	0.07	0.21	0.43
	3	206	205.11	205.11	166.67	0	0.12	0.05	0.23	0.40
	2	250	250.00	205.11	250.00	0	0.08	0.02	0.04	0.14
u500_05	7	206	205.09	205.09	71.43	0	0.18	0.12	0.57	0.87
	6	206	205.09	205.09	83.33	0	0.18	0.11	0.42	0.71
	5	206	205.09	205.09	100.00	0	0.17	0.10	0.27	0.54
	4	206	205.09	205.09	125.00	0	0.16	0.08	0.28	0.52
	3	206	205.09	205.09	166.67	0	0.13	0.05	0.10	0.28
	2	250	250.00	205.09	250.00	0	0.08	0.02	0.04	0.14
u500_06	7	207	206.91	206.89	71.43	0	0.17	0.08	1.08	1.33
	6	207	206.91	206.89	83.33	0	0.18	0.08	0.30	0.55
	5	207	206.91	206.89	100.00	0	0.17	0.07	1.78	2.03
	4	207	206.91	206.89	125.00	0	0.16	0.06	0.48	0.70
	3	207	206.91	206.89	166.67	0	0.13	0.04	0.82	1.00
	2	250	250.00	206.89	250.00	0	0.08	0.02	0.11	0.22
u500_07	7	204	203.98	203.98	71.43	0	0.18	0.11	1.96	2.25
	6	204	203.98	203.98	83.33	0	0.18	0.10	0.85	1.13
	5	204	203.98	203.98	100.00	0	0.17	0.10	0.21	0.49
	4	204	203.98	203.98	125.00	0	0.16	0.08	1.20	1.43
	3	204	203.98	203.98	166.67	0	0.13	0.05	0.12	0.30
	2	250	250.00	203.98	250.00	0	0.08	0.02	0.03	0.14
u500_08	7	196	195.68	195.68	71.43	0	0.18	0.10	0.68	0.96
	6	196	195.68	195.68	83.33	0	0.18	0.10	0.60	0.88
	5	196	195.68	195.68	100.00	0	0.18	0.10	0.43	0.71
	4	196	195.68	195.68	125.00	0	0.16	0.07	0.35	0.58
	3	196	195.68	195.68	166.67	0	0.13	0.05	0.22	0.40
	2	250	250.00	195.68	250.00	0	0.08	0.02	0.07	0.18
u500_09	7	202	201.06	201.06	71.43	0	0.18	0.11	0.35	0.65
	6	202	201.06	201.06	83.33	0	0.18	0.10	0.39	0.67
	5	202	201.06	201.06	100.00	0	0.18	0.09	0.21	0.48
	4	202	201.06	201.06	125.00	0	0.16	0.08	0.20	0.43
	3	202	201.06	201.06	166.67	0	0.14	0.05	0.11	0.30
	2	250	250.00	201.06	250.00	0	0.09	0.02	0.03	0.14
u500_10	7	200	199.07	199.07	71.43	0	0.18	0.10	0.54	0.82
	6	200	199.07	199.07	83.33	0	0.18	0.09	0.31	0.58
	5	200	199.07	199.07	100.00	0	0.18	0.09	0.26	0.53
	4	200	199.07	199.07	125.00	0	0.16	0.08	0.22	0.46
	3	200	199.07	199.07	166.67	0	0.13	0.05	0.12	0.30
	2	250	250.00	199.07	250.00	0	0.09	0.02	0.06	0.17
u500_11	7	200	199.43	199.43	71.43	0	0.18	0.11	0.38	0.67
	6	200	199.43	199.43	83.33	0	0.18	0.10	0.46	0.75
	5	200	199.43	199.43	100.00	0	0.18	0.09	0.50	0.77
	4	200	199.43	199.43	125.00	0	0.16	0.07	0.41	0.64
	3	200	199.43	199.43	166.67	0	0.14	0.05	0.15	0.33
	2	250	250.00	199.43	250.00	0	0.09	0.02	0.03	0.14
u500_12	7	199	198.62	198.62	71.43	0	0.18	0.10	0.50	0.78
	6	199	198.62	198.62	83.33	0	0.18	0.10	0.55	0.84
	5	199	198.62	198.62	100.00	0	0.18	0.09	0.48	0.76
	4	199	198.62	198.62	125.00	0	0.16	0.08	0.34	0.58

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instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	3	199	198.62	198.62	166.67	0	0.13	0.05	0.57	0.75
	2	250	250.00	198.62	250.00	0	0.08	0.02	0.06	0.16
u500_13	7	196	195.59	195.59	71.43	0	0.18	0.09	0.48	0.75
	6	196	195.59	195.59	83.33	0	0.18	0.11	1.00	1.29
	5	196	195.59	195.59	100.00	0	0.17	0.09	0.36	0.62
	4	196	195.59	195.59	125.00	0	0.16	0.07	0.22	0.46
	3	196	195.59	195.59	166.67	0	0.13	0.05	0.11	0.29
	2	250	250.00	195.59	250.00	0	0.08	0.02	0.05	0.16
u500_14	7	204	203.03	203.03	71.43	0	0.18	0.10	0.43	0.71
	6	204	203.03	203.03	83.33	0	0.18	0.11	0.53	0.83
	5	204	203.03	203.03	100.00	0	0.18	0.10	0.31	0.58
	4	204	203.03	203.03	125.00	0	0.16	0.08	0.30	0.54
	3	204	203.03	203.03	166.67	0	0.13	0.05	0.18	0.37
	2	250	250.00	203.03	250.00	0	0.09	0.02	0.04	0.15
u500_15	7	201	200.13	200.13	71.43	0	0.18	0.11	0.47	0.76
	6	201	200.13	200.13	83.33	0	0.17	0.10	0.25	0.53
	5	201	200.13	200.13	100.00	0	0.17	0.10	0.18	0.45
	4	201	200.13	200.13	125.00	0	0.16	0.08	0.48	0.72
	3	201	200.13	200.13	166.67	0	0.13	0.05	0.25	0.43
	2	250	250.00	200.13	250.00	0	0.09	0.02	0.06	0.17
u500_16	7	202	201.01	201.01	71.43	0	0.18	0.10	0.40	0.68
	6	202	201.01	201.01	83.33	0	0.18	0.10	0.21	0.49
	5	202	201.01	201.01	100.00	0	0.18	0.09	0.31	0.58
	4	202	201.01	201.01	125.00	0	0.16	0.08	0.25	0.49
	3	202	201.01	201.01	166.67	0	0.13	0.05	0.13	0.31
	2	250	250.00	201.01	250.00	0	0.08	0.02	0.04	0.15
u500_17	7	198	197.43	197.43	71.43	0	0.18	0.10	0.39	0.67
	6	198	197.43	197.43	83.33	0	0.18	0.10	0.23	0.51
	5	198	197.43	197.43	100.00	0	0.17	0.09	0.46	0.73
	4	198	197.43	197.43	125.00	0	0.16	0.08	0.74	0.98
	3	198	197.43	197.43	166.67	0	0.13	0.05	0.11	0.30
	2	250	250.00	197.43	250.00	0	0.08	0.02	0.06	0.16
u500_18	7	202	201.29	201.29	71.43	0	0.18	0.10	0.41	0.68
	6	202	201.29	201.29	83.33	0	0.18	0.10	0.27	0.54
	5	202	201.29	201.29	100.00	0	0.18	0.09	0.39	0.65
	4	202	201.29	201.29	125.00	0	0.16	0.07	0.19	0.42
	3	202	201.29	201.29	166.67	0	0.13	0.04	0.15	0.33
	2	250	250.00	201.29	250.00	0	0.09	0.02	0.03	0.14
u500_19	7	196	195.63	195.63	71.43	0	0.18	0.11	0.54	0.83
	6	196	195.63	195.63	83.33	0	0.18	0.12	0.51	0.80
	5	196	195.63	195.63	100.00	0	0.17	0.09	0.53	0.80
	4	196	195.63	195.63	125.00	0	0.16	0.07	0.45	0.68
	3	196	195.63	195.63	166.67	0	0.14	0.05	0.26	0.45
	2	250	250.00	195.63	250.00	0	0.09	0.02	0.03	0.14
u1000_00	7	399	398.43	398.43	142.86	0	0.18	0.10	0.34	0.62
	6	399	398.43	398.43	166.67	0	0.18	0.10	0.50	0.78
	5	399	398.43	398.43	200.00	0	0.17	0.10	0.39	0.66
	4	399	398.43	398.43	250.00	0	0.16	0.08	0.36	0.59
	3	399	398.43	398.43	333.33	0	0.13	0.05	0.21	0.39
	2	500	500.00	398.43	500.00	0	0.09	0.02	0.07	0.18

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instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
u1000_01	7	406	405.25	405.25	142.86	0	0.18	0.10	0.48	0.76
	6	406	405.25	405.25	166.67	0	0.18	0.10	0.49	0.77
	5	406	405.25	405.25	200.00	0	0.18	0.09	0.33	0.61
	4	406	405.25	405.25	250.00	0	0.16	0.08	0.36	0.60
	3	406	405.25	405.25	333.33	0	0.13	0.05	0.21	0.39
	2	500	500.00	405.25	500.00	0	0.08	0.02	0.03	0.14
u1000_02	7	411	410.20	410.20	142.86	0	0.18	0.10	0.53	0.81
	6	411	410.20	410.20	166.67	0	0.18	0.10	0.38	0.66
	5	411	410.20	410.20	200.00	0	0.18	0.10	0.51	0.79
	4	411	410.20	410.20	250.00	0	0.16	0.07	0.38	0.61
	3	411	410.20	410.20	333.33	0	0.14	0.05	0.15	0.33
	2	500	500.00	410.20	500.00	0	0.09	0.02	0.04	0.15
u1000_03	7	411	410.87	410.87	142.86	0	0.18	0.10	0.80	1.08
	6	411	410.87	410.87	166.67	0	0.18	0.11	0.35	0.64
	5	411	410.87	410.87	200.00	0	0.18	0.10	0.33	0.61
	4	411	410.87	410.87	250.00	0	0.16	0.08	0.24	0.49
	3	411	410.87	410.87	333.33	0	0.14	0.05	0.23	0.41
	2	500	500.00	410.87	500.00	0	0.09	0.02	0.04	0.15
u1000_04	7	397	396.74	396.74	142.86	0	0.19	0.09	0.70	0.98
	6	397	396.74	396.74	166.67	0	0.18	0.10	0.66	0.95
	5	397	396.74	396.74	200.00	0	0.18	0.10	0.51	0.79
	4	397	396.74	396.74	250.00	0	0.16	0.07	0.37	0.61
	3	397	396.74	396.74	333.33	0	0.14	0.05	0.31	0.50
	2	500	500.00	396.74	500.00	0	0.09	0.02	0.09	0.20
u1000_05	7	399	398.49	398.49	142.86	0	0.18	0.10	0.54	0.82
	6	399	398.49	398.49	166.67	0	0.18	0.10	0.35	0.64
	5	399	398.49	398.49	200.00	0	0.17	0.09	0.44	0.71
	4	399	398.49	398.49	250.00	0	0.16	0.07	0.20	0.43
	3	399	398.49	398.49	333.33	0	0.13	0.05	0.18	0.36
	2	500	500.00	398.49	500.00	0	0.09	0.02	0.03	0.14
u1000_06	7	395	394.21	394.21	142.86	0	0.18	0.10	0.52	0.80
	6	395	394.21	394.21	166.67	0	0.18	0.11	0.49	0.78
	5	395	394.21	394.21	200.00	0	0.17	0.08	0.42	0.67
	4	395	394.21	394.21	250.00	0	0.16	0.08	0.25	0.49
	3	395	394.21	394.21	333.33	0	0.13	0.05	0.18	0.36
	2	500	500.00	394.21	500.00	0	0.08	0.02	0.06	0.17
u1000_07	7	404	403.16	403.16	142.86	0	0.18	0.09	0.53	0.80
	6	404	403.16	403.16	166.67	0	0.18	0.10	0.39	0.68
	5	404	403.16	403.16	200.00	0	0.18	0.10	0.21	0.49
	4	404	403.16	403.16	250.00	0	0.16	0.07	0.45	0.68
	3	404	403.16	403.16	333.33	0	0.13	0.05	0.21	0.39
	2	500	500.00	403.16	500.00	0	0.09	0.02	0.06	0.17
u1000_08	7	399	398.43	398.43	142.86	0	0.18	0.09	0.44	0.72
	6	399	398.43	398.43	166.67	0	0.18	0.10	0.39	0.68
	5	399	398.43	398.43	200.00	0	0.17	0.09	0.39	0.65
	4	399	398.43	398.43	250.00	0	0.16	0.08	0.78	1.03
	3	399	398.43	398.43	333.33	0	0.14	0.05	0.25	0.43
	2	500	500.00	398.43	500.00	0	0.08	0.02	0.06	0.17
u1000_09	7	397	396.93	396.93	142.86	0	0.18	0.09	2.25	2.53
	6	397	396.93	396.93	166.67	0	0.18	0.10	4.81	5.09

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instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	5	397	396.93	396.93	200.00	0	0.17	0.09	1.53	1.79
	4	397	396.93	396.93	250.00	0	0.16	0.07	1.33	1.56
	3	397	396.93	396.93	333.33	0	0.13	0.04	0.15	0.32
	2	500	500.00	396.93	500.00	0	0.09	0.02	0.04	0.15
u1000_10	7	400	399.34	399.34	142.86	0	0.18	0.10	0.38	0.66
	6	400	399.34	399.34	166.67	0	0.18	0.10	0.46	0.74
	5	400	399.34	399.34	200.00	0	0.17	0.10	0.48	0.75
	4	400	399.34	399.34	250.00	0	0.16	0.08	0.36	0.60
	3	400	399.34	399.34	333.33	0	0.14	0.05	0.21	0.39
	2	500	500.00	399.34	500.00	0	0.09	0.02	0.04	0.15
u1000_11	7	401	400.52	400.52	142.86	0	0.18	0.10	0.56	0.85
	6	401	400.52	400.52	166.67	0	0.19	0.11	0.52	0.81
	5	401	400.52	400.52	200.00	0	0.18	0.09	0.49	0.76
	4	401	400.52	400.52	250.00	0	0.16	0.07	0.33	0.57
	3	401	400.52	400.52	333.33	0	0.14	0.05	0.25	0.44
	2	500	500.00	400.52	500.00	0	0.09	0.02	0.04	0.15
u1000_12	7	393	392.24	392.24	142.86	0	0.18	0.09	0.37	0.65
	6	393	392.24	392.24	166.67	0	0.18	0.10	0.51	0.79
	5	393	392.24	392.24	200.00	0	0.18	0.09	0.24	0.51
	4	393	392.24	392.24	250.00	0	0.16	0.07	0.28	0.51
	3	393	392.24	392.24	333.33	0	0.13	0.05	0.13	0.31
	2	500	500.00	392.24	500.00	0	0.08	0.02	0.06	0.17
u1000_13	7	396	395.27	395.27	142.86	0	0.18	0.09	0.57	0.85
	6	396	395.27	395.27	166.67	0	0.18	0.10	0.42	0.71
	5	396	395.27	395.27	200.00	0	0.18	0.09	0.47	0.74
	4	396	395.27	395.27	250.00	0	0.16	0.07	0.34	0.57
	3	396	395.27	395.27	333.33	0	0.14	0.05	0.24	0.42
	2	500	500.00	395.27	500.00	0	0.09	0.02	0.05	0.16
u1000_14	7	394	393.89	393.89	142.86	0	0.18	0.09	3.83	4.10
	6	394	393.89	393.89	166.67	40	0.18	0.10	8.86	9.14
	5	394	393.89	393.89	200.00	0	0.18	0.08	5.36	5.62
	4	394	393.89	393.89	250.00	0	0.16	0.08	0.31	0.55
	3	394	393.89	393.89	333.33	0	0.14	0.05	0.21	0.39
	2	500	500.00	393.89	500.00	0	0.09	0.02	0.06	0.17
u1000_15	7	402	401.81	401.81	142.86	0	0.19	0.10	1.20	1.48
	6	402	401.81	401.81	166.67	0	0.18	0.10	0.69	0.98
	5	402	401.81	401.81	200.00	0	0.18	0.08	0.47	0.74
	4	402	401.81	401.81	250.00	0	0.16	0.07	0.37	0.60
	3	402	401.81	401.81	333.33	0	0.14	0.05	0.55	0.74
	2	500	500.00	401.81	500.00	0	0.09	0.02	0.04	0.16
u1000_16	7	404	403.03	403.03	142.86	0	0.19	0.10	0.36	0.64
	6	404	403.03	403.03	166.67	0	0.18	0.11	0.46	0.76
	5	404	403.03	403.03	200.00	0	0.18	0.10	0.50	0.78
	4	404	403.03	403.03	250.00	0	0.16	0.08	0.20	0.44
	3	404	403.03	403.03	333.33	0	0.14	0.05	0.11	0.29
	2	500	500.00	403.03	500.00	0	0.08	0.02	0.04	0.14
u1000_17	7	404	403.80	403.80	142.86	0	0.18	0.10	0.61	0.88
	6	404	403.80	403.80	166.67	0	0.18	0.10	1.37	1.66
	5	404	403.80	403.80	200.00	0	0.17	0.09	0.70	0.96
	4	404	403.80	403.80	250.00	0	0.16	0.07	0.41	0.65

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instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	3	404	403.80	403.80	333.33	0	0.14	0.05	0.21	0.39
	2	500	500.00	403.80	500.00	0	0.09	0.02	0.04	0.15
u1000_18	7	399	398.19	398.19	142.86	0	0.18	0.10	0.29	0.57
	6	399	398.19	398.19	166.67	0	0.18	0.10	0.45	0.73
	5	399	398.19	398.19	200.00	0	0.17	0.08	0.24	0.49
	4	399	398.19	398.19	250.00	0	0.17	0.07	0.33	0.57
	3	399	398.19	398.19	333.33	0	0.14	0.05	0.21	0.39
	2	500	500.00	398.19	500.00	0	0.09	0.02	0.03	0.15
u1000_19	7	400	399.33	399.33	142.86	0	0.19	0.09	0.43	0.71
	6	400	399.33	399.33	166.67	0	0.18	0.10	0.47	0.75
	5	400	399.33	399.33	200.00	0	0.18	0.09	0.28	0.55
	4	400	399.33	399.33	250.00	0	0.16	0.07	0.24	0.47
	3	400	399.33	399.33	333.33	0	0.14	0.05	0.11	0.29
	2	500	500.00	399.33	500.00	0	0.08	0.02	0.05	0.16

Table A.17: Cardinality constrained bin packing results on triplets classes.

instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
t60_00	4	20	20.00	20.00	15.00	0	0.17	0.05	0.08	0.30
	3	20	20.00	20.00	20.00	0	0.17	0.05	0.08	0.30
	2	30	30.00	20.00	30.00	0	0.07	0.01	0.02	0.09
t60_01	4	20	20.00	20.00	15.00	0	0.20	0.06	0.45	0.72
	3	20	20.00	20.00	20.00	0	0.20	0.06	0.45	0.71
	2	30	30.00	20.00	30.00	0	0.07	0.01	0.02	0.11
t60_02	4	20	20.00	20.00	15.00	0	0.17	0.05	0.06	0.28
	3	20	20.00	20.00	20.00	0	0.17	0.05	0.06	0.28
	2	30	30.00	20.00	30.00	0	0.06	0.01	0.01	0.09
t60_03	4	20	20.00	20.00	15.00	0	0.16	0.05	0.09	0.30
	3	20	20.00	20.00	20.00	0	0.16	0.05	0.09	0.29
	2	30	30.00	20.00	30.00	0	0.06	0.01	0.01	0.09
t60_04	4	20	20.00	20.00	15.00	0	0.15	0.04	0.31	0.50
	3	20	20.00	20.00	20.00	0	0.15	0.04	0.31	0.50
	2	30	30.00	20.00	30.00	0	0.06	0.01	0.01	0.09
t60_05	4	20	20.00	20.00	15.00	0	0.17	0.06	0.07	0.29
	3	20	20.00	20.00	20.00	0	0.17	0.06	0.07	0.30
	2	30	30.00	20.00	30.00	0	0.06	0.01	0.01	0.09
t60_06	4	20	20.00	20.00	15.00	0	0.17	0.05	0.62	0.83
	3	20	20.00	20.00	20.00	0	0.16	0.05	0.61	0.81
	2	30	30.00	20.00	30.00	0	0.07	0.01	0.02	0.09
t60_07	4	20	20.00	20.00	15.00	0	0.17	0.05	0.09	0.31
	3	20	20.00	20.00	20.00	0	0.16	0.05	0.09	0.31
	2	30	30.00	20.00	30.00	0	0.06	0.01	0.01	0.09
t60_08	4	20	20.00	20.00	15.00	0	0.15	0.05	0.07	0.27
	3	20	20.00	20.00	20.00	0	0.15	0.05	0.07	0.27
	2	30	30.00	20.00	30.00	0	0.06	0.01	0.01	0.08
t60_09	4	20	20.00	20.00	15.00	0	0.16	0.05	0.07	0.28
	3	20	20.00	20.00	20.00	0	0.16	0.05	0.07	0.28
	2	30	30.00	20.00	30.00	0	0.06	0.01	0.01	0.09

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instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
t60_10	4	20	20.00	20.00	15.00	0	0.18	0.05	0.29	0.52
	3	20	20.00	20.00	20.00	0	0.17	0.05	0.29	0.51
	2	30	30.00	20.00	30.00	0	0.07	0.01	0.02	0.10
t60_11	4	20	20.00	20.00	15.00	0	0.14	0.04	0.05	0.23
	3	20	20.00	20.00	20.00	0	0.14	0.04	0.06	0.24
	2	30	30.00	20.00	30.00	0	0.06	0.01	0.01	0.08
t60_12	4	20	20.00	20.00	15.00	0	0.18	0.06	0.09	0.33
	3	20	20.00	20.00	20.00	0	0.18	0.06	0.09	0.33
	2	30	30.00	20.00	30.00	0	0.07	0.01	0.02	0.10
t60_13	4	20	20.00	20.00	15.00	0	0.18	0.06	0.07	0.30
	3	20	20.00	20.00	20.00	0	0.18	0.06	0.07	0.30
	2	30	30.00	20.00	30.00	0	0.07	0.01	0.02	0.10
t60_14	4	20	20.00	20.00	15.00	0	0.18	0.06	0.07	0.31
	3	20	20.00	20.00	20.00	0	0.18	0.06	0.07	0.31
	2	30	30.00	20.00	30.00	0	0.07	0.01	0.01	0.09
t60_15	4	20	20.00	20.00	15.00	0	0.19	0.06	0.40	0.65
	3	20	20.00	20.00	20.00	0	0.19	0.06	0.40	0.65
	2	30	30.00	20.00	30.00	0	0.07	0.01	0.02	0.10
t60_16	4	20	20.00	20.00	15.00	0	0.16	0.04	0.07	0.27
	3	20	20.00	20.00	20.00	0	0.16	0.04	0.07	0.27
	2	30	30.00	20.00	30.00	0	0.06	0.01	0.01	0.08
t60_17	4	20	20.00	20.00	15.00	0	0.17	0.04	0.12	0.33
	3	20	20.00	20.00	20.00	0	0.17	0.04	0.12	0.33
	2	30	30.00	20.00	30.00	0	0.06	0.01	0.01	0.09
t60_18	4	20	20.00	20.00	15.00	0	0.17	0.05	0.11	0.33
	3	20	20.00	20.00	20.00	0	0.17	0.05	0.11	0.33
	2	30	30.00	20.00	30.00	0	0.07	0.01	0.02	0.09
t60_19	4	20	20.00	20.00	15.00	0	0.18	0.06	0.07	0.32
	3	20	20.00	20.00	20.00	0	0.18	0.06	0.07	0.32
	2	30	30.00	20.00	30.00	0	0.07	0.01	0.02	0.09
t120_00	4	40	40.00	40.00	30.00	0	0.35	0.12	1.25	1.72
	3	40	40.00	40.00	40.00	0	0.36	0.12	1.28	1.76
	2	60	60.00	40.00	60.00	0	0.15	0.03	0.05	0.23
t120_01	4	40	40.00	40.00	30.00	0	0.32	0.10	0.54	0.95
	3	40	40.00	40.00	40.00	0	0.31	0.11	1.98	2.40
	2	60	60.00	40.00	60.00	0	0.15	0.03	0.09	0.27
t120_02	4	40	40.00	40.00	30.00	0	0.38	0.12	0.25	0.74
	3	40	40.00	40.00	40.00	0	0.40	0.13	3.01	3.54
	2	60	60.00	40.00	60.00	0	0.16	0.04	0.05	0.26
t120_03	4	40	40.00	40.00	30.00	0	0.36	0.13	3.42	3.91
	3	40	40.00	40.00	40.00	0	0.36	0.13	3.47	3.96
	2	60	60.00	40.00	60.00	0	0.14	0.03	0.06	0.24
t120_04	4	40	40.00	40.00	30.00	0	0.40	0.14	3.69	4.22
	3	40	40.00	40.00	40.00	0	0.41	0.14	3.70	4.25
	2	60	60.00	40.00	60.00	0	0.16	0.04	0.05	0.26
t120_05	4	40	40.00	40.00	30.00	0	0.38	0.14	0.46	0.98
	3	40	40.00	40.00	40.00	0	0.37	0.13	0.44	0.94
	2	60	60.00	40.00	60.00	0	0.15	0.04	0.05	0.24
t120_06	4	40	40.00	40.00	30.00	0	0.34	0.11	0.24	0.69

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instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	3	40	40.00	40.00	40.00	0	0.34	0.11	0.24	0.69
	2	60	60.00	40.00	60.00	0	0.14	0.03	0.06	0.24
t120_07	4	40	40.00	40.00	30.00	0	0.35	0.10	0.44	0.89
	3	40	40.00	40.00	40.00	0	0.34	0.12	2.22	2.68
	2	60	60.00	40.00	60.00	0	0.15	0.04	0.05	0.24
t120_08	4	40	40.00	40.00	30.00	0	0.35	0.10	0.24	0.69
	3	40	40.00	40.00	40.00	0	0.35	0.10	0.25	0.70
	2	60	60.00	40.00	60.00	0	0.15	0.03	0.05	0.23
t120_09	4	40	40.00	40.00	30.00	0	0.32	0.09	0.53	0.94
	3	40	40.00	40.00	40.00	0	0.33	0.12	2.12	2.57
	2	60	60.00	40.00	60.00	0	0.14	0.03	0.05	0.22
t120_10	4	40	40.00	40.00	30.00	0	0.34	0.10	3.28	3.72
	3	40	40.00	40.00	40.00	0	0.33	0.09	3.27	3.69
	2	60	60.00	40.00	60.00	0	0.14	0.03	0.07	0.24
t120_11	4	40	40.00	40.00	30.00	0	0.36	0.14	1.56	2.06
	3	40	40.00	40.00	40.00	0	0.35	0.13	1.60	2.08
	2	60	60.00	40.00	60.00	0	0.15	0.03	0.05	0.22
t120_12	4	40	40.00	40.00	30.00	0	0.32	0.09	0.22	0.63
	3	40	40.00	40.00	40.00	0	0.33	0.10	2.41	2.84
	2	60	60.00	40.00	60.00	0	0.14	0.03	0.05	0.22
t120_13	4	40	40.00	40.00	30.00	0	0.34	0.11	2.03	2.48
	3	40	40.00	40.00	40.00	0	0.35	0.12	2.05	2.51
	2	60	60.00	40.00	60.00	0	0.14	0.03	0.10	0.28
t120_14	4	40	40.00	40.00	30.00	0	0.33	0.12	0.69	1.14
	3	40	40.00	40.00	40.00	0	0.34	0.12	0.71	1.18
	2	60	60.00	40.00	60.00	0	0.14	0.03	0.06	0.23
t120_15	4	40	40.00	40.00	30.00	0	0.33	0.10	1.82	2.25
	3	40	40.00	40.00	40.00	0	0.33	0.10	1.83	2.26
	2	60	60.00	40.00	60.00	0	0.13	0.03	0.08	0.24
t120_16	4	40	40.00	40.00	30.00	0	0.34	0.11	0.31	0.77
	3	40	40.00	40.00	40.00	0	0.35	0.12	0.32	0.79
	2	60	60.00	40.00	60.00	0	0.15	0.03	0.06	0.24
t120_17	4	40	40.00	40.00	30.00	0	0.38	0.13	1.44	1.95
	3	40	40.00	40.00	40.00	0	0.39	0.13	1.43	1.95
	2	60	60.00	40.00	60.00	0	0.16	0.04	0.06	0.26
t120_18	4	40	40.00	40.00	30.00	0	0.34	0.10	0.22	0.65
	3	40	40.00	40.00	40.00	0	0.34	0.11	1.00	1.45
	2	60	60.00	40.00	60.00	0	0.15	0.03	0.06	0.24
t120_19	4	40	40.00	40.00	30.00	0	0.35	0.10	2.68	3.13
	3	40	40.00	40.00	40.00	0	0.34	0.10	2.72	3.17
	2	60	60.00	40.00	60.00	0	0.15	0.03	0.10	0.28
t249_00	4	83	83.00	83.00	62.25	0	0.63	0.22	0.55	1.40
	3	83	83.00	83.00	83.00	0	0.64	0.25	10.57	11.46
	2	125	124.50	83.00	124.50	0	0.32	0.09	0.24	0.65
t249_01	4	83	83.00	83.00	62.25	0	0.66	0.24	1.99	2.89
	3	83	83.00	83.00	83.00	0	0.64	0.28	2.96	3.87
	2	125	124.50	83.00	124.50	0	0.36	0.11	0.24	0.70
t249_02	4	83	83.00	83.00	62.25	0	0.64	0.25	5.83	6.72
	3	83	83.00	83.00	83.00	0	0.66	0.28	8.98	9.93

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instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	2	125	124.50	83.00	124.50	0	0.35	0.10	0.25	0.70
t249_03	4	83	83.00	83.00	62.25	0	0.67	0.24	1.17	2.08
	3	83	83.00	83.00	83.00	8	0.68	0.29	11.96	12.93
	2	125	124.50	83.00	124.50	0	0.37	0.11	0.25	0.73
t249_04	4	83	83.00	83.00	62.25	0	0.64	0.23	4.74	5.61
	3	83	83.00	83.00	83.00	8	0.64	0.25	12.75	13.64
	2	125	124.50	83.00	124.50	0	0.33	0.10	0.24	0.66
t249_05	4	83	83.00	83.00	62.25	0	0.70	0.27	8.85	9.81
	3	83	83.00	83.00	83.00	0	0.71	0.31	14.17	15.19
	2	125	124.50	83.00	124.50	0	0.37	0.11	0.28	0.77
t249_06	4	83	83.00	83.00	62.25	0	0.63	0.21	0.49	1.34
	3	83	83.00	83.00	83.00	0	0.63	0.24	5.15	6.02
	2	125	124.50	83.00	124.50	0	0.34	0.10	0.24	0.69
t249_07	4	83	83.00	83.00	62.25	0	0.64	0.23	0.52	1.39
	3	83	83.00	83.00	83.00	0	0.64	0.28	0.91	1.83
	2	125	124.50	83.00	124.50	0	0.33	0.10	0.19	0.62
t249_08	4	83	83.00	83.00	62.25	0	0.65	0.24	2.58	3.48
	3	83	83.00	83.00	83.00	0	0.64	0.28	5.11	6.03
	2	125	124.50	83.00	124.50	0	0.35	0.10	0.23	0.68
t249_09	4	83	83.00	83.00	62.25	0	0.65	0.25	0.66	1.55
	3	83	83.00	83.00	83.00	0	0.67	0.29	4.49	5.45
	2	125	124.50	83.00	124.50	0	0.36	0.11	0.26	0.72
t249_10	4	83	83.00	83.00	62.25	0	0.68	0.24	3.47	4.38
	3	83	83.00	83.00	83.00	0	0.67	0.30	4.37	5.35
	2	125	124.50	83.00	124.50	0	0.36	0.10	0.25	0.71
t249_11	4	83	83.00	83.00	62.25	0	0.67	0.27	1.78	2.73
	3	83	83.00	83.00	83.00	0	0.68	0.27	1.82	2.77
	2	125	124.50	83.00	124.50	0	0.35	0.11	0.25	0.70
t249_12	4	83	83.00	83.00	62.25	0	0.66	0.24	4.32	5.22
	3	83	83.00	83.00	83.00	0	0.66	0.27	2.37	3.30
	2	125	124.50	83.00	124.50	0	0.36	0.11	0.23	0.70
t249_13	4	83	83.00	83.00	62.25	0	0.64	0.23	4.35	5.22
	3	83	83.00	83.00	83.00	0	0.65	0.26	5.18	6.08
	2	125	124.50	83.00	124.50	0	0.35	0.10	0.26	0.71
t249_14	4	83	83.00	83.00	62.25	0	0.69	0.32	7.30	8.31
	3	83	83.00	83.00	83.00	0	0.69	0.32	7.36	8.37
	2	125	124.50	83.00	124.50	0	0.39	0.12	0.26	0.76
t249_15	4	83	83.00	83.00	62.25	0	0.71	0.27	1.01	1.98
	3	83	83.00	83.00	83.00	0	0.70	0.31	6.61	7.62
	2	125	124.50	83.00	124.50	0	0.36	0.10	0.27	0.73
t249_16	4	83	83.00	83.00	62.25	0	0.71	0.26	4.77	5.74
	3	83	83.00	83.00	83.00	0	0.71	0.29	3.40	4.40
	2	125	124.50	83.00	124.50	0	0.38	0.11	0.27	0.76
t249_17	4	83	83.00	83.00	62.25	0	0.69	0.26	2.78	3.73
	3	83	83.00	83.00	83.00	0	0.70	0.29	2.78	3.77
	2	125	124.50	83.00	124.50	0	0.37	0.11	0.27	0.75
t249_18	4	83	83.00	83.00	62.25	0	0.66	0.24	0.61	1.51
	3	83	83.00	83.00	83.00	0	0.66	0.27	14.16	15.09
	2	125	124.50	83.00	124.50	0	0.35	0.10	0.24	0.69

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instance	C	z^*	$lb^{\rm lp}$	$lb^{\rm sp}$	$lb^{\rm crd}$	$n^{\rm bb}$	$t^{\rm pp}$	$t^{\rm lp}$	$t^{\rm ip}$	$t^{\rm tot}$
t249_19	4	83	83.00	83.00	62.25	0	0.65	0.22	0.67	1.54
	3	83	83.00	83.00	83.00	0	0.64	0.23	1.16	2.03
	2	125	124.50	83.00	124.50	0	0.33	0.10	0.22	0.65
t501_00	4	167	167.00	167.00	125.25	0	1.04	0.43	4.53	6.00
	3	167	167.00	167.00	167.00	0	1.03	0.43	21.68	23.14
	2	251	250.50	167.00	250.50	0	0.62	0.22	0.43	1.27
t501_01	4	167	167.00	167.00	125.25	0	1.13	0.45	3.31	4.89
	3	167	167.00	167.00	167.00	0	1.13	0.47	24.26	25.86
	2	251	250.50	167.00	250.50	0	0.63	0.22	0.63	1.48
t501_02	4	167	167.00	167.00	125.25	0	1.05	0.45	1.38	2.88
	3	167	167.00	167.00	167.00	0	1.08	0.57	26.20	27.86
	2	251	250.50	167.00	250.50	0	0.62	0.22	0.68	1.52
t501_03	4	167	167.00	167.00	125.25	0	1.17	0.51	15.95	17.62
	3	167	167.00	167.00	167.00	0	1.19	0.67	29.03	30.89
	2	251	250.50	167.00	250.50	0	0.68	0.19	0.77	1.64
t501_04	4	167	167.00	167.00	125.25	0	1.13	0.44	7.34	8.92
	3	167	167.00	167.00	167.00	77	1.14	0.54	61.19	62.87
	2	251	250.50	167.00	250.50	0	0.68	0.23	0.49	1.40
t501_05	4	167	167.00	167.00	125.25	0	1.12	0.46	14.89	16.48
	3	167	167.00	167.00	167.00	0	1.13	0.53	21.60	23.26
	2	251	250.50	167.00	250.50	0	0.65	0.23	0.48	1.36
t501_06	4	167	167.00	167.00	125.25	0	1.15	0.46	10.58	12.19
	3	167	167.00	167.00	167.00	0	1.18	0.50	29.11	30.79
	2	251	250.50	167.00	250.50	0	0.68	0.23	0.77	1.69
t501_07	4	167	167.00	167.00	125.25	0	1.05	0.42	10.54	12.01
	3	167	167.00	167.00	167.00	0	1.08	0.49	5.34	6.91
	2	251	250.50	167.00	250.50	0	0.66	0.22	0.57	1.44
t501_08	4	167	167.00	167.00	125.25	0	1.15	0.44	3.81	5.40
	3	167	167.00	167.00	167.00	0	1.17	0.56	11.14	12.87
	2	251	250.50	167.00	250.50	0	0.68	0.24	0.57	1.50
t501_09	4	167	167.00	167.00	125.25	0	1.03	0.41	14.25	15.69
	3	167	167.00	167.00	167.00	0	1.02	0.50	12.48	14.00
	2	251	250.50	167.00	250.50	0	0.64	0.22	0.73	1.59
t501_10	4	167	167.00	167.00	125.25	0	1.05	0.44	9.65	11.14
	3	167	167.00	167.00	167.00	0	1.07	0.51	23.12	24.69
	2	251	250.50	167.00	250.50	0	0.65	0.22	0.56	1.43
t501_11	4	167	167.00	167.00	125.25	0	1.15	0.45	11.77	13.37
	3	167	167.00	167.00	167.00	0	1.16	0.49	14.36	16.01
	2	251	250.50	167.00	250.50	0	0.66	0.22	0.66	1.54
t501_12	4	167	167.00	167.00	125.25	0	1.02	0.42	2.32	3.76
	3	167	167.00	167.00	167.00	0	1.04	0.46	5.49	6.99
	2	251	250.50	167.00	250.50	0	0.64	0.20	0.75	1.59
t501_13	4	167	167.00	167.00	125.25	0	1.18	0.46	2.55	4.18
	3	167	167.00	167.00	167.00	0	1.19	0.55	20.38	22.12
	2	251	250.50	167.00	250.50	0	0.70	0.24	0.75	1.69
t501_14	4	167	167.00	167.00	125.25	0	1.25	0.47	12.60	14.32
	3	167	167.00	167.00	167.00	0	1.26	0.58	13.61	15.46
	2	251	250.50	167.00	250.50	0	0.75	0.26	0.71	1.72
t501_15	4	167	167.00	167.00	125.25	0	1.15	0.46	15.40	17.01

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instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	3	167	167.00	167.00	167.00	8	1.19	0.58	33.68	35.45
	2	251	250.50	167.00	250.50	0	0.69	0.25	0.48	1.42
t501_16	4	167	167.00	167.00	125.25	0	1.17	0.48	10.46	12.10
	3	167	167.00	167.00	167.00	8	1.16	0.53	27.30	28.98
	2	251	250.50	167.00	250.50	0	0.67	0.23	0.60	1.51
t501_17	4	167	167.00	167.00	125.25	0	1.14	0.45	4.74	6.33
	3	167	167.00	167.00	167.00	0	1.14	0.52	11.13	12.80
	2	251	250.50	167.00	250.50	0	0.68	0.25	0.54	1.47
t501_18	4	167	167.00	167.00	125.25	0	1.12	0.45	2.74	4.32
	3	167	167.00	167.00	167.00	0	1.22	0.49	12.07	13.78
	2	251	250.50	167.00	250.50	0	0.66	0.22	0.48	1.37
t501_19	4	167	167.00	167.00	125.25	0	1.07	0.43	4.20	5.69
	3	167	167.00	167.00	167.00	0	1.06	0.52	8.22	9.80
	2	251	250.50	167.00	250.50	0	0.66	0.23	0.47	1.36

Table A.18: Two-constraint bin packing results for $n = 25$.

class	inst.	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{d1}	lb^{d2}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
1	1	6	5.69	5.41	5.65	5.64	5.65	0	1.27	0.29	2.97	4.53
	2	7	6.23	5.70	5.07	6.23	5.30	0	0.83	0.16	1.70	2.69
	3	7	6.16	5.07	5.93	5.30	6.16	0	0.80	0.18	1.47	2.44
	4	7	6.16	5.93	5.22	6.16	5.45	0	0.86	0.18	1.51	2.55
	5	7	6.02	5.22	5.79	5.45	6.02	0	0.83	0.17	1.42	2.42
	6	7	6.02	5.79	4.95	6.02	5.31	0	1.04	0.20	2.29	3.54
	7	7	6.08	4.95	5.85	5.31	6.08	0	0.84	0.19	1.60	2.63
	8	7	6.08	5.85	4.99	6.08	5.35	0	0.96	0.20	2.06	3.22
	9	7	6.10	4.99	5.87	5.35	6.10	0	0.81	0.18	1.55	2.54
	10	7	6.10	5.87	4.89	6.10	5.25	0	0.96	0.20	1.75	2.90
2	1★	13	12.50	12.00	11.00	10.48	10.53	0	0.07	0.01	0.03	0.11
	2★	14	14.00	11.00	13.00	10.53	11.04	0	0.04	0.00	0.01	0.05
	3★	14	13.50	13.00	12.00	11.04	11.43	0	0.05	0.01	0.02	0.08
	4★	14	14.00	12.00	13.00	11.43	10.93	0	0.03	0.00	0.01	0.04
	5★	13	13.00	13.00	11.00	10.93	10.61	0	0.05	0.01	0.02	0.08
	6★	14	14.00	11.00	13.50	10.61	11.37	0	0.04	0.01	0.01	0.05
	7★	14	13.50	13.50	10.25	11.37	9.95	0	0.06	0.01	0.02	0.09
	8★	15	15.00	10.25	13.50	9.95	11.92	0	0.04	0.01	0.01	0.05
	9★	15	14.50	13.50	10.00	11.92	9.68	0	0.05	0.01	0.02	0.07
	10★	16	16.00	10.00	14.50	9.68	12.46	0	0.03	0.00	0.01	0.04
3	1★	13	12.50	12.25	11.62	11.29	11.31	0	0.02	0.00	0.00	0.03
	2★	14	14.00	11.62	13.00	11.31	11.62	0	0.02	0.00	0.00	0.02
	3★	14	13.50	13.00	12.38	11.62	11.85	0	0.02	0.00	0.00	0.02
	4★	14	14.00	12.38	13.00	11.85	11.55	0	0.02	0.00	0.00	0.02
	5★	13	13.00	13.00	11.88	11.55	11.36	0	0.02	0.00	0.00	0.02
	6★	14	14.00	11.88	13.50	11.36	11.82	0	0.02	0.00	0.00	0.02
	7★	14	13.50	13.50	11.38	11.82	10.96	0	0.02	0.00	0.00	0.02
	8★	15	15.00	11.38	14.50	10.96	12.15	0	0.02	0.00	0.01	0.02
	9★	15	14.50	14.50	11.25	12.15	10.80	0	0.02	0.00	0.00	0.02
	10★	16	16.00	11.25	15.50	10.80	12.47	0	0.01	0.00	0.01	0.03
6	1	10	9.33	7.00	7.50	8.90	8.93	0	0.07	0.01	0.03	0.10

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class	inst.	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{d1}	lb^{d2}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	2	10	9.60	7.50	6.83	8.93	9.20	0	0.05	0.01	0.02	0.07
	3	10	9.88	6.83	8.04	9.20	9.41	0	0.05	0.01	0.02	0.08
	4	10	9.80	8.04	7.33	9.41	9.14	0	0.05	0.01	0.02	0.07
	5	10	9.47	7.33	7.57	9.14	8.97	0	0.06	0.01	0.02	0.09
	6	10	9.60	7.57	7.67	8.97	9.38	0	0.05	0.01	0.02	0.07
	7	10	9.50	7.67	7.18	9.38	8.61	0	0.06	0.01	0.02	0.09
	8	10	9.80	7.18	7.67	8.61	9.67	0	0.05	0.01	0.02	0.07
	9	10	9.88	7.67	6.88	9.67	8.47	0	0.05	0.01	0.02	0.08
	10	11	10.10	6.88	8.00	8.47	9.96	0	0.04	0.01	0.01	0.06
7	1	9	8.94	7.00	7.87	8.90	8.63	0	0.11	0.03	0.07	0.20
	2	9	8.98	7.50	8.35	8.93	8.73	0	0.11	0.03	0.08	0.23
	3	10	9.33	6.83	8.49	9.20	9.05	0	0.10	0.02	0.06	0.18
	4	10	9.54	8.04	8.83	9.41	9.19	0	0.09	0.02	0.06	0.17
	5	10	9.33	7.33	8.32	9.14	8.88	0	0.10	0.02	0.06	0.18
	6	10	9.07	7.57	8.45	8.97	8.81	0	0.11	0.03	0.10	0.23
	7	10	9.50	7.67	8.48	9.38	9.03	0	0.08	0.01	0.04	0.13
	8	9	8.67	7.18	7.97	8.61	8.53	0	0.13	0.04	0.11	0.28
	9	10	9.83	7.67	8.77	9.67	9.29	0	0.07	0.01	0.03	0.11
	10	9	8.57	6.88	7.90	8.47	8.46	0	0.15	0.05	0.17	0.37
8	1★	13	12.50	7.00	7.87	8.90	10.06	0	0.02	0.00	0.00	0.03
	2★	13	12.50	7.50	9.95	8.93	10.07	0	0.02	0.00	0.00	0.03
	3★	13	12.50	6.83	8.10	9.20	9.82	0	0.02	0.00	0.00	0.02
	4★	13	12.50	8.04	9.42	9.41	9.58	0	0.02	0.00	0.00	0.03
	5★	13	12.50	7.33	8.10	9.14	9.83	0	0.02	0.00	0.00	0.02
	6★	13	12.50	7.57	9.92	8.97	10.05	0	0.02	0.00	0.00	0.03
	7★	13	12.50	7.67	7.43	9.38	9.54	0	0.02	0.00	0.00	0.03
	8★	13	12.50	7.18	10.58	8.61	10.43	0	0.02	0.00	0.00	0.02
	9★	13	12.50	7.67	7.65	9.67	9.23	0	0.02	0.00	0.00	0.03
	10★	13	12.50	6.88	10.11	8.47	10.61	0	0.02	0.00	0.00	0.02
9	1	7	6.06	5.75	6.00	6.00	6.00	0	0.82	0.15	1.65	2.62
	2	7	6.07	6.00	5.76	6.00	6.00	0	0.61	0.12	1.06	1.79
	3	7	6.05	5.76	6.00	6.00	6.00	0	0.83	0.16	1.76	2.74
	4	7	6.06	6.00	5.75	6.00	5.99	0	0.64	0.12	1.14	1.90
	5	7	6.06	5.75	6.00	5.99	6.00	0	0.79	0.16	1.62	2.57
	6	7	6.05	6.00	5.76	6.00	5.99	0	0.69	0.15	1.15	1.98
	7	7	6.04	5.76	6.00	5.99	6.00	0	0.80	0.17	1.57	2.54
	8	8	7.13	7.00	6.39	6.99	6.99	0	0.23	0.04	0.21	0.48
	9	8	7.12	6.39	6.87	6.99	6.99	0	0.26	0.05	0.28	0.59
	10	8	7.15	6.87	6.40	6.99	6.99	0	0.23	0.04	0.23	0.50

Table A.19: Two-constraint bin packing results for $n = 50$.

class	inst.	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{d1}	lb^{d2}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
1	1	13	12.60	11.65	10.62	12.60	11.21	0	18.91	11.24	142.83	172.98
	2	13	12.80	10.62	11.85	11.21	12.80	0	12.32	4.20	86.31	102.83
	3	13	12.80	11.85	10.61	12.80	11.20	0	16.30	7.93	156.57	180.80

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class	inst.	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{d1}	lb^{d2}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	4	13	12.76	10.61	11.82	11.20	12.76	0	12.60	4.46	88.83	105.90
	5	13	12.76	11.82	10.53	12.76	11.13	0	16.39	8.43	124.88	149.70
	6	14	13.03	10.53	12.08	11.13	13.03	0	11.93	4.16	80.64	96.73
	7	14	13.03	12.08	10.41	13.03	11.01	0	14.88	6.53	98.28	119.69
	8	14	13.12	10.41	12.31	11.01	13.12	0	11.62	3.72	75.16	90.49
	9	14	13.12	12.31	10.50	13.12	11.10	0	13.07	4.60	92.27	109.93
	10	14	13.36	13.13	10.67	13.36	11.27	0	10.24	3.04	70.18	83.46
2	1★	30	30.00	28.50	20.53	25.35	20.74	0	0.36	0.06	0.47	0.89
	2★	31	30.50	20.53	29.50	20.74	26.02	0	0.13	0.02	0.06	0.21
	3★	31	31.00	29.50	20.50	26.02	20.71	0	0.27	0.05	0.29	0.61
	4★	31	30.50	20.50	29.50	20.71	25.91	0	0.14	0.02	0.06	0.23
	5★	31	31.00	29.50	20.17	25.91	20.45	0	0.28	0.05	0.32	0.65
	6★	32	31.50	20.17	30.50	20.45	26.78	0	0.14	0.02	0.06	0.22
	7★	32	32.00	30.50	19.75	26.78	20.07	0	0.27	0.04	0.29	0.60
	8★	32	32.00	19.75	31.00	20.07	27.09	0	0.14	0.02	0.06	0.22
	9★	33	32.50	31.00	20.02	27.09	20.35	0	0.21	0.03	0.20	0.45
	10★	32	32.00	20.02	31.00	20.35	26.95	0	0.13	0.02	0.05	0.20
3	1★	30	30.00	29.50	22.00	25.20	22.42	0	0.05	0.01	0.01	0.07
	2★	31	30.50	22.00	30.50	22.42	25.60	0	0.04	0.00	0.01	0.05
	3★	31	31.00	30.50	22.00	25.60	22.41	0	0.05	0.01	0.01	0.06
	4★	31	30.50	22.00	30.50	22.41	25.53	0	0.04	0.00	0.01	0.05
	5★	31	31.00	30.50	21.75	25.53	22.25	0	0.05	0.01	0.01	0.06
	6★	32	31.50	21.75	31.50	22.25	26.06	0	0.04	0.00	0.01	0.05
	7★	32	32.00	31.50	21.50	26.06	22.02	0	0.05	0.01	0.01	0.06
	8★	32	32.00	21.50	32.00	22.02	26.24	0	0.04	0.00	0.01	0.05
	9★	33	32.50	32.00	21.75	26.24	22.19	0	0.04	0.01	0.01	0.06
	10★	32	32.00	21.75	32.00	22.19	26.16	0	0.04	0.00	0.00	0.05
6	1	21	20.30	14.17	13.21	20.16	17.70	0	0.25	0.05	0.18	0.48
	2	21	20.70	13.21	14.54	17.70	20.52	0	0.17	0.03	0.11	0.32
	3	21	20.68	14.54	13.19	20.52	17.68	0	0.22	0.04	0.15	0.41
	4	21	20.61	13.19	14.54	17.68	20.46	0	0.18	0.03	0.22	0.43
	5	21	20.61	14.54	13.04	20.46	17.54	0	0.22	0.05	0.17	0.44
	6	22	21.08	13.04	14.54	17.54	20.93	0	0.17	0.03	0.11	0.31
	7★	22	21.07	14.54	12.51	20.93	17.33	0	0.21	0.04	0.14	0.39
	8	22	21.21	12.51	14.54	17.33	21.09	0	0.17	0.03	0.10	0.30
	9	22	21.24	14.54	12.22	21.09	17.49	0	0.19	0.03	0.13	0.35
	10	22	21.13	12.22	14.45	17.49	21.01	0	0.17	0.03	0.11	0.31
7	1	21	20.27	14.17	16.76	20.16	19.58	0	0.29	0.10	0.41	0.80
	2	18	17.89	13.21	15.29	17.70	17.82	0	0.56	0.66	1.15	2.36
	3	21	20.67	14.54	17.30	20.52	19.93	0	0.27	0.10	0.37	0.74
	4	18	17.87	13.19	15.26	17.68	17.79	0	0.55	0.71	3.39	4.66
	5	21	20.67	14.54	16.74	20.46	19.84	0	0.27	0.09	0.33	0.69
	6	18	17.82	13.04	14.73	17.54	17.77	0	0.57	0.64	1.19	2.39
	7★	22	21.17	14.54	16.55	20.93	20.25	0	0.23	0.07	0.25	0.55
	8	18	17.65	12.51	14.26	17.33	17.61	0	0.58	0.70	1.70	2.97
	9	22	21.33	14.54	16.91	21.09	20.45	0	0.21	0.06	0.23	0.50
	10	18	17.80	12.22	14.02	17.49	17.74	0	0.56	0.61	1.35	2.53
8	1★	25	25.00	14.73	12.29	20.16	17.73	0	0.07	0.01	0.01	0.09
	2★	25	25.00	13.38	15.67	17.70	20.53	0	0.06	0.01	0.01	0.08
	3★	25	25.00	15.09	12.41	20.52	17.37	0	0.07	0.01	0.01	0.09

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class	inst.	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{d1}	lb^{d2}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	4★	25	25.00	13.37	15.67	17.68	20.54	0	0.06	0.01	0.01	0.08
	5★	25	25.00	15.09	12.14	20.46	17.41	0	0.07	0.01	0.01	0.09
	6★	25	25.00	13.22	15.92	17.54	20.74	0	0.06	0.01	0.01	0.08
	7★	25	25.00	15.09	11.52	20.93	16.91	0	0.07	0.01	0.01	0.09
	8★	25	25.00	12.69	16.33	17.33	20.97	0	0.06	0.01	0.01	0.08
	9★	25	25.00	15.09	11.92	21.09	16.77	0	0.07	0.01	0.01	0.09
	10★	25	25.00	12.22	16.58	17.49	20.81	0	0.06	0.01	0.01	0.08
9	1	14	13.04	12.01	12.30	12.99	12.99	0	8.68	2.81	51.44	62.93
	2	14	13.05	12.30	12.03	12.99	12.99	0	6.00	1.64	31.78	39.42
	3	14	13.04	12.03	12.31	12.99	13.00	0	7.87	2.48	45.18	55.53
	4	14	13.05	12.31	12.03	13.00	13.00	0	6.02	1.75	31.23	39.00
	5	14	13.06	12.03	12.30	13.00	13.00	0	7.73	2.54	45.00	55.27
	6★	15	14.07	13.24	12.97	13.99	13.99	0	3.11	0.75	11.78	15.64
	7	15	14.07	12.97	13.23	13.99	13.99	0	3.81	0.94	15.64	20.38
	8	15	14.07	13.23	13.12	13.99	13.99	0	3.17	0.70	11.33	15.20
	9	15	14.07	13.12	13.24	13.99	13.99	0	3.73	0.96	15.07	19.77
	10	15	14.07	13.24	13.12	13.99	13.99	0	3.14	0.69	11.28	15.11

Table A.20: Two-constraint bin packing results for $n = 100$.

class	inst.	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{d1}	lb^{d2}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
2	1★	62	62.00	53.50	60.00	49.62	55.38	0	0.53	0.09	0.72	1.34
	2★	57	57.00	49.00	52.00	47.60	50.34	0	1.93	0.33	5.29	7.55
	3★	56	56.00	52.00	49.00	50.34	47.75	0	1.47	0.28	3.78	5.54
	4★	57	57.00	49.00	51.00	47.75	49.76	0	2.35	0.41	7.48	10.24
	5★	56	56.00	51.00	49.00	49.76	47.66	0	1.48	0.30	3.78	5.55
	6★	57	57.00	49.00	51.00	47.66	50.05	0	2.36	0.43	7.44	10.23
	7★	56	56.00	51.00	49.00	50.05	47.88	0	1.47	0.30	3.65	5.42
	8★	58	58.00	49.00	51.50	47.88	50.42	0	2.29	0.42	7.36	10.07
	9★	57	57.00	51.50	49.75	50.42	48.30	0	1.26	0.24	3.16	4.66
	10★	58	58.00	49.75	51.50	48.30	50.45	0	2.02	0.36	5.69	8.06
3	1★	56	56.00	49.50	49.00	49.91	48.53	0	0.18	0.02	0.09	0.29
	2★	57	57.00	49.00	50.50	48.53	50.17	0	0.17	0.02	0.09	0.28
	3★	57	56.50	50.50	49.00	50.17	48.62	0	0.16	0.02	0.08	0.27
	4★	57	57.00	49.00	49.50	48.62	49.82	0	0.18	0.02	0.10	0.29
	5★	56	56.00	49.50	49.00	49.82	48.56	0	0.16	0.02	0.08	0.26
	6★	57	57.00	49.00	49.50	48.56	49.99	0	0.18	0.02	0.11	0.31
	7★	56	56.00	49.50	49.00	49.99	48.70	0	0.16	0.02	0.08	0.25
	8★	58	58.00	49.00	50.00	48.70	50.22	0	0.17	0.02	0.10	0.29
	9★	57	57.00	50.00	48.50	50.22	48.95	0	0.16	0.02	0.07	0.25
	10★	58	58.00	48.50	50.00	48.95	50.24	0	0.17	0.02	0.09	0.27
6	1★	41	40.26	23.45	21.59	39.90	38.68	0	0.76	0.28	1.10	2.14
	2	41	40.52	21.59	23.45	38.68	40.13	0	0.76	0.30	0.81	1.87
	3	41	40.50	23.45	21.41	40.13	38.75	0	0.73	0.25	1.04	2.02
	4★	41	40.28	21.41	23.45	38.75	39.83	0	0.83	0.33	0.81	1.96
	5★	41	40.25	23.45	20.83	39.83	38.71	0	0.75	0.28	0.77	1.79

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class	inst.	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{d1}	lb^{d2}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	6★	41	40.42	20.83	23.45	38.71	39.98	0	0.81	0.35	1.35	2.51
	7★	41	40.41	23.45	20.29	39.98	38.83	0	0.74	0.27	0.78	1.79
	8	41	40.62	20.29	23.45	38.83	40.18	0	0.78	0.30	1.04	2.12
	9	41	40.69	23.45	20.01	40.18	39.05	0	0.70	0.25	2.77	3.72
	10	41	40.75	20.01	23.46	39.05	40.19	0	0.77	0.28	2.21	3.25
7	1★	41	40.08	23.83	26.03	39.90	39.55	0	1.22	1.72	6.94	9.89
	2★	39	38.93	22.87	23.40	38.68	38.71	0	1.58	2.77	40.21	44.56
	3	41	40.35	23.83	26.03	40.13	39.81	0	1.12	1.81	7.22	10.15
	4★	39	38.97	22.70	23.40	38.75	38.71	0	1.52	2.86	24.98	29.36
	5★	41	40.03	23.83	26.27	39.83	39.49	0	1.23	1.84	3.58	6.65
	6★	39	38.94	22.10	22.35	38.71	38.70	0	1.56	3.07	46.15	50.78
	7★	41	40.20	23.83	26.27	39.98	39.67	0	1.15	1.84	5.87	8.86
	8★	40	39.10	21.57	21.88	38.83	38.87	0	1.52	2.91	3.28	7.71
	9	41	40.43	23.83	26.27	40.18	39.92	0	1.10	1.57	8.94	11.61
	10	40	39.35	20.77	22.11	39.05	39.10	0	1.52	2.65	3.94	8.11
8	1★	50	50.00	24.37	21.92	39.90	36.29	0	0.20	0.04	0.06	0.29
	2★	50	50.00	23.41	21.48	38.68	37.71	0	0.18	0.03	0.05	0.26
	3★	50	50.00	24.37	22.19	40.13	36.07	0	0.20	0.04	0.06	0.30
	4★	50	50.00	23.24	21.48	38.75	37.59	0	0.18	0.03	0.05	0.26
	5★	50	50.00	24.37	22.19	39.83	36.37	0	0.20	0.04	0.06	0.30
	6★	50	50.00	22.65	21.48	38.71	37.66	0	0.18	0.03	0.05	0.26
	7★	50	50.00	24.37	22.05	39.98	36.23	0	0.19	0.03	0.06	0.29
	8★	50	50.00	22.12	21.48	38.83	37.57	0	0.17	0.03	0.05	0.25
	9★	50	50.00	24.37	22.05	40.18	36.06	0	0.20	0.04	0.06	0.29
	10★	50	50.00	21.83	21.71	39.05	37.34	0	0.18	0.03	0.05	0.26

Table A.21: Two-constraint bin packing results for $n = 200$.

class	inst.	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{d1}	lb^{d2}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
6	1★	81	80.35	29.29	29.94	79.35	79.19	0	3.53	4.38	22.20	30.11
	2★	81	80.50	29.53	28.95	79.19	79.36	0	3.44	4.60	7.61	15.66
	3★	81	80.41	29.29	29.94	79.36	79.30	0	3.51	4.04	7.18	14.73
	4★	81	80.52	29.53	28.95	79.30	79.29	0	3.44	4.82	9.59	17.85
	5★	81	80.21	29.29	29.35	79.29	78.93	0	3.49	4.22	7.30	15.01
	6★	81	80.67	28.95	28.95	78.93	79.74	0	3.43	4.87	83.22	91.52
	7★	81	80.49	29.29	29.35	79.74	78.83	0	3.52	4.33	7.27	15.12
	8	81	80.84	28.95	28.95	78.83	80.05	0	3.36	5.29	33.05	41.71
	9	81	80.84	29.29	29.07	80.05	79.07	0	3.39	4.02	85.49	92.89
	10★	82	81.25	28.66	28.95	79.07	80.45	0	3.25	4.63	7.86	15.74
7	1	80	79.55	29.49	34.33	79.35	79.15	0	4.52	10.31	300.34	315.17
	2	80	79.37	31.53	30.53	79.19	79.03	0	4.97	11.83	152.64	169.43
	3	80	79.57	29.49	34.33	79.36	79.19	0	4.53	10.51	222.15	237.18
	4	80	79.48	31.53	30.53	79.30	79.13	0	4.97	12.41	244.41	261.79
	5	80	79.49	29.49	34.33	79.29	79.03	0	4.51	11.53	250.51	266.55
	6★	80	79.14	31.16	30.53	78.93	78.87	0	4.86	13.34	17.36	35.57
	7★	80	79.95	29.49	34.33	79.74	79.45	0	4.32	10.72	59.31	74.36

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class	inst.	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{d1}	lb^{d2}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	8★	80	79.07	31.16	30.53	78.83	78.85	0	4.90	14.07	159.18	178.15
	9	81	80.29	29.49	34.33	80.05	79.81	0	4.13	9.52	275.89	289.54
	10	80	79.38	30.87	30.53	79.07	79.19	0	4.76	11.50	139.04	155.29
8	1★	100	100.00	30.90	29.86	79.35	73.23	0	0.56	0.15	0.25	0.96
	2★	100	100.00	33.61	31.44	79.19	73.42	0	0.50	0.13	0.20	0.83
	3★	100	100.00	30.90	29.86	79.36	73.23	0	0.57	0.16	0.24	0.97
	4★	100	100.00	33.61	31.44	79.30	73.29	0	0.48	0.12	0.19	0.80
	5★	100	100.00	30.90	30.15	79.29	73.25	0	0.58	0.16	0.20	0.94
	6★	100	100.00	32.81	31.44	78.93	73.72	0	0.50	0.13	0.17	0.80
	7★	100	100.00	31.29	30.15	79.74	72.79	0	0.56	0.16	0.22	0.94
	8★	100	100.00	32.81	31.44	78.83	73.85	0	0.50	0.13	0.20	0.83
	9★	100	100.00	31.79	30.43	80.05	72.52	0	0.56	0.15	0.24	0.95
	10★	100	100.00	31.98	31.44	79.07	73.67	0	0.48	0.13	0.18	0.80

Table A.22: Two-constraint bin packing results on the class 10.

n	inst.	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{d1}	lb^{d2}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
24	1	8	8.00	6.49	6.23	8.00	8.00	0	0.06	0.01	0.02	0.09
	2	8	8.00	6.12	5.88	8.00	8.00	0	0.06	0.01	0.02	0.09
	3	8	8.00	6.23	6.55	8.00	8.00	0	0.06	0.01	0.02	0.09
	4	8	8.00	5.88	5.81	8.00	8.00	0	0.06	0.01	0.02	0.09
	5	8	8.00	6.55	6.46	8.00	8.00	0	0.06	0.01	0.02	0.09
	6	8	8.00	5.81	6.19	8.00	8.00	0	0.07	0.01	0.02	0.09
	7	8	8.00	6.46	6.99	8.00	8.00	0	0.06	0.01	0.02	0.09
	8	8	8.00	6.19	5.82	8.00	8.00	0	0.06	0.01	0.02	0.09
	9	8	8.00	6.99	6.43	8.00	8.00	0	0.07	0.01	0.02	0.10
	10	8	8.00	5.82	5.92	8.00	8.00	0	0.07	0.01	0.02	0.10
51	1	17	17.00	9.14	9.31	17.00	17.00	0	0.27	0.10	0.15	0.52
	2	17	17.00	8.36	8.24	17.00	17.00	0	0.27	0.11	0.15	0.53
	3	17	17.00	9.74	9.54	17.00	17.00	0	0.27	0.07	0.14	0.48
	4	17	17.00	8.24	8.74	17.00	17.00	0	0.26	0.07	0.12	0.45
	5	17	17.00	9.28	9.58	17.00	17.00	0	0.28	0.10	0.15	0.53
	6	17	17.00	8.74	9.07	17.00	17.00	0	0.30	0.13	0.15	0.58
	7	17	17.00	9.92	9.28	17.00	17.00	0	0.27	0.07	0.14	0.48
	8	17	17.00	9.07	8.81	17.00	17.00	0	0.26	0.08	0.13	0.46
	9	17	17.00	9.28	9.47	17.00	17.00	0	0.29	0.11	0.17	0.57
	10	17	17.00	8.81	9.07	17.00	17.00	0	0.30	0.13	0.15	0.58
99	1	33	33.00	11.39	11.40	33.00	33.00	0	1.03	1.69	0.91	3.63
	2	33	33.00	11.02	11.56	33.00	33.00	0	1.05	1.92	1.26	4.23
	3	33	33.00	11.57	11.77	33.00	33.00	0	0.99	1.28	0.86	3.13
	4	33	33.00	11.11	10.61	33.00	33.00	0	1.03	1.22	0.62	2.87
	5	33	33.00	11.39	11.29	33.00	33.00	0	0.99	1.47	1.13	3.59
	6	33	33.00	10.29	11.20	33.00	33.00	0	1.09	2.09	1.47	4.65
	7	33	33.00	11.46	11.77	33.00	33.00	0	0.94	1.21	0.76	2.91
	8	33	33.00	11.29	10.61	33.00	33.00	0	1.02	1.12	0.70	2.84
	9	33	33.00	11.39	11.97	33.00	33.00	0	0.97	1.56	0.87	3.40

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n	inst.	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{d1}	lb^{d2}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	10	33	33.00	10.29	10.88	33.00	33.00	0	1.11	1.98	1.17	4.26
201	1	67	67.00	14.55	15.68	66.81	67.00	24	3.99	16.80	104.67	125.46
	2	67	67.00	12.81	14.51	66.96	67.00	530	3.98	15.02	131.74	150.74
	3	67	67.00	13.55	14.32	66.86	67.00	32	3.57	15.57	92.46	111.61
	4	67	67.00	13.28	13.03	66.68	67.00	0	3.99	11.58	34.05	49.62
	5	67	67.00	14.26	16.11	66.95	67.00	198	3.94	17.44	116.98	138.35
	6	67	67.00	12.85	14.17	66.81	67.00	22	4.09	13.98	238.77	256.83
	7	67	67.00	13.99	14.40	66.80	67.00	0	4.00	17.99	39.14	61.12
	8	67	67.00	13.52	13.03	66.84	67.00	0	4.06	13.45	30.79	48.31
	9	67	67.00	13.95	16.11	66.93	67.00	75	3.96	18.84	110.01	132.82
	10	67	67.00	12.85	14.17	66.76	67.00	0	4.20	14.06	107.21	125.46

★ - previously open instance