An improved formulation of the underground mine scheduling optimization problem when considering selective mining

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Abstract The use of mixed integer programming is a modeling approach well suited to formulate the mine scheduling optimization problem for both open pit and underground mining. The resolution applied for discretizing the problem, however, has a direct effect on both the level of selectivity that can be applied to improve profitability, as well as the computational feasibility. The proposed model allows for a balance in reducing the resolution used in discretizing the underground mine scheduling problem, while maintaining enough detail that will allow the generation of mine production schedules that improve profitability through selective mining. As a secondary contribution, an improved formulation set within a resource production/consumption framework is presented, which can potentially simplify notation used in formulating underground mine scheduling optimization problems.

Keywords: Underground mining; integer programming; aggregation

1 Introduction

The method of extracting valuable minerals from the earth is dependant on the characteristics of the mineral deposits, referred to as the ore body. The two primary methods of mining are open pit mining and underground mining. For the latter, specifically in deep underground gold mining, the ore body is recognized to be a thin sheet or layer-like deposit referred to as a reef, which in some cases are just a few centimeters thick. Depending on its inclination with the surface, a reef could reach depths of up to several kilometers. Open pit mining makes economically more sense when the ore body is more concentrated and closer to the surface.

Irrespective of the mining method, the tasks of planning and executing mining activities are very complex. The optimal use of resources and the timing of production activities could have an enormous effect on the profitability of a mining operation. Furthermore, since the life of a mine can be as long as 60 years, long term planning is essential to make the right investment choices along the
way in order to fully unlock the value of an ore body. The use of mine planning systems to aid decision making is therefore imperative. Specifically, the optimal design of the mine layout and the optimal scheduling of mining activities while taking resource constraints and economic factors into account, are the main driving forces for continued research into realistic mathematical models and efficient solution approaches.

In recent years the use of project scheduling techniques within mining has been applied effectively using readily available project scheduling software tailored for the mining environment. Some of these systems, however, were previously not capable of producing schedules that would maximize a predefined objective function such as Net Present Value (NPV) while still satisfying production constraints. Therefore, the term being used in this study namely, mine scheduling optimization, refers to the generation of an optimal schedule or plan for executing mining activities in future, by taking activity precedence and other side constraints, e.g. capacity constraints, as input.

The use of mixed integer programming (MIP) is a modeling approach well suited to formulate the mine scheduling optimization problem. Compared to open pit applications (see e.g. [14], [4], [2] and [1]), the work done on underground mine scheduling problems is limited. Earlier references to the use of MIP formulations can be found in [5] and [13], although no model formulations were provided in these two papers. In the paper by [11] a Benders decomposition approach is followed to improve the solution time of solving the MIP problem for small to medium sized randomly generated problem instances. Also in an attempt to improve computing times, the approach followed by both [8] and [9] is to aggregate production activities that follow a natural continuous sequence which results in a reduced number of variables. Similarly, the approach by [10] is to heuristically generate feasible solutions by aggregating time periods. By doing this improvements on computing times are achieved but to the detriment of optimality.

The approach proposed in this paper is related to the work by [8], [9] and [10], in the sense that the number of variables in our formulation is reduced by introducing a lower time period resolution while maintaining enough information to enable the optimization model to boost profits through selective mining. As a secondary contribution, a generic formulation of the mine scheduling optimization problem cast within a resource production/consumption framework is introduced, with the purpose of simplifying notation.

The model presented in this study is applicable to underground selective mining where the ore grade is highly variable and by selecting out only high grade areas to mine a reduction in cost can be achieved, thus improving profitability. The scope of this work is not limited to a specific mineral.

2 Technical aspects of underground mining

Figure 1 is a simplified illustration of a typical layout of an underground mine. The main vertical tunnel is called the *shaft* from which horizontal tunnels are
excavated to give access to the ore body. The layers of tunnels at different depths are referred to as levels and on each level several smaller tunnels, called raise lines, give access to the valuable minerals along the ore body. Blocks of ore-bearing rock alongside raise lines and in between two levels, are called stoping blocks and are demarcated into smaller pieces to form stoping panels. The activity of excavating stoping panels is referred to as stoping whereas the excavation of tunnels giving access to the ore body is referred to as development. We differentiate between off-reef development and on-reef development where the latter refers to the excavation of tunnels (e.g. raise lines) within the ore body with the result that some minerals are also mined out but with a high dilution factor. Off-reef development, on the other hand, refers to the excavation of tunnels through waste rock to give access to the ore body.

Different mining methods can be applied for the excavation of stoping panels. For the purpose of this paper reference is made to two of the most commonly used mining methods in shallow dip reef mining, namely, sequential and pillar mining. For a more complete reference on different mining methods see [6]. Figure 2 illustrates the differences between sequential and pillar mining. For the latter, parts of the stoping block are left behind as pillars for safety purposes, whereas sequential mining would result in clearing out the entire stoping block. The step-like pattern maintained in sequential mining is due to safety reasons.

The schedule optimization problem for underground mining boils down to deciding when to execute a specific mining activity in future, where an activity could relate to e.g. the excavation of part of an underground tunnel per unit time or the placement of machinery that will enable the excavation. This is done for all activities in order to maximize the NPV of the project while taking into account constraints that relate to the physical infrastructure of the mine, e.g. hoisting capacity, as well as resource constraints such as the available labor force at any given time. The most important set of constraints, however, has to do with activity precedence. That is, except for the very first mining activity that would be executed in the mine plan, the execution of all other mining activities will depend on whether their predecessors activities have been completed. For this purpose we rely on a precedence graph that is constructed as input to our optimization model. The structure of such a precedence graph will ultimately depend on the mining method employed. Figure 3 shows the precedence graph (implied by the arrows) for simulating a sequential mining method. Notice that in order to get the step-wise effect of the sequential mining method, the first activity of the second panel (row) is only allowed to start once the second activity of the first panel is completed, etc.

For a short term plan the unit of time would typically be a month or even a week, whereas for a long term plan a unit of time could be for instance a year. Clearly, with an increase in time period resolution an increase in computing times could be expected due to an increase in the number of variables that have to be considered.

In this paper it is not the intention to model the underground mine production problem into the finest detail, nor is the focus on specific mining methods
Figure 1. A typical underground mine layout
Figure 2. Sequential vs. Pillar mining (adapted from [7])

Figure 3. Precedence graph for sequential mining
that would result in very specific side constraints. The objective of this study is to provide a generic formulation that has the primary constraint ingredients such as activity precedence, but that gives a good trade-off between reducing time period resolution and maintaining enough information that would allow selective mining based on the grade variability of the ore body.

3 Model formulation and notation

The underground mine scheduling optimization problem is formulated in this paper as a MIP. In the section below the proposed formulation is set within a generic resource production/consumption framework in an attempt to simplify notation. Thereafter, amendments to the formulation are proposed that will result a reduction in the number of variables, but at the same time will maintain enough information to allow the simulation of selective mining.

3.1 The resource model

The basic idea of the resource production/consumption framework is that we view each activity as either being resource consuming or resource producing or both. By creating a set of different resources which are either produced (e.g. minerals) or consumed (e.g. labor hours), we can simplify the notation used for the problem formulation:

- Let \( R \) denote the set of resources. A potential set of resources could e.g. include the number of tonnes from stoping production, the amount of minerals produced through processing the ore, the amount of explosives consumed for blasting, the number of labor hours consumed for excavation, etc. Note that a resource \( r \in R \) is not an actual numerical value but rather the name of a specific resource.
- Let \( A \) denote the index set of all activities. An activity \( a \in A \) could relate to e.g. the excavation of part of an underground tunnel per unit time or the placement of machinery that will enable the excavation, etc. Note that the size of the set \( A \) is a function of the time period resolution used in the discretization of the problem. For instance, for a high resolution discretization where each activity \( a \in A \) relate to a task being performed within, say, a single week, the size of \( A \) would be considerably larger compared to a lower resolution discretization where each activity \( a \in A \) relate to a task being performed within, say, a month.
- Let \( (a - 1) \) denote the predecessor activity of \( a \in A \). For the purpose of simplifying the presentation of our approach, we assume that each activity will only have a single predecessor.
- Let \( A(r) \) denote the set of activities that either consume or produce the resource \( r \in R \).
- Let \( \delta_{ra} \geq 0 \) be a numerical value for the quantity of resource \( r \in R \) being produced/consumed by activity \( a \in A \). Note that this quantity is dependent on the time period resolution and would be determined as part of a
preprocessing step. If we consider a constant rate at which activity \( a \in A \) is being performed, an increase in the time period size would result in a linear increase in \( \delta_{ra} \).

- Let \( T = \{1, 2, \ldots, |T|\} \) denote the time period indices.
- Let \( (t - 1) \) denote the predecessor time period for \( t \in T \).
- Let \( c_{rt} \) be the cost per unit of consuming/producing a resource \( r \in R \) in a time period \( t \in T \). Note that the coefficient \( c_{rt} \) could either be negative or positive, depending on the type of resource. For instance, if we have a maximization problem, the coefficient associated with a resource that denotes the tonnes of rock from stoping production would be negative, since the production of the resource would incur costs. However, the coefficient associated with a resource that denotes, e.g., the amount of minerals, would be treated as positive since revenue is incurred by the production of this resource.
- Let \( p \) denote the time period resolution parameter expressed as the number of months contained within a single period. That is, if \( p = 1 \) we are considering a monthly calendar with each period \( t \in T \) being exactly one month and if we, for example, let \( p = 12 \) then we are considering an annual calendar where each period \( t \in T \) is taken to be one year.
- Let \( U_{rt} \) be the upper limit on the quantity of resource \( r \in R \) that may be consumed/produced for time period \( t \in T \) expressed as a value per month. Generalization of \( U_{rt} \) to other time period resolutions is obtained by multiplying with the parameter \( p \).
- Let \( L_{rt} \) be the lower limit on the quantity of resource \( r \in R \) that may be consumed/produced for time period \( t \in T \) expressed as a value per month. Generalization of \( L_{rt} \) to other time period resolutions is obtained by multiplying with the parameter \( p \).
- Let \( d(p) \) be the effective NPV rate, for a period of size \( p \), at which future cash-flows will be discounted with. This will ensure that the discounting is done according to the correct period sizes when we compare schedules generated with different time period resolutions.

As a result of formulating our problem within a resource production/consumption framework, it is not necessary to distinguish between different types of mining activities when defining the decision variables. The use of the parameters \( \delta_{ra} \) will translate the decision to execute the activity \( a \in A \) into a measurable quantity related to the resource \( r \in R \). Consequently, if we define the variable \( z_{at} \in \{0, 1\} \) to take on a value of one if the mining activity \( a \in A \) is scheduled to be executed in time period \( t \in T \), we obtain the following binary integer programming problem referred to as the Resource based Mine Scheduling Optimization Problem (RMSOP):
\[
\max \sum_{t \in T} (1 + d^p)^{-t} \sum_{r \in R} \sum_{a \in A(r)} c_{rt} \delta_{ra} z_{at} \tag{1}
\]

s.t.
\[
z_{at} \leq \sum_{\substack{k \in T \\k < t}} z_{(a-1)k} \forall a \in A, \forall t \in T \tag{2}
\]
\[
\sum_{a \in A} z_{at} \leq 1 \forall t \in T \tag{3}
\]
\[
pL_{rt} \leq \sum_{a \in A(r)} \delta_{ra} z_{at} \leq pU_{rt} \forall r \in R, \forall t \in T \tag{4}
\]

For the above formulation, the objective function (1) would maximize NPV at a discount rate of \(d^p\), provided that we have defined appropriate cost coefficients \(c_{rt}\) for each of the relevant resources \(r \in R\) and for each time period \(t \in T\). For example, if we define the resource \(r^1 = \text{MINERAL} \in R\) to denote the kilograms of minerals produced, then the corresponding coefficient \(c_{r^1t}\) should reflect the mineral prices per kilogram as positive for the resource \(r^1\) in time period \(t \in T\). If we define the resource \(r^2 = \text{STOPE} \in R\) to denote the tonnes of stoping rock produced, then the corresponding coefficient \(c_{r^2t}\) should reflect the stoping cost per tonne as negative for the resource \(r^2\) in time period \(t \in T\).

The constraint set (2) from the above formulation enforces the precedence relationship. An activity \(a \in A\) can only be performed once its predecessor \((a-1) \in A\) has been completed. The constraint set (3) will only allow an activity to be scheduled once. Note that the inequality sign in (3) enables the model to simulate selective mining since an activity is allowed not to be scheduled at all. The constraint set (4) provides upper and lower limits on the consuming and producing of resources respectively. From an implementation point of view, the benefit of having these generic constraints is that we can easily add limits on resource consumption/production by simply adding the said resources to \(R\) and by specifying the limits \(L_{rt}\) and \(U_{rt}\), without having to explicitly define additional constraint for the model.

An important property of the RMSOP formulation is that computational complexity worsens with an increase in the time period resolution. That is, by making the time periods smaller, e.g. from a monthly calendar to a weekly calendar, we increase the number of variables in the model. On the positive side, by increasing the time period resolution, selectivity is improved due to more detail that becomes available by discretizing the problem into smaller activity pieces. That is, instead of scheduling a monthly activity for mining with a diluted grade, we can reduce the period size to a week and only schedule part of the same activity that coincides with a higher grade, leaving behind the low grade portion. Therefore, a clear trade-off exist between more information that will allow improved selective mining vs. a worsening of computing times as a result of increasing time period resolution.
3.2 A preprocessing step for variable reduction

The preprocessing performed on the RMSOP in order to reduce the number of variables is based on the approach described in [10]. The basic idea is that each activity has an earliest possible starting time and a latest possible finishing time. This is determined by the chain of predecessors and successors to the activity. For example, if an activity \( a^4 \in A \) has the chain of predecessor activities \( \{ a^3, a^2, a^1 \} \) where \( a^3 \) is the immediate predecessor of \( a^4 \), \( a^2 \) is the immediate predecessor of \( a^3 \) and \( a^1 \) is the immediate predecessor of \( a^2 \), then activity \( a^4 \) can start at the earliest in the fourth period if each of the predecessors in the chain are scheduled to be performed in each of the preceding time periods. By having an earliest starting time \( t^0_a \in T \) and a latest finishing time \( t^1_a \in T \) for each activity \( a \in A \), we can remove all the variables \( z_{at} \) for all \( t < t^0_a \) and \( t > t^1_a \) from the problem formulation.

3.3 Selectivity and mining methods

The application of selectivity in mining is motivated by high variability in grade. Leaving low grade stoping areas behind leads to improved profitability. However, the mining method employed could limit the level of selectivity applied. By considering the allowable sequencing for sequential mining depicted in Figure 3, it is clear that selectivity is spatially constrained. That is, by not scheduling the stoping activities in the south-east corner, we are also forced to leave behind the rest of the stoping block. Consequently, we will mostly benefit from selectivity within a sequential mining context when stoping activities in the north-western area of a stoping block having a lower grade are left behind and not scheduled to be mined.

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<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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Table 1. Example data for creating a spatial grade tonnage curve for monthly stoping activities when considering a sequential mining method.

Central to the practice of selective mining is the concept of a grade tonnage curve which has been used for decades by mining practitioners. It represents the
relationship between the level of selectivity and the expected grade. This relationship is represented as a cumulative graph, obtained by ranking demarcated ore blocks according to their grade in descending order, cumulatively adding their volume and plotting the cumulative volume vs. the average grade over the accumulated blocks. This method does not, however, take the mining method into account and assumes the effect of cherry picking whereby high grade portions can be removed even though the mining method may not permit it. For our purpose we therefore make use of a \textit{mining-method-dependent grade tonnage curve} (henceforth MMD grade tonnage curve), which is surprisingly not very often used in mine planning. To the best of our knowledge the only reference available on the topic is \cite{12}. The construction of an MMD grade tonnage curve is illustrated by means of example data listed in Table 1, where we consider a single stoping block comprising 10 stoping panels being mined over a period of 12 months.

![Figure 4](image)

\textbf{Figure 4.} an MMD grade tonnage curve created with example data from Table 1

The entries for the matrix formed by the period vs panel intersection, reflects the mineral content obtained by mining part of a panel in a specific time period. That is, the entries correspond to the mineral content associated with each stoping activity. Furthermore, from Table 1 we can see that mining of the second panel only starts one period after the first panel, due to the sequential mining method adopted. The column on the right-hand side of the table labeled \textit{Cumulative content (kg)} is obtained by summing the mineral content for each period over all the panels. Since we assumed for this example that all stoping activities are of the same dimension, i.e. they all correspond to the excavation
of exactly the same tonnages per time period, the column on the far right-hand side labeled *Cumulative tonnages (t)* is obtained by summing the tonnages for each period over all the panels.

The MMD grade tonnage curve depicted in Figure 4 is obtained by plotting the columns *Cumulative tonnages (t)* vs *Cumulative content (kg)*. From the graph there is a clear change in the slope of the function as mining advances through the stoping block, implying that profitability could be improved by leaving behind a portion of the stoping block due to a decline in grade. The benefit of having such a summarized view of the stoping block is that instead of having variables that relate to each of the activities that could be scheduled in one of many alternative periods, we could simply introduce a single variable that will denote the extraction of part of the stoping block.

### 3.4 The low resolution resource model with micro selectivity

The use of an MMD grade tonnage curve introduced above facilitates the formulation of the mine scheduling optimization problem with a lower time period resolution, resulting in a reduced number of activities and eventually a reduced number of variables. However, since the MMD grade tonnage curve is constructed out of a higher resolution discretized problem, enough information is taken into account to allow selectivity on a micro level.

In summary, the approach is to first discretize our problem into, say, monthly activities. The monthly stoping activities are then grouped by stoping blocks and an MMD grade tonnage curve is then constructed for each stoping block by using the monthly stoping activities as was illustrated in Table 1. It should be noted that the monthly discretization should be performed without taking any capacity constraints into account such that the only factors influencing the shape of the MMD grade tonnage curves would be the mining method and the grade distribution. The next step is to discretize our problem according to a lower time period resolution, e.g. by using annual activities. These activities will be used as the set $\mathcal{A}$ in our problem formulation. The last step is then to associate the annual stoping activities with the stoping blocks such that we can map several annual stoping activities to a single MMD grade tonnage curve. Note that in order to maintain linearity we approximate each MMD grade tonnage curve with a piece-wise linear approximation. Figure 5 illustrates the approximation of an MMD grade tonnage curve using three line segments. The points $(x_0, y_0), (x_1, y_1), (x_2, y_2)$ and $(x_3, y_3)$ are called the knots of the piece-wise linear function. In order to formulate this into our model we require the following notation:

- Let $\mathcal{B}$ denote the set of all stoping blocks.
- Let $\mathcal{A}(b)$ denote the set of stoping activities associated with a stoping block $b \in \mathcal{B}$.
- Let the points $(x_{bi}, y_{bi})$, $i \in \mathcal{I} = \{0, 1, 2, \ldots, N-1\}$ be the knots for the piece-wise linear approximation of the MMD grade tonnage curve associated with the stoping block $b \in \mathcal{B}$. 
To incorporate the MMD grade tonnage curve into our problem formulation we introduce the auxiliary variables $x_b \geq 0$, denoting the tonnes of the stoping block $b \in B$ that will be extracted and $y_b \geq 0$ denoting the corresponding mineral content that will be produced according to the MMD grade tonnage curve associated with stoping block $b \in B$. To account for the mineral content and tonnes extracted per period, the variables $x_{at} \geq 0$ and $y_{at} \geq 0$ are defined such that $x_b = \sum_{a \in A(b)} \sum_{t \in T} x_{at}$ and $y_b = \sum_{a \in A(b)} \sum_{t \in T} y_{at}$ for all $b \in B$. To align these newly introduced auxiliary variables with the proposed generic resource based framework, the subset of resources $R^s \subseteq R$ is introduced which is exclusively being associated with stoping activities. For the subsequent formulations, we denote $r^1 \in R^s$ to be the resource associated with the kilograms of minerals produced and $r^2 \in R^s$ as the resource associated with the tonnes of stoping rock produced. By associating the variable $y_{at}$ with the resource $r^1$ and the variable $x_{at}$ with the resource $r^2$, we will be able to correctly calculate the revenue and costs respectively. These associations are established through the use of the quantities $\delta_{r^1a}$ and $\delta_{r^2a}$ denoting the actual kilograms of minerals produced and the tonnes of stoping rock produced by executing activity $a \in A$, respectively. The coefficients $c_{r^1t}$ is defined to reflect the (positive) mineral prices and $c_{r^2t}$ to reflect the (negative) stoping production cost, for all $t \in T$.

The modeling technique employed here to represent the MMD grade tonnage curves as piece-wise linear approximations (see [3] for a detailed description) requires the introduction of the auxiliary variables $\lambda_i \geq 0$, with $i \in I$ and $l_j \in \{0, 1\}$, with $j \in I \setminus \{0\} = \{1, 2, \ldots, N - 1\}$. The latter is for selecting the most appropriate line segment for local approximation with respect to the objective
function, whereas the former is needed to express the decision variables $x_b$ and $y_b$ as convex combinations of the knots $(x_{bi}, y_{bi}), i = 0, 1, \ldots, N - 1$.

The formulation of the RMSOP is adapted to obtain the Low Resolution model with Micro Selectivity (LRMS):

$$
\max_{t \in \mathcal{T}} \sum_{t \in \mathcal{T}} (1 + d^{(p)})^{-t} \left\{ \sum_{r \in \mathcal{R}\setminus \mathcal{R}^s} \sum_{a \in \mathcal{A}(r)} c_{rt} \delta_{ra} z_{at} + \sum_{a \in \mathcal{A}(r^1)} c_{rt^1} y_{at} + \sum_{a \in \mathcal{A}(r^2)} c_{rt^2} x_{at} \right\}
$$

s.t.

$$
z_{at} \leq \sum_{k \in \mathcal{T}, k < t} z_{(a-1)k} \quad \forall a \in \mathcal{A}, \forall t \in \mathcal{T} \tag{6}
$$

$$
\sum_{a \in \mathcal{A}} z_{at} \leq 1 \quad \forall t \in \mathcal{T} \tag{7}
$$

$$
p_{Lrt} \leq \sum_{a \in \mathcal{A}(r)} \delta_{ra} z_{at} \leq p_{Urt} \quad \forall r \in \mathcal{R}, \forall t \in \mathcal{T} \tag{8}
$$

$$
y_{at} \leq \delta_{r^1a} \sum_{k \in \mathcal{T}, k \leq t} z_{ak} \quad \forall b \in \mathcal{B}, \forall a \in \mathcal{A}(b), \forall t \in \mathcal{T}, \ r^1 \in \mathcal{R}^s \tag{9}
$$

$$
x_{at} \leq \delta_{r^2a} \sum_{k \in \mathcal{T}, k \leq t} z_{ak} \quad \forall b \in \mathcal{B}, \forall a \in \mathcal{A}(b), \forall t \in \mathcal{T}, \ r^2 \in \mathcal{R}^s \tag{10}
$$

$$
\delta_{r^1(a-1)} z_{at} \leq \sum_{k \in \mathcal{T}, k < t} y_{(a-1)k} \quad \forall b \in \mathcal{B}, \forall a \in \mathcal{A}(b), \forall (a-1) \in \mathcal{A}(b), \forall t \in \mathcal{T}, \ r^1 \in \mathcal{R}^s \tag{11}
$$

$$
x_b = \sum_{a \in \mathcal{A}(b)} \sum_{t \in \mathcal{T}} x_{at} \quad \forall b \in \mathcal{B} \tag{12}
$$

$$
y_b = \sum_{a \in \mathcal{A}(b)} \sum_{t \in \mathcal{T}} y_{at} \quad \forall b \in \mathcal{B} \tag{13}
$$

$$
x_b = \sum_{i \in \mathcal{I}} \lambda_{ib} x_b \quad \forall b \in \mathcal{B} \tag{14}
$$

$$
y_b = \sum_{i \in \mathcal{I}} \lambda_{ib} y_b \quad \forall b \in \mathcal{B} \tag{15}
$$

$$
\sum_{i \in \mathcal{I}} \lambda_{ib} = 1 \quad \forall b \in \mathcal{B} \tag{16}
$$

$$
\lambda_{0b} \leq l_{1b} \quad \forall b \in \mathcal{B} \tag{17}
$$

$$
\lambda_{ib} \leq l_{(i+1)b} + l_{i(b)} \quad \forall b \in \mathcal{B}, \forall i \in \mathcal{I} \setminus \{0, N - 1\} \tag{18}
$$

$$
\lambda_{(N-1)b} \leq l_{(N-1)b} \quad \forall b \in \mathcal{B} \tag{19}
$$
The objective function (5) comprises the costs associated with non-stopping activities indexed through the resource set $\mathcal{R} \setminus \mathcal{R}^*$ and revenues (costs) associated with the stoping activities indexed through the resource $r^1 \in \mathcal{R}^*$ ($r^2 \in \mathcal{R}^*$). Note that the second and third terms in the objective function do not need the multipliers $\delta_{ra}$ for the variables $y_{at}$ and $x_{at}$, since these variables already reflect the mineral content and stoping tonnages respectively, obtained through the MMD grade tonnage curve relationship.

The constraint sets (6), (7) and (8) are exactly the same as the constraint sets (2), (3) and (4) from the RMSOP and will have the same purpose within the LRMS formulation. The constraint set (9) is responsible for allowing the mineral content variables $y_{at}$ to take on a value only if the corresponding activity has been scheduled to be executed. Furthermore, these constraints will also limit the variables $y_{at}$ to a level of $\delta_{r^1 a}$ to maintain the proportionality of the activity $a \in \mathcal{A}(b)$ with respect to the stoping block $b$. Analogous to this we have the constraint set (10) to govern the values that the variables $x_{at}$ may take on relative to the activity variables $z_{at}$. The purpose of the constraint set (11) is to allow an activity to be executed only if its predecessor as been completed, i.e. if the mineral content variable $y_{(a-1)t}$ has reached its limit $\delta_{a(a-1)}$. The constraint sets (12) and (13) aggregate the time dependent variables $x_{at}$ and $y_{at}$ to the stoping block totals $x_b$ and $y_b$. The constraint sets (14) and (15) in turn express these stoping block totals as convex combinations of the piece-wise linearization knots of the MMD grade tonnage curve. Evidently, the constraint set (16) maintains the necessary convexity conditions and the constraint sets (17), (18) and (19) are responsible for enabling the appropriate convexity variable $\lambda_{ib}$ to take on a value based on the selection of a specific line segment $l_{ib}$.

An important feature of the LRMS model, compared to the RMSOP model, is that precedence relationships might be distorted during aggregation. Specifically, in cases where branching occur within the precedence of a high resolution discretization, it might be that for the corresponding low resolution discretization the branches are now either allowed to be executed during the same period as its predecessor or much later after the total aggregation has been completed, depending on the aggregation policy applied.

## 4 Empirical results

Empirical tests were performed to determine whether the LRMS model would improve on computing times by considering a lower time period resolution, while enabling micro selectivity through the use of MMD grade tonnage curves. The data used were generated randomly for a range of problem instances with a biased grade distribution that will favor selective mining. That is, the grade tonnage curves were created to reflect a decrease in grade as excavation advances through the stoping block over time making it economically more attractive to be selective. Note, however, that selective mining becomes less attractive with an increase in mineral price (provided constant costs) due to improved profit.
margins. The interplay between the level of selectivity and increasing mineral prices were also examined as part of this empirical study.

<table>
<thead>
<tr>
<th>Problem instance Resolution Blocks</th>
<th>Mineral price (x1) N / P / gap(%)</th>
<th>Mineral price (x1.25) N / P / gap(%)</th>
<th>Mineral price (x1.5) N / P / gap(%)</th>
<th>Mineral price (x1.75) N / P / gap(%)</th>
<th>Mineral price (x2) N / P / gap(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly (RMSOP)</td>
<td>10 2E+07 2.8 7.4E+07 1.8 1.3E+08 2.5 1.9E+08 1.1 2.7E+08 0.7</td>
<td>20 2E+07 12.9 6.6 1.1E+08 6.4 1.7E+08 5.4 4.0E+08 2.4 5.6E+08 3.3</td>
<td>30 1E+08 23.5 11.6 2.5E+08 8.4 4.1E+08 8.4 6.0E+08 4.2 8.2E+08 2.3</td>
<td>40 2E+08 25.8 14.1 3.4E+08 11.6 5.4E+08 11.6 7.8E+08 7.0 1.1E+09 3.8</td>
<td>50 2E+08 29.2 17.5 4.3E+08 14.5 6.7E+08 14.5 9.5E+08 9.8 1.3E+09 5.6</td>
</tr>
<tr>
<td>Annually (RMSOP)</td>
<td>10 0 0.0 700683 0.0 6.1E+07 0.0 1.4E+08 0.0 2.2E+08 0.0</td>
<td>20 0 0.0 1.8E+07 0.0 1.3E+08 0.0 2.8E+08 0.0 4.4E+08 0.0</td>
<td>30 0 0.0 2.8E+07 0.0 1.9E+08 0.0 4.0E+08 0.0 6.3E+08 0.0</td>
<td>40 0 0.0 3.3E+07 0.0 2.4E+08 0.0 5.1E+08 0.0 8.0E+08 0.0</td>
<td>50 0 0.0 4.0E+07 0.0 2.8E+08 0.0 6.1E+08 0.0 9.5E+08 0.0</td>
</tr>
<tr>
<td>Annually with MMD grade tonnage curve (RMSOP)</td>
<td>10 2E+07 2.6 12.8 2.7E+08 6.6 4.0E+08 5.4 5.6E+08 2.4 5.6E+08 3.3</td>
<td>20 6E+07 12.9 6.6 1.6E+08 11.6 4.1E+08 8.4 6.0E+08 4.2 8.2E+08 2.3</td>
<td>30 1E+08 23.5 11.6 2.5E+08 11.6 4.1E+08 8.4 6.0E+08 4.2 8.2E+08 2.3</td>
<td>40 2E+08 25.8 14.1 3.4E+08 14.5 5.4E+08 14.5 7.8E+08 7.0 1.1E+09 3.8</td>
<td>50 2E+08 29.2 17.5 4.3E+08 17.5 6.7E+08 17.5 9.5E+08 9.8 1.3E+09 5.6</td>
</tr>
</tbody>
</table>

Table 2. Objective values and integrality gaps for solving the mine scheduling optimization problem with a time limit of two hours.

A problem instance is defined by the time period resolution, i.e. either monthly or annually, and in the case of an annual resolution whether an MMD grade tonnage curve was used or not and lastly, the size of the problem instance expressed in terms of the number of stoping blocks considered. Table 2 shows the results for solving the mine scheduling optimization problem for a range of problem instances and a range of mineral prices. The column labeled “Mineral Price (x1)” is an arbitrarily selected base price at which a high degree of selectivity can be observed. By increasing the mineral price by increments of 25% results are obtained for the columns labeled “Mineral Price (x1.25)” up to “Mineral Price (x2)”. An execution time limit of two hours were imposed on all problem instances and the objective function values listed under the columns labeled “NPV” are, therefore, the best net present values found within this time limit. The entries listed under the columns labeled “gap(%)” are the optimality gaps found within the time limit, calculated as $|Z_L - Z_U| / Z_U$ with $Z_L$ and $Z_U$ the best lower and upper bounds respectively. Evidently an optimality gap of zero would imply that the specific problem instance was solved to optimality within the time limit.

All problem instances were solved using CPLEX 12.1. The most notable result from Table 2 is the inability of CPLEX to generate feasible solutions within the set time limit for the RMSOP model when considering a monthly
resolution and problem sizes exceeding 50 stoping blocks. In contrast, much larger problem instances could be solved with the LRMS model for all mineral price cases. Also very prominent is the inferior results produced by the RMSOP model with an annual time period resolution. Specifically, for the base mineral price (x1), optimization terminated optimally but with only a zero NPV for all problem sizes due to the inability of the RMSOP model to effectively harness the potential of selectivity as a result of the aggregation of information. In these cases the costs overshadowed revenue with the result that the most economical plan is to refrain from scheduling any mining activities. For other mineral price cases the RMSOP model with an annual time period resolution did provide non-zero NPV values that were, however, much lower compared to the corresponding results for both the RMSOP model with monthly periods as well as the LRMS model.

The benefits of employing an MMD grade tonnage curve in conjunction with the LRMS model is evident from the base mineral case (x1). All NPV entries for the LRMS model exceed that of the other two cases where the RMSOP model was used for both a monthly and an annual time period resolution. For an increased mineral price the benefits of using the LRMS model diminishes to such an extent that for the (x2) mineral price case no improvements in objective values are obtained over the result from the RMSOP model with monthly periods. However, for the said mineral price case, the RMSOP model with a monthly time period is still unable to produce any feasible integer solutions within the two hour time limit for problem sizes exceeding 50 stoping blocks, whereas with the LRMS model solutions could be computed.

In summary, the major benefit of using the LRMS model with MMD grade tonnage curves can be observed for problem instances where low profit margins drive selectivity and where the size of a problem instance in terms of the number of stoping blocks exceeds the computable capacity of a high resolution model formulation.

5 Summary and conclusion

The primary contribution of this paper is the introduction of the LRMS model which is based on a low resolution time discretization, but with the capability of harnessing information captured within grade tonnage curves to allow selectivity on a micro level. The LRMS model outperforms high resolution discretized problem formulations specifically in low profit margin cases were selectivity is needed to boost profitability and in cases where the dimension of a problem instance is detrimental to computational feasibility.
References


