

Optimization Problems in Natural Gas Transportation Systems: A State-of-the-Art Review

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Abstract

This paper provides a review on the most relevant research works conducted to solve natural gas transportation problems via pipeline systems. The literature reveals three major groups of gas pipeline systems, namely gathering, transmission, and distribution systems. In this work, we aim at presenting a detailed discussion of the efforts made in optimizing natural gas transmission lines.

There is certainly a vast amount of research done over the past few years on many decision-making problems in the natural gas industry and, specifically, in pipeline network optimization. In this work, we present a state-of-the-art survey focusing on specific categories that include short-term basis storage (line-packing problems), gas quality satisfaction (pooling problems), and compressor station modeling (fuel cost minimization problems). We discuss both steady-state and transient optimization models highlighting the modeling aspects and the most relevant solution approaches known to date.

Although the literature on natural gas transmission system problems is quite extensive, this is, to the best of our knowledge, the first comprehensive review or survey covering this specific research area on natural gas transmission from an operations research perspective. The paper includes a discussion of the most important and promising research areas in this field. Hence, this paper can serve as a useful tool to gain insight into the evolution of the many real-life applications and most recent advances in solution methodologies arising from this exciting and challenging research area of decision-making problems.

Keywords: Operations research; Natural gas; Pipeline optimization; Transmission systems; Non-linear programming; Mixed-integer linear programming; Mixed-integer nonlinear programming; Steady-state models; Transient models; Line-packing problem; Pooling problem; Fuel cost minimization problem.

1 Introduction

Natural gas [93] is believed by many to be an essential energy source for the future. The abundance of natural gas coupled with its environmental soundness and multiple applications across all sectors, means that natural gas will continue to play an increasingly important role in meeting demand for energy in many countries.

While short term factors can significantly affect the demand for natural gas, it is the long term demand factors that reflect the basic trends for natural gas use into the future.

In order to analyze those factors that affect the long term demand for natural gas, it is most beneficial to examine natural gas demand by sector. The three most important sectors are: residential and commercial demand, industrial demand, and electric generation demand.

Residential and commercial: According to the U.S. Energy Information Administration (EIA), the energy demand in the U.S. residential and commercial sectors for example, is expected to increase 6% between 2009 and 2035. Natural gas consumption in the residential sector accounts for 21% of all consumption in the U.S.

Probably the most important long term driver of natural gas demand in the residential sector is future residential heating applications. Between 1991 and 1999, 66% of new homes, and 57% of multifamily buildings constructed used natural gas heating. In 2010, 54% of new single family homes constructed used natural gas. The increase in the number of new homes using natural gas for heat over the next 20 years is expected to provide a strong driver for residential natural gas demand (see Figure 1.)

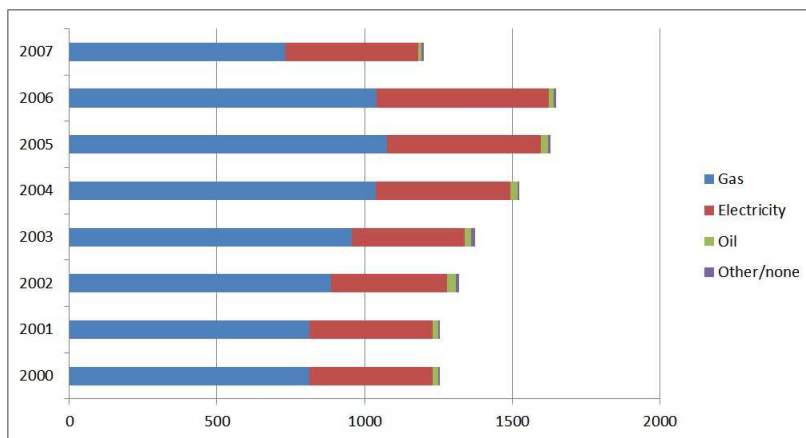


Figure 1: New houses by heating fuel type 2000 – 2007 (Source: EIA - US Natural Gas Markets: Mid-Term Prospects for Natural Gas Supply - 2008)

Regarding the energy demand in the commercial sector, the EIA expects that it increases at an average annual rate of 1.1% through 2035. Commercial floor space is expected to increase at a rate of 1.2% per year over the same period, so the energy demand per area of commercial

floor space is actually expected to decrease 0.1% per year reflecting improvements in energy efficiency. Natural gas currently supplies 15% of the energy consumed in the commercial sector and will do so through 2035.

Industrial: The EIA projects that industrial energy demand will increase at an average rate of 0.9% per year to 2035. The estimate represents energy requirements for both energy-intensive manufacturing industries (which are expected to decline), and non energy-intensive manufacturing industries (which are expected to grow). Industrial demand accounts for 27% of natural gas demand. The primary force shaping the demand for natural gas in the industrial sector is the movement away from energy-intensive manufacturing processes. There are two driving forces behind this shift: the increased energy efficiency of equipment and processes used in the industrial sector, as well as a shift to the manufacture of goods that require less energy input. This trend is expected to hold into the future, and is the reason for modest increases in energy demand for the industrial sector.

Electric generation: The demand for electricity is predicted by the EIA to increase by an average rate of 1% per year through 2035. In order to meet this growing demand, EIA predicts that 223 gigawatts of new electric generation capacity will be needed by 2035. Because of the relatively low capital requirements for building natural gas-fired combined cycle generation plants, as well as the reduction of emissions that can be earned from using natural gas as opposed to other fossil fuels, the EIA expects 60% of new electric generation capacity built by 2035 will be natural gas combined-cycle or combustion turbine generation.

While natural gas-fired electric generation accounted for 16% of all generation in 2002, the EIA predicts it will account for 24% of all generation in 2035. In addition to increased demand for natural gas powered central station generation, distributed electricity generation (as discussed for residential, commercial, and industrial sectors) may serve to increase the demand for natural gas for electricity generation purposes in the future.

The EIA, in its Annual Energy Outlook 2011 [121], projects that natural gas demand in the United States could be 26.55 trillion cubic feet (Tcf) by the year 2035. That is an increase of 16% over 2009 demand levels. In comparison total energy consumption is expected to increase by 20% (from 94.79 quadrillion British thermal units to 114.19) by 2035. The EIA predicts an annual energy demand increase of 0.7% over the next 26 years.

On an international arena, assessments in natural gas demand, production and reserves also foresee a continual increase in the decades ahead. According to global projections provided by the EIA in its International Energy Outlook 2011 [122], natural gas consumption increases at an average rate of 1.6% per year through 2035. Models used to generate these projections are based on macroeconomic assumptions and obey the Organization for Economic Cooperation and

Development members (OECD) and non-members (non-OECD). OECD countries are split into three subgroups: OECD Americas (United States, Canada, and Mexico/Chile), OECD Europe, and OECD Asia (Japan, South Korea, and Australia/New Zealand). Non-OECD countries are split into five regional subgroups: non-OECD Europe and Eurasia (which includes Russia); non-OECD Asia (which includes China and India); Middle East; Africa; and Central and South America (which includes Brazil). Figure 2 shows global projections in natural gas consumption for OECD and non-OECD members. Results indicate that growth in natural gas consumption is most concentrated in non-OECD countries while accounting for 76% of the total world increment in natural gas consumption.

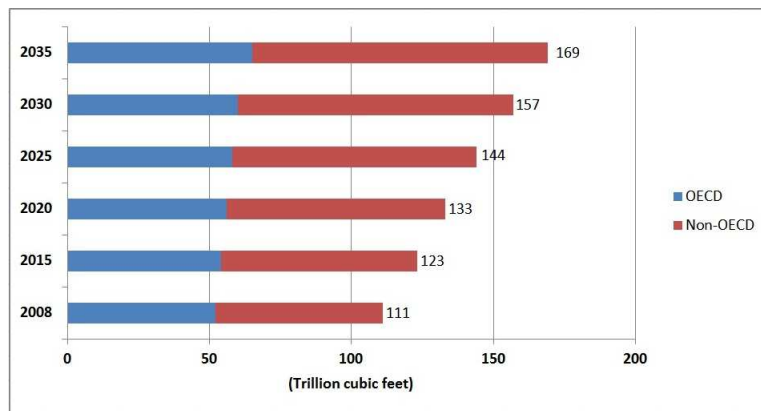


Figure 2: Annual projection values of world natural gas consumption through 2035 for OECD and non-OECD members (Source: Oil and Gas Journal and EIA)

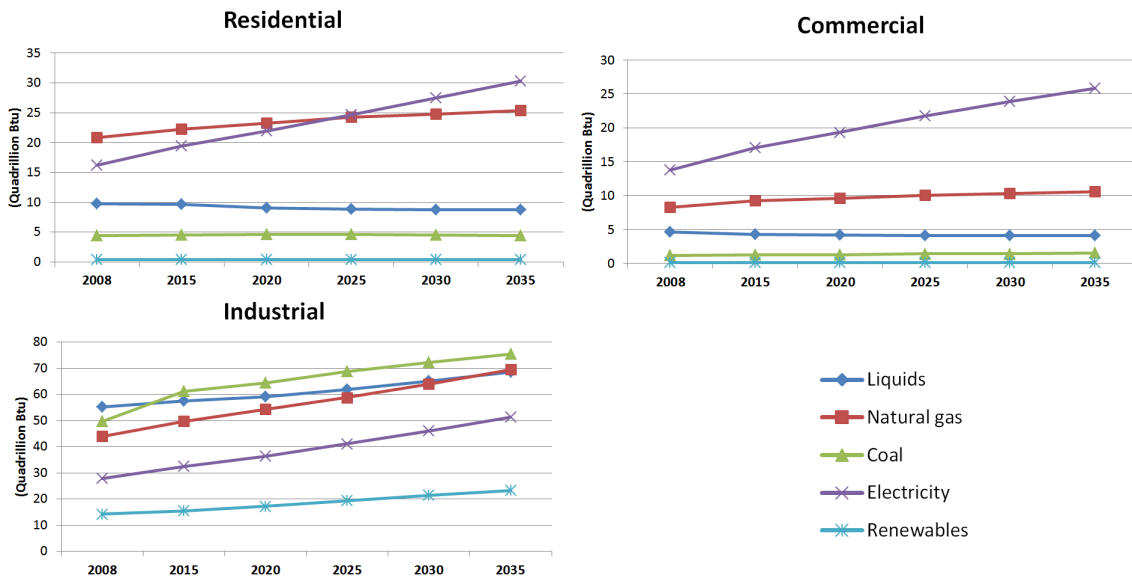


Figure 3: Projected world energy consumption by end-use sector and fuel (Source: EIA - International Energy Statistics database, March 2011)

World projections of energy consumption by end-use sector and fuel through 2035 are shown in Figure 3. Electric power sector projections are shown separately in Figure 4, with the addition of nuclear power projections. Figures reveal that most of natural gas consumption is concentrated in the industrial and electric power sectors, accounting for 87% of the total world natural gas consumption. By comparison with other hydrocarbon fuels, figures also indicate that natural gas continues to be the fuel of choice through 2035. Natural gas consumption growth is observed at an average of 1.7% and 2.0% per year in the industrial and electric power sectors, respectively.

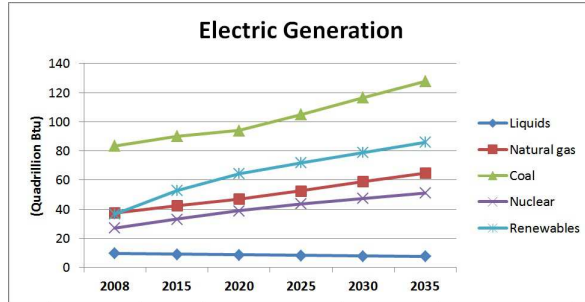


Figure 4: Projected world energy consumption of the electric power sector (Source: EIA - International Energy Statistics database, March 2011)

Factors such as low greenhouse gas emissions and relatively reduced capital costs make natural gas position competitive in most of the sectors among other energy sources, particularly for new power generation facilities. In addition, global projections in natural gas reserve levels support growth in markets through 2035. Figure 5 shows global projections in natural gas reserves by geographic regions. In the figure, the largest concentrations are observed in Eurasia and the Middle East, where as of January 2011 four of non-OECD Middle East’s more important producers, namely Qatar, Iran, Saudi Arabia and the United Arab Emirates, claim together more than 40% of the world’s proved natural gas reserves.

Regarding world natural gas production estimates, OECD and non-OECD countries reveal increases at an average rate of 0.9% and 2.0% per year through 2035, respectively (see Figures 6–7). Projected values for non-OECD countries also indicate a production growth that accounts for more than 81% of the total growth in world natural gas production, which exceeds their own projected consumption growth. As a result, export trades to OECD countries are also projected through 2035.

Among OECD subgroups, despite the 84% projected increase in Norway’s natural gas production, OECD Europe shows a decline in its projections while falling from 10.6 trillion cubic feet in 2008 to 8.3 trillion cubic feet in 2035. OECD Asia reveals the strongest growth in natural gas production among OECD regions, with an average rate of 4.5% per year.

Among non-OECD subgroups, Middle East projects the largest growth in natural gas production while increasing its values from 13.5 trillion cubic feet in 2008 to 28.8 trillion cubic feet in 2035.

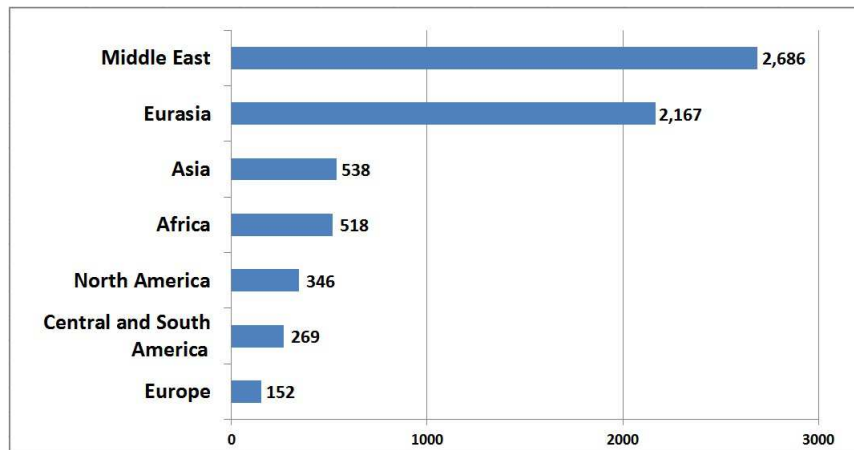


Figure 5: Projected world natural gas reserves by geographic region (trillion cubic feet) – Sources: Oil&Gas Journal and EIA

Europe and Eurasia also project a strong growth in natural gas production with a 32% increase from 2008 to 2035, where Russia is the dominant natural gas producer, contributing with more than 75% of the production estimates. Readers interested in a detailed analysis on global trends in natural gas demand, production and reserves are referred to [122].

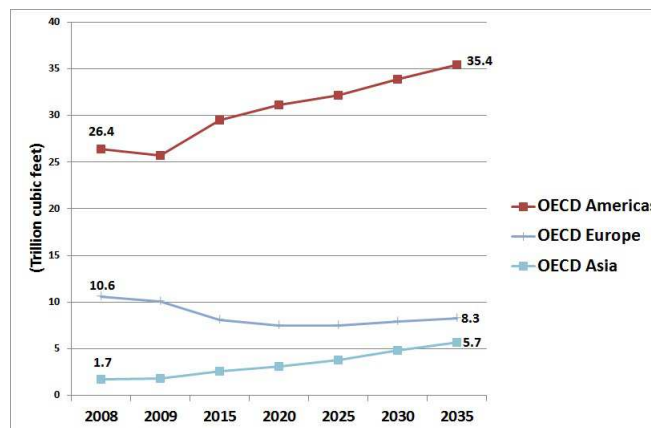


Figure 6: Projected world natural gas production by OECD subgroups (trillion cubic feet)

It is important to note that the projected global steady climb on trends in demand, production and reserves levels for natural gas could increase as climate change legislation grows demand for low-carbon fuels such as clean natural gas. While forecasts made by different federal agencies may differ in their exact expectations on natural gas global tendencies, one result is recurrent across studies: natural gas consumption, production and reserves will continue to increase for the foreseeable future.

The following stages are involved in the process of getting the natural gas out of the ground, and driven to its end destination.

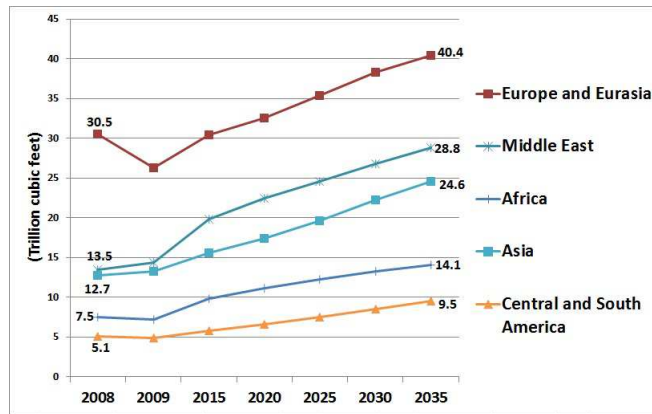


Figure 7: Projected world natural gas production by non-OECD subgroups (trillion cubic feet)

Exploration: In this stage, the issue of how natural gas is found and how companies decide where to drill wells for it is addressed.

Extraction: This stage deals with the drilling process, and how natural gas is brought from its underground reservoirs to the surface.

Production: In this stage the processing of natural gas once is brought out from the underground takes place.

Transport: The natural gas is transported from the processing plant to local distribution companies across a pipeline network in this stage.

Storage: This stage accounts for the storage of natural gas.

Distribution: In this stage, natural gas is delivered from the major pipelines to the end users.

Marketing: This stage involves the buying/selling activity from the natural gas marketers.

The efficient and effective movement of natural gas from producing regions to consumption regions requires an extensive and elaborate transportation system. In many instances, natural gas produced from a particular well will have to travel a great distance to reach its point of use. The transportation system for natural gas consists of a complex network of pipelines, designed to quickly and efficiently transport natural gas from its origin, to areas of high natural gas demand. Transportation of natural gas is closely linked to its storage: should the natural gas being transported not be immediately required, it can be put into storage facilities for when it is needed.

There are three major types of pipelines along the transportation route: the gathering system, the interstate pipeline system, and the distribution system. The gathering system consists of low pressure, small diameter pipelines that transport raw natural gas from the wellhead to the processing plant. Should natural gas from a particular well have high sulfur and carbon dioxide

contents (sour gas), a specialized sour gas gathering pipe must be installed. Sour gas is corrosive, thus its transportation from the wellhead to the sweetening plant must be done carefully.

The field of Operations Research has taken a major role in the natural gas industry as a number of important and relevant problems in design, extraction, production, transportation, storage, distributing, and marketing, have been successfully tackled by OR models and techniques over the past 40 years. Zheng et al. [137] provide a recent survey on optimization models in the natural gas industry while focusing on three specific aspects: production, transportation, and market. The authors basically discuss a mathematical formulation of the underlying problem and provide a literature revision of the existing optimization techniques that solve it. Their study covers six general problems, namely the production scheduling problem, the maximal recovery problem, the network design problem, the fuel cost minimization problem, and the regulated and deregulated market problems. Our goal in this paper is to discuss the most relevant research work that has been done in the natural gas transport industry from the operations research perspective. The paper covers works on three specific optimization areas posed by the gas transport industry when optimizing its transmission systems. This involves problems in short-term basis storage, pipeline resistance and gas quality satisfaction, and fuel cost minimization via pipeline transmission networks.

The paper is organized as follows. We provide next a more in-depth insight into the natural gas industry. Particularly, we emphasize the main components of natural gas transmission pipeline systems and highlight those stages that define the focus of the present work. Some concepts regarding the modeling and optimization of natural gas transportation problems used throughout the text are also discussed. In Section 3 we address the effective application of the optimization theory on the transport and storage of natural gas towards the contractual demands satisfaction. The section particularly discusses the existing, although very limited literature on the efficient transport and short-term basis storage of natural gas along transmission lines, also referred to as the line-packing problem. In Section 4 we deal with the pipeline resistance and gas quality problems in natural gas transportation systems. The research works discussed here are based on pipeline resistance studies, also referred to as the maximum flow capacity in a pipeline, and gas blending-type constraints to meet gas quality requirements, also referred to as the pooling problem. In Section 5 we discuss the problem of how to transport the gas through a pipeline network at minimum cost referred to as pipeline optimization. Final remarks and discussion on major challenges in the field are presented in Section 6.

2 Background: Network Properties and Classification

Natural gas industry is a fast-growing infrastructure. It provides consumers with a virtually almost-free natural gas access in its front end. By simply turning on the main valves (taps) for delivering gas from a pipeline system, end users can make use of a wide list of home or industrial gas appliances.

Nevertheless, the long trip that natural gas covers from the wellheads (as a raw material) to get to residential or businesses (as a clean and efficient source of energy, i.e., as we know it, see Figure 8), entails a considerable number of complex tasks. Such tasks correspond to different transitional stages of natural gas that can be classified into two primary groups:

- (a) Exploration, drilling, extraction, production and long-term storage of natural gas.
- (b) Gathering, short-term storage, transportation and distribution of natural gas.

This classification obeys the key instrument used for the service to be achievable, namely *pipeline network systems*. Unlike group (a), long pipelines of various diameters are essentially required for the group (b) dynamics.



Figure 8: Natural gas.

Moreover, the 20th century witnessed the outbreak of business giants that captured most of the transitional stages involved with natural gas from the wellheads to end users. Yet the changing nature of the natural gas infrastructure called for new upgrades. Nowadays, small and medium size companies are transforming the natural gas industry worldwide. Unlike the former corporations, these companies derive profit by focusing their efforts on one or two specific transitional stages of natural gas. As a result, the monopolistic control of the natural gas industry observed back in the last century has been declining to some extent.

In this study, we focus primarily on works related to gas transport industry problems. Particularly, we discuss the most relevant works in the field of optimization that deal with short-term storage, quality, and compressor fuel cost of natural gas pipeline network systems. Hence, we provide next some insights into the gas transportation via pipelines.

2.1 Natural Gas Transportation Via Pipelines

The natural gas transportation is a crucial activity performed by the gas industry in which the gas has to be moved from one location to another. Several types of transportation means might be applied to transport the gas, yet it is well known that pipelines represent the most economical means

to transport large quantities of natural gas. In addition, the advent of metallurgical improvements and welding techniques, coupled with the exponential increase of pipeline networks during the last decades all over the world, have made the gas transportation via pipelines more economically attractive.

Currently, pipelines are used both offshore and onshore, with a remarkable difference in terms of security and construction prices. Building pipeline systems under the sea is highly costly and technically demanding, a lot more than onshore. For example, according to *Gazprom*¹, the *Nord Stream*² (41 in) pipeline project is expected to cost around € 14.8 billion [96], of which 40.5% [40] corresponds to the 965.7 km long onshore pipeline system on Russian and German territories, whereas the remaining 59.5% is destined to the 259.4 km long offshore section of the project. Hence, when financial, political or environmental issues arise, gas transportation operators look for different alternatives to perform this task. This includes tanker ships and flatboats, by which natural gas can be transported as LNG (liquefied natural gas), MLG (medium conditioned liquefied gas), or CNG (compressed natural gas). More detailed information on commercially applicable methods for natural gas storage and transport can be found in [24, 47, 58, 50].

Moreover, it is a common practice that the location, construction and operation of natural gas pipeline systems are regulated by federal and state regulations. Nonetheless, in several countries, including USA, Canada, and Brazil, pipeline systems are fully-privatized, i.e., they are private company-owned and thus operated independently. In these scenarios, the fuel cost minimization problem (see Section 5) has a priority focus. This, however, is not exactly the case in most of the European countries. For example, in Nordic countries like Norway and Denmark, the compressor stations located in transmission lines are usually kept on to their maximum capacity for long periods of time, thus this problem is either completely neglected or becomes a lesser matter.

There are essentially three major types of pipelines (usually buried underground) along the transportation lines, ranging in size from 4 inches to 48 inches in diameter (100 to 1,220 mm): gathering systems, transmission systems, and distribution systems. Gathering pipeline systems gather raw natural gas from production wells. Transmission pipeline systems transport natural gas thousands of miles across the world to bring natural gas from the pre-processing plants or storage facilities to distribution systems. Distribution pipeline systems can be found in communities and distribute natural gas to homes and businesses.

¹*OAO Gazprom* (Open Joint Stock Company) is the Russian state-owned energy monopoly established as the largest extractor of natural gas in the world.

²*Nord Stream* is a two-line gas submarine pipeline to link Russia and the European Union via the Baltic Sea.



Figure 9: Transportation pipelines.

The main differences among these systems are the physical properties of the pipelines used, such as diameter, stiffness, material, etc., and the specifications of the maximum and minimum upstream and downstream pressures. For instance, gathering and transmission lines are constructed from steel pipe (see Figure 9), whereas distribution lines can be constructed from steel or modern plastic pipe. A more detailed description of these pipeline systems is presented next.

2.2 Gathering Systems

Gathering lines are pipelines that collect gas from multiple flow lines, which in turn are connected to a single wellhead in a production field. Flow lines are composed of narrow pipelines typically buried 4 ft underground and working at a roughly 250 psi pressure. According to an Environmental Protection Agency (EPA) study [123], flow lines represent one of the largest sources of emissions in the gas industry due to methane leakage [61].

Gathering systems are composed of medium size steel pipelines, usually no more than 18 inches in diameter, and working at a roughly 700 psi pressure. These systems basically carry raw oil or nonodorous raw gas to centralized points, such as production and long-term storage facilities, or marine docks. Submarine pipelines that collect gas from the deep water production platforms are also considered gathering systems.

At the production facilities, also known as processing plants, the raw gas undertakes a number of purification procedures. These procedures typically remove impurities like water, carbon oxide (CO), carbon dioxide (CO₂), mercury (Hg) and sulfur (S) to prevent corrosion of the pipelines, or inert gases like helium (He) to increase the energy value of the gas. Heavier hydrocarbons like propane (C₃H₈) and butane (C₄H₁₀) are also removed and treated as by-product of natural gas (a

typical mixture known as liquefied petroleum gas, LPG, or LP gas).

After pre-processing, the gathering systems are also used to transport natural gas from the production facilities to the transmission lines for its final sell and distribution. In some cases, the raw gas is directly transported from the wellheads to a main transmission line or to a long-term storage facility for its further treatment and use.

2.3 Transmission Systems

A transmission line is a pipeline mainly used to transport large amounts of natural gas across long distances from a gathering, processing or storage facility to a distribution system, which may include far-away storage facilities and large-volume customers. Note that the large-volume customer category are those factories, power plants, petrochemical facilities, and gas institutional users that are not downstream from a distribution center.

Several devices such as valves, regulators, pipelines, compressors, pressure gauges, junction nodes, among others, play a key role in natural gas transmission systems to efficiently perform the gas transportation task. These systems, federally regulated, are characterized for having long steel pipelines (ranging from 3 inches to 48 inches in diameter) that may cross cities, states or even countries. Large amounts of natural gas are typically transported by means of compressor stations installed at strategic points along the transmission line. These compressor stations usually work at a pressure of approximately 200 psi to 1,400 psi.

2.4 Distribution Systems

Distribution lines, also known as “main”, are pipelines used to supply natural gas to end users. They are part of a network system of piping located downstream of a natural gas transmission line, i.e., they are the middle step between high pressure pipelines and low pressure service pipelines.

Natural gas (local) distribution companies (LDCs) are establishments in charge of distributing natural gas to end users. These companies usually face a number of management problems related to natural gas purchase under conditions of uncertain demands and frequent price changes (variable price rate). A good example is that of the Chilean local distribution system built in 1994, which was aimed at serving residential, commercial, and industrial customers while lacked of local storage facilities. Yet, it started its operations in 1997 and grew quite fast during the next decade, becoming one of the largest LDC in Chile while serving gas to more than 200,000 residential customers [32]. In [32], Contesse et al. present a MIP model to optimize the purchasing and transportation contracts for this Chilean LDC in the absence of local storage resources.

Natural gas distribution systems are small- to mid-size pipelines (ranging from 2 inches to 20 inches in diameter) which are constructed out of plastic, cast iron, and steel. They are usually installed underground along streets and roadways. Due to safety reasons, distribution pipelines

typically operate below their capacity and work at a pressure of approximately 0.5 psi up to 200 psi.

These systems connect with the main transmission lines at “city gate” stations. The key function of a city gate station is to reduce the incoming gas pressure in order to meet safety regulations and to allow gas distribution through primarily underground mains and service lines. According to the U.S. Environmental Protection Agency (EPA)’s 2011 report [124], for example, there were more than 1,200,000 miles of distribution lines in USA in 2009. This reflects an increase of roughly 5 times over the US distribution system observed back in 1990.

2.5 Technicalities of Gas Transmission Network Components

Compressor Stations

As mentioned above, compressor stations play a crucial role in the natural gas industry. They are considered one of the most important assets in the considerably high number of transmission pipeline network systems worldwide. Note that a compressor station may be composed of several compressor units connected in series or in parallel.

A simple definition of a compressor is that of a device to increase the pressure of natural gas by reducing its volume, thus providing the required propel force or boost to keep it moving along the transmission line. These are devices strategically installed along the gas transmission lines that provide enough energy to natural gas for its transmission. More precisely, a compressor unit along a natural gas transmission system is a large mechanical device that receives the gas at pressures ranging from 200 psi to 600 psi, and compresses it back up to 1000 psi to 1400 psi. (As a reference, the typical vehicle tires work with compressed air at roughly 30 to 50 pounds of pressure per square inch.) As a result, natural gas overcomes frictional losses and maintains required pressures to keep moving through the transportation line towards another compressor station or end users.

Several types of gas compressors units can be found in the gas industry. Among the most frequently used are those compressors characterized by a centrifugal dynamic movement or by means of reciprocating positive displacements. The latter is a compressor in which the compressing element is a piston having a reciprocating motion in a cylinder.

In many cases, the decision whether to use a centrifugal or reciprocating compressor unit includes, among other factors, a thorough analysis of operating conditions, hydraulic pipeline studies, emission requirements, and general lifecycle cost estimates [91, 92]. In addition, the pressure and flow characteristics of pipelines, coupled with the steady-state and transient capabilities of the network system, also influence the design and arrangement of compressor units in a station [6, 75]. Readers interested in design and arrangement of compressor stations in natural gas transmission systems are referred to the works of Akhtar [6], Kurz [74], Mokhatab, Santos, and Cleveland [92], and Santos [113].

Gate Settings

Gate settings are installed periodically (about every 10 miles) along the natural gas transmission system to help control the gas flow when, e.g., a particular section of the pipeline must be isolated for safety reasons or due to a scheduled maintenance. These are specific locations where the gas streams can be stopped by closing a valve, and are usually bounded by fences to prevent unauthorized operation of the valves.

When multiple lines are located parallel to each other in the same rights-of-way, crossover piping that connects all of the pipelines together is built at the gate sites. This allows diversion of the flow of the natural gas between the adjacent pipelines. The flexibility to divert gas from one pipeline to an adjacent pipeline is extremely useful to allow one pipeline to be isolated and worked on while gas continues to flow down the parallel pipeline system.

Rights-of-Way Corridors

Most natural gas transmission pipelines are located underground in rights-of-way corridors. This terminology has to do with the fact that the pipeline owner has to obtain permissions from affected landowners that give the operator the right to construct, operate, and maintain the pipeline for its entire length. That is, a rights-of-way consists of consecutive property easements acquired by, or granted to, the pipeline company. The main goal is to obtain enough space to perform unforeseen or scheduled maintenance (or inspections) of the pipelines.

Since pipelines have a long life span, typically many decades long, the original owner or owners of the land who gave permission to build the pipeline in its current location may not be the current owners of the land. In the life of a pipeline, the landowners may change numerous times. Keeping up with who the current owners are is a major task for the pipeline operator.

Valves and Regulators

Valves and regulators are typical components in a pipeline system. They are installed for operational and safety reasons. That is, by means of a valve, the gas operator can restrict or direct the gas flow from one point to another so that scheduled maintenance or demand requirements are met. The operator may also require to completely shut down the gas flow through a specific pipeline section due to malfunctions in order to prevent loss of fluid. In real-practice, regulatory requirements [87] force the transporter to install mainline block valves at certain fixed spacing along the transmission line.

Valves are constructed of steel due to regulations and specifications imposed by the American Petroleum Institute (API), the American National Standards Institute (ANSI), and the American Gas Association (AGA) [87].

For several decades the scientific community in the field of gas pipeline optimization has been significantly challenged with the large number of complex issues arisen from this particular real-life decision making problem. Such aspects are thoroughly discussed in the following sections.

3 Short-Term Basis Storage

Reasons for success in different arenas of the natural gas industry are due to both the efficient management of resources and equipment, as well as the effective implementation of the appropriate analytical strategies. The natural gas transport and storage industry is no exception. Because of the substantial increase in both natural gas demand and its reserves in recent decades, coupled with the expected promising growth in its production and distribution in the years ahead, the gas industry has become more aware of the need for a sustainable infrastructure that may lead to increases in revenue.

It is well known that natural gas can be stored and transported in its different states of matter [24]. However, the natural gas transportation means other than pipelines, e.g., truck, train, or ship, are usually not economically feasible [32]. For example, natural gas in its liquid form at approximately -163°C (-260°F), also known as liquefied natural gas or LNG [47, 50], can be transported in cryogenic containers while its volume is increased about 1/600th the volume in its gaseous state. The liquefaction and re-gasification processes, as well as the specially designed cryogenic vessels (LNG carriers) or cryogenic tankers have shown to be very costly. However, LNG can become economically appealing for the gas transport industry when the distance over which natural gas is transported is significantly long. According to [50], shipping natural gas in its liquid form is more beneficial than transporting it in its gaseous state via onshore and offshore pipelines when the distance exceeds 700 miles and 2,200 miles, respectively.

Natural gas in its gaseous form is still considered the most economical way to be stored and transported. In this section, we focus on those research works that address the effective application of the transport and storage of natural gas that leads to contractual demands satisfaction over a given planning horizon. More precisely, we discuss the existing, although very limited literature on the efficient transport and short-term basis storage of natural gas along transmission lines, also known as the line-packing problem. Readers interested in empirical models, theoretical foundations and applications of long-term basis storage of natural gas are referred to the works of Zwitterloot and Radloff [140], Neumann and Zachmann [94], Holland [66, 67], and the references therein.

The line-packing problem in natural gas transmission pipeline systems basically entails the optimization of gas refill in pipelines in periods of low demand or sufficient capacity, and the gas withdrawals in periods of shortfall. This is done by, e.g., closing (or throttling) a downstream valve while upstream compressors continue sending gas into the pipeline, i.e., packing more gas in the pipelines by increasing the pressure. A more complete description of the line-packing problem is

provided next.

3.1 The Line-Packing Problem

Gas pipelines have proven to be the most suitable transportation means for the gas industry since the advent of metallurgical improvements and welding techniques after World War II. Since then, dependable and economic pipeline systems have become essential in preserving the continuous business growth of the gas transport industry in national and international arenas. Nevertheless, a common denominator in the transportation process is that a number of unpredictable or scheduled events do occur on a daily basis. Among these events we can find, e.g., the break down of flow capacities elsewhere in the system due to malfunctions, routine maintenance or inspection; failures in upstream process capacity; shortfall in downstream capacity; demand uncertainty; and high fluctuation in demand due to seasons (in the winter the demand is usually higher than in the summer). However, gas producers must be able to supply gas to their customers despite such difficulties.

As a strategy to some extent alleviate the consequences of those events, natural gas operators must take into account one key fact: Gas pipelines do not only serve as transportation links between producer and consumer, but they also represent potential storage units for safety stocks. That is, due to the compressible nature of dry gas, large reserves can be stored on a short-term basis inside the pipeline through a process called line packing. This is accomplished by injecting more gas into the pipelines during off peak times by increasing the gas pressure, and by withdrawing larger amounts of gas during periods of high demand when flow capacities elsewhere in the system break down. Hence, the problem of keeping a sufficient level of line-pack during a given planning horizon becomes critical to the gas transporter.

To conceptualize this problem, let us see the simplest example. Let us suppose that there is a unique transmission line between one producer and one costumer, and let us assume that the amount of gas required by the client during several consecutive periods can easily be satisfied with only 70% of the maximum capacity. An obvious solution is simply to send the required amount for the mentioned periods. However, let us assume that the demand increases up to 130% of maximum capacity for some subsequent period. Here, the producer can not meet such requirement, thus leading to considerable economic losses. Hence, the strategic idea would be to send for instance 100% of the maximum capacity, then consuming just the required demand in each period, and storing the remaining gas to satisfy future extraordinary requirements.

Figure 10 shows a typical network instance that is composed of 10 source nodes, 10 sink nodes and 10 pipeline segments. As we can observe from the figure, a highly potential storage unit can be inferred by each pipeline in order to meet multiple gas demands during a given planning horizon.

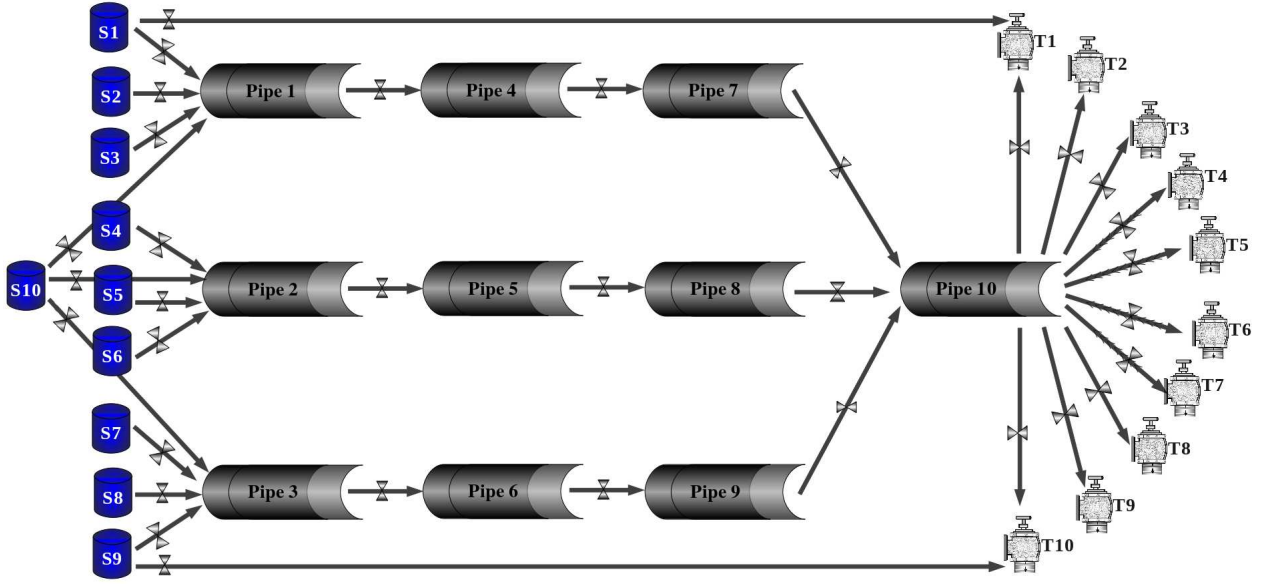


Figure 10: Potential network elements for short-term storage.

3.2 Literature Survey

Related works facing problems of seasonal peak demands can be traced back since 1970. However, the literature reveals that these works were more focused on solving this problem from a managerial perspective, rather than from an operative perspective. Welch et al. [127], for example, proposed to deal with this problem by using other fuels and optimizing a number of scheduled interruptions whenever the gas flow broke down. In addition, they showed that the availability of large industrial contracts was an important factor in containing the peak demand. More recently, Contesse, Ferrer, and Maturana [32] conduct a study on the natural gas supply chain, in which they infer that the changes in the gas industry regulatory system have lead to several alternatives for absorbing demand fluctuations based on contractual strategies of, for example, the use of storage facilities. They mainly refer to two types of contracts: (a) a sale customer contract on a supply interruptible basis in which customers have their gas supply shortened during periods of peak demands in exchange for a lower price; and (b) the firm transportation contract, which allows shippers to reserve a portion of the pipeline’s total delivery capacity for their own use.

From the mathematical programming perspective, some attempts, although few, have been made in the direction of mathematical planning models for the line-packing problem [37, 28, 73, 54, 18].

For instance, de Nevers and Day [37] examine the natural gas pipeline inventory from a mathematical perspective to match time-varying demands with supplies in an unsteady-state pipeline network system. Their study is based on two dimensionless parameters for the packing and drafting behavior. As a result, their model is capable of showing the limits of the line-packing and

line-drafting for a single pipeline segment.

Carter and Rachford [28] discuss several control strategies to operate pipeline network systems through periods of fluctuating loads. Their study aims at finding an optimal schedule for the line-pack under uncertain demand assumptions. As a result, they provide a number of possible scenarios with specific schedules for modifying the set-point values of compressor stations.

Krishnaswami, Chapman, and Abbaspour [73] present a simulation approach for optimizing pressure units of compressor stations to meet a specific line-packing along transient, non-isothermal pipeline network systems. They first formulate an implicit finite difference model to provide a flow capacity analysis, and then propose a nonlinear programming model to minimize the average fuel consumption rate of each compressor station over a given planning horizon. The model is solved by applying a sequential unconstrained minimization technique based on a directed grid search method that solves the unconstrained subproblems. Due to the complexity of problem, their study is, however, limited to a linear (gun-barrel) pipeline network system with two compressor stations composed of three compressor units each.

Frimannslund and Haugland [54] follow the ideas presented in the work of Carter and Rachford [28], and propose a mathematical formulation to cope with line-packing levels for a pipeline network system in steady-state conditions. Their study is based on homogeneous gas batches, a concept introduced in [28]. The concept refers to the creation of a number of batches (gas packages) inside the pipelines for their future scheduled withdrawal. The “homogeneous” term in turn establishes that all gas batches are made of the same gas composition no matter when they are constructed, thus implying the assumption that all gas sources in the network provide gas of the same quality. Due to this assumption, no quality constraints on the transported and delivered gas was required. According to [54], a blending process between the batches inside the pipeline seems to be unrealistic unless a long lasting shortfall in downstream capacity takes place.

Borraz-Sánchez [18], motivated by the work of Frimannslund and Haugland [54], proposes and implements a mixed-integer nonlinear programming (MINLP) model and a global optimizer-based mathematical programming algorithm for solving large-scale natural gas transmission networks problems under steady-state assumptions. Unlike Frimannslund and Haugland’s work, the key idea behind Borraz-Sánchez’s MINLP model is to build up ‘heterogeneous’ batches (i.e., gas packages of possibly different composition) for a multiple-time period planning horizon. This strategy basically allows the model to account for gas sources that may provide gas of different quality, thus resulting in a more sophisticated model. An essential assumption of Borraz-Sánchez’s work is to consider that no blending process among the batches takes place inside the pipelines [54], which is a rather common practice in the gas industry. Moreover, a fundamental part of Borraz-Sánchez’s model is also its capability to keep track of energy content and gas quality to ensure that contract terms are met. The model assumes a specific gas quality at the sources (which may be determined by producers), and satisfies the gas quality imposed at the terminals. Here, several gas streams of

different composition may be blend at junction points of the network in order to meet the quality requirements. The inherent problem in satisfying natural gas quality requirements, which directly introduces an NP-hard problem known as the pooling problem [53, 62], is addressed in Section 4.

More recently, Zavala [136] presents a stochastic model to solve the line-packing problem. The model also captures the network dynamics by discretizing the governing partial differential equations in time and space. Zavala considers a gas network with links comprising long pipelines and nodes consisting of junction points and compressors. The proposed model is a representation of a stochastic optimal control model that considers conservation and momentum equations, typical operational constraints, and uncertainty in demands. The author performs a degrees-of-freedom (DOF) analysis to verify the consistency of the model and uses the underlying results to derive consistent initial conditions and non-anticipativity constraints. In addition, the author also incorporates a risk metric into the objective function to mitigate cost variance and system volatility. The computational study demonstrates the benefits obtained with the stochastic formulation against the deterministic and robust counterparts.

4 Pipeline Resistance and Gas Quality Satisfaction

We start by distinguishing two disjoint research groups encompassing network flow problems. One group, which may be recalled as the classical group, defines constant arc capacities for transporting solid goods, whereas the second group, committed to optimizing fluid flows, defines non-constant arc capacities. In this section, we focus on those works contributed by the second group, which are committed to optimize transmission pipeline systems of natural gas transport in steady-state.

Two major characteristics of steady-state network flow models are the strong dependence between the pipeline flow and the pressure drop along the transmission line, and the inclusion of pressure values as a state variable at interconnection points. Furthermore, the pressure values in the pipeline system are determined by the flow and pressure values of upstream network elements of the evaluated component. Consequently, more refined modeling techniques are required to compute the resistance of the pipelines.

The fundamental flow equation, based on derived solutions from partial differential equations, has been universally accepted as the full statement to describe fluid flows under various boundary conditions. This mathematical description of fluid flow is based on a physical principle and models of physical behavior, namely the law of conservation of mass, Darcy’s law, and equations of state [87]. Basically, the bigger, shorter, and colder the pipeline containing lighter gasses is, the more flow is permitted.

Menon [87] establishes that the pipeline resistance, also referred to as the maximum flow capacity in a pipeline, is strongly dependent on the physical properties of pipelines and the composition of the gas. Thus, during the last century several equations were proposed to simulate compressible

gas flow in long pipelines, including the Weymouth equation (developed in 1912), the Panhandle A equation (developed in 1940), and the Panhandle B equation (developed in 1956). The equations were developed from the fundamental energy equation for compressible flow, but each has a special representation of the friction factor to allow the equations be solved analytically. In addition, they differ from each other by the method used to create them and the number of parameters used to define them. For low pressures and short pipeline, they may not be applicable. The works of Osiadacz [99], Crane [35], and Modisette [89] provide complete details of these equations.

Due to its simplicity and its accuracy when applied to gas flows at high pressures, coupled with the fact that it has been around the longest, the Weymouth equation is, however, the most-widely used to model flow capacities. The equation, which basically defines the relationship between the flow and the pressure drop through a horizontal pipeline segment, is given by:

$$x_{uv}^2 = W_{uv} (p_u^2 - p_v^2), \quad (1)$$

where x_{uv} is the mass flow rate through the horizontal pipeline segment (u, v) , p_u and p_v are the upstream and downstream pressure, respectively, and W_{uv} , referred to as the Weymouth factor, is a parameter that depends on gas and pipeline properties as given by

$$W_{uv} = \frac{d_{uv}^5}{K z_{uv} g_u T f_{uv} L_{uv}},$$

where z_{uv} is the compressibility of the flow in pipeline (u, v) , g_u is the specific gravity of the flow arriving at node u , T is the gas temperature, f_{uv} is the (Darcy-Weisbach) friction factor in pipeline (u, v) , L_{uv} is the length of pipeline (u, v) , d_{uv} the inside diameter of pipeline (u, v) , and K is a global constant with value defined by the units used.

Note that (1) typically expresses the mass flow rate in terms of standard ft³/day, which is not a measure of volume per time, but a measure of mass per time, i.e., the mass contained in one cubic foot of stated specific gravity at standard conditions given by the pressure base P_b (psia)(Kilopascals) and the temperature base T_b (°R). Such standard conditions are determined by gas sales contracts and are different from those conditions usually defined by the scientific community. Moreover, flow estimates become more accurate when the variability of the gas specific gravity is introduced into the calculation, which results in an alteration of the standard measure.

4.1 Ideal Gas Law

An ideal gas is a theoretical gas composed of molecules, in which there are no intermolecular attractive forces and all the internal energy is in the form of kinetic energy. From the kinetic theory, it can be deduced the so-called *ideal gas Law*, which can be written

$$z = \frac{PV}{NkT}, \quad (2)$$

where z is the gas compressibility, P is the absolute pressure, V is the volume, N is the number of molecules, k is the *Boltzmann constant* (1.38066×10^{-23} J/K), and T is the absolute temperature.

Eq. (2) defines the relationship between the state variables that characterize an ideal gas: P , V , and T . Contrary to real gases, an essential assumption of an ideal gas is that z is equal to 1 for any pressure value at a given temperature. However, real gases exhibit a clear dependence on current pressure and temperature conditions, thus changing the compressibility. This is studied next.

4.2 Gas Compressibility

As observed in (2), the gas compressibility, also referred to as the z -factor, can be considered as the deviation from ideal gas. More formally, it is defined as the relative change in gas volume in response to a change in pressure and temperature. The importance of accurate estimates of this parameter is obvious from (1) and the definition of W_{uv} .

The literature on gas metering reveals a number of diverse methods for approximating the z -factor, including experimental measurements, equations of state methods [43], empirical correlations [71], and regression analysis methods [44, 59].

For instance, in Chapter 4 of Katz et al. [71] a graphical correlation for the z -factor as a function of pseudo-reduced temperature and pressure based on experimental data is presented. As a result, the Standing-Katz z -factor chart has been used to obtain natural gas compressibility factors for more than 40 years. Dranchuk and Abou-Kassem [43] used the equation of the state to fit the Standing-Katz data and extrapolated to higher reduced pressure. This was accomplished by a simple mathematical description of the Standing-Katz z -factor chart.

In addition, several equations have been introduced to compute the z -factor, including the CNGA method, the AGA-NX19 and DPR methods.

Methods for Computing the z -Factor

The CNGA method [36] was developed by the California Natural Gas Association (CNGA) to compute the z -factor based on the gas specific gravity, temperature and pressure values. This method has been in use since the last century. One of its first applications is reported by Davisson [36], who makes use of the method in a computer program for precise flow calculations. More recently, Borraz-Sánchez and Haugland [21] make use of the CNGA method to compute gas compressibility values in pipelines along natural gas transmission systems.

The AGA-NX19 method is used to compute z -factor values based on the gas specific gravity and the average temperature and pressure conditions. The method was developed in a research project supported by the American Gas Association (AGA) between 1956 and 1962 [114].

The Dranchuk, Purvis, and Robinson (DPR) method uses the Benedict-Webb-Rubin equation of state [17] to correlate the Standing-Katz chart in order to approximate z as a function of (g, T, P) [44]. The DPR method uses several coefficients in a polynomial function of the reduced density, ρ_r .

Note that all these methods have a domain where they are reasonably accurate, and may break down outside. A brief description of these methods is provided next. A complete survey can be found in Menon [87].

4.3 Gas Specific Gravity Estimates

The specific gravity, g , is a dimensionless unit defined as the ratio between the density (mass per unit volume) of the actual gas and the density of air at the same temperature. Menon [87] provides a complete list of specific gravity values and other properties of various hydrocarbon gases. Published values of the specific gravity of natural gas range from 0.554 to 0.870.

Specific gravity estimates in natural gas transmission systems are based on the principle established for the pooling problem, i.e., quality constraints.

The Pooling Problem

In the oil and natural gas industry, the pooling problem refers to the scenario in which a number of different sources in a network system provide gas or oil with different quality attributes (i.e., the product being transported is made of different composition) and which flow streams must be blended in a series of pools in order to meet given customer requirements. The problem is an extension of the minimum cost flow problem on networks describing three sets of nodes: sources, pools and terminals. Here, the resulting quality of the end products, after they might have been blended each other a certain number of times, depend on what sources they originated from, and in what proportion. Typically, an expected range in the quality of the product being transported is imposed at terminals.

As specified by Adhya, Tawarmalani, and Sahinidis [3] and Foulds, Haugland, and Jørnsten [51], the mathematical model includes bilinear and nonconvex quadratic programming constraints, also referred to as quality constraints, that make it hard to solve. The nonlinearity appears in two types of constraints, namely the quality balance at pools or network elements, and quality bound at the terminals.

THEOREM 1. *The pooling problem is NP-hard even in the case of single-layer of pools.*

Proof: A poly reduction from the 3-dimensional matching problem to the single-layered pooling problem. Interested readers in the complete proof are referred to [7].

Haverly [63] presents one of the pioneering works in this field, and since then the scientific com-

munity has put special attention into proposing optimization techniques, models and applications for the pooling problem. Aggarwal and Floudas [4] present a Bender’s decomposition-based algorithm to search for global solutions to the pooling problem. Floudas and Visweswaran [49], Adhya, Tawarmalani, and Sahinidis [3] and Almutairi and Elhedhli [8] pursue the goal of the previous work and apply Lagrangian relaxation techniques.

Three main formulations of the pooling problem, based on nonlinear programming, can be found in the literature [10, 88]: i) the p-formulation (Haverly [63]) which consists of flow and quality variables; ii) the q-formulation (Ben-Tal et al. [16]) which replaces the quality variables of the p-formulation with variables representing flow proportions; and iii) the pq-formulation (Tawarmalani and Sahinidis [119]) which adds RLT cuts to the q-formulation (RLT –Reformulation-linearization technique suggested by Sherali and Adams [115].) The pq-formulation dominates (i.e., is tighter than) both the q- and p-formulations, and the p-formulation is in turn dominated by the q-formulation. However, the pq-formulation cannot be applied to networks with multi-layered of pools. For this type of networks, dry gas pipeline transportation and multi-period inventory models are applied. Audet et al. [10] propose a branch-and-cut quadratic programming algorithm to solve the pooling problem, and study two mathematical models: a flow variables-based model and a model based on flow proportions entering pools. As a result, they propose a hybrid model based on the two tested models for general pooling problems. More recently, Alfaki and Haugland [7] propose a formulation based on source and terminal proportions (denoted the STP-formulation) that is stronger than the pq-formulation proposed by Tawarmalani and Sahinidis [119], and suggest a branching strategy for solving it. The STP-formulation basically combines the source and terminal flow proportions, and defines specific flow streams on diverse paths of the network.

The literature reveals a good selection of pooling problems published by Haverly [63], Ben-Tal et al. [16], Foulds, Haugland, and Jørnsten [51], and Adhya, Tawarmalani, and Sahinidis [3].

Moreover, when uncertainty plays a key role in the design and operation of the pooling system, stochastic optimization models are applied. For instance, Li et al. [78, 79] propose a duality-based decomposition method to guarantee finding an ϵ -optimal solution for the stochastic pooling problem. The method basically decomposes the stochastic nonconvex mixed-integer nonlinear program into a series of primal bounding subproblems by convexifying and underestimating the bilinear functions. Since the resulting master problem is typically hard to solve, they apply relaxation and dualization techniques to solve a sequence of primal bounding problems, feasibility problems and relaxed master problem. The sequence of subproblems are submitted to version 8.1.5 of the global optimizer BARON. BARON [118] is an implementation of a branch-and-bound algorithm where a convex relaxation of the submitted problem is solved in each node of the search tree.

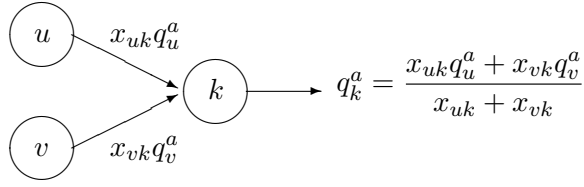


Figure 11: A flow balance imposed by quality constraints

Quality Constraints

Let u and v be two network elements such that there is a link from u to v , and let x_{uv} be the flow variable through the link. Let q_u^a denote the quality a of the flow stream leaving element u , i.e., the relative content of some gas component like e.g. CO₂. For the flow stream q_v^a entering element v , the corresponding quality constraint is given by

$$q_v^a = \frac{\sum_{u \in N_v} x_{uv} q_u^a}{\sum_{u \in N_v} x_{uv}}, \quad (3)$$

where N_v is the set of upstream elements that connect v with a direct link.

For convenience, equation (3) can be written in a non-fractional form as:

$$q_v^a \sum_{u \in N_v} x_{uv} = \sum_{u \in N_v} x_{uv} q_u^a. \quad (4)$$

As we can observe, the left-hand side of (4) is given by a bilinear term for each upstream process stage u of v . The main challenge of the quality constraints is that their terms are products of two unknown variables and cannot be linearized. Figure 11 shows the flow balance imposed by the quality constraint for the case where a network element k has two upstream neighbors, u and v .

Let $G = (V, A)$ be a directed acyclic gas transmission network, where V and A are the node and arc sets, respectively. Let $V_s \subseteq V$ be the set of supply nodes (sources), and let $V_d \subseteq V$ be the set of demand nodes (sinks). By convention, for any node $i \in V$, let $V_i^- = \{j \in V : (j, i) \in A\}$ be the set of start nodes of incoming arcs.

For a source node $i \in V_s$, it is assumed that g_i is known, and for nodes $j \in V \setminus V_s$, the gas specific gravity value is given by

$$g_j = \frac{\sum_{i \in V_j^-} g_i x_{ij}}{\sum_{i \in V_j^-} x_{ij}}. \quad (5)$$

Equation (5) expresses the specific gravity value of a blend of different flow streams as the weighted average of specific gravities of entering flows. The flow values constitute the respective weights.

The equation of specific gravity balance is obtained by multiplying (5) by the total flow:

$$g_j \sum_{i \in V_j^-} x_{ij} - \sum_{i \in V_j^-} g_i x_{ij} = 0, \quad j \in V. \quad (6)$$

4.4 Flow Accurate Estimates: Related Work

Since the last century, a substantial research work has been done in optimizing flow along arc capacity networks [5]. Among those works, the majority of those ones related to steady-state gas flow problems, have modeled the resistance of a pipeline as a function of state variables ever since the inception of gas pipeline optimization (around the middle of the 20th century). The work of Wong and Larson [130] when minimizing the total fuel cost incurred by compressor stations is a good example. They suggested to apply the well-known Weymouth equation [99] to compute the pipeline capacity. The same principle is followed by more sophisticated works, as those presented by Carter [26], Ríos-Mercado et al. [111], De Wolf and Smeers [38, 39], Borraz-Sánchez and Ríos-Mercado [23], Bakhouya and De Wolf [13], Kalvelagen [70], and Borraz-Sánchez and Haugland [19].

Carter [26] propose a non-sequential dynamic technique that outperforms hybrid methods for cyclic networks and allows rapid turnaround of optimization runs for steady-state flow models. Ríos-Mercado et al. [111] propose a reduction technique for natural gas transmission network optimization problems that substantially decreases the size of the network without altering with its mathematical properties. De Wolf and Smeers [39] present a model to solve the problem of distributing gas at a minimum cost through a pipeline network under nonlinear flow-pressure relations constraints, material balance equations, and pressure bounds. The solution method is based on piecewise linear approximations of the nonlinear flow-pressure relations. Borraz-Sánchez and Ríos-Mercado [23], motivated by the work of Carter [26], propose a non-sequential dynamic programming algorithm for optimizing large-size cyclic network systems under steady-state assumptions.

Several works, such as those presented by O’Neill et al. [97], Wilson, Wallace, and Furey [128] and De Wolf and Smeers in [38, 39], propose various MINLP models to describe the operating settings of the compressor stations. Their models, however, integrate transportation functions with gas sale planning functions. O’Neill et al. [97] and Wilson et al. [128] apply a *Successive Linear Programming*, whereas De Wolf and Smeers [38, 39] implement piecewise linear approximations solved by an extension of the Simplex algorithm [52]. Bakhouya and De Wolf [13] separate the integrated model proposed by previous works, and focus only on minimizing the total power consumption at compressor stations. They solve the problem by applying a two-phase method to Belgian and French gas transmission networks. Kalvelagen [70] proposes an improved model for the MINLP gas transportation problem and solves it using GAMS [33]. Borraz-Sánchez and Haugland [19] efficiently tackle the fuel cost minimization problem in steady-state natural gas transmission networks by proposing and implementing a dynamic programming-based tree decomposition algorithm.

All the cited works neglect the fact that the parameter in the Weymouth equation depends

not only on pipeline characteristics, but also on thermodynamic and physical gas properties. This includes temperature, specific gravity (relative density) and compressibility (z -factor), which are assumed as universal constants in these works. In instances where the network elements show no or only modest variation in these properties, it may be valid to neglect their variability and to represent them by global constants. This does however not seem to be the case in all real-life instances.

Examples where the assumption is unrealistic exist. The pipeline network connecting wells on the Norwegian continental shelf with the European continent is supplied by gas from sources of relatively lean gas, situated in the North Sea, and sources located in e.g. the Haltenbank area. Since the latter area generally has richer gas, in the sense that it consists of components of higher specific gravity, the assumption of constant properties may be unrealistic. Also, gas compressibility depends on current temperature and pressure conditions, which also vary along the transmission line.

The literature on optimization models for pipeline gas transportation does not seem to be very rich on models with variable specific gravity or compressibility, and most works focus on models for transient flow. Abbaspour and Chapman [2], for example, analyze non-isothermal transient flow of gas in natural gas pipeline. Their work is based on z -factor estimates as a function of pressure and temperature. They assume a steady-state heat flow between the gas in the pipeline and the surroundings. Chaczykowski [29] studies one-dimensional, non-isothermal gas flow model to simulate slow and fast fluid transients. Their work is based on unsteady heat transfer term in the energy equation. Simulations of two gas transmission pipeline networks were conducted to show that the unsteady heat transfer model hinders the gas temperature changes while considering the heat in the surroundings.

In steady-state flow models, Belyaev, Gushchin, and Shenbrot [14] provide arguments on potential error causes in gas metering by studying a real-test case from the Russian Federation. Belyaev and Patrikeev [15] present a study on the influence of variations of the gas composition by using correction factors that depend on the density under standard conditions. Their work is based on readings of all the instruments involved in commercial operations during a certain period. Bahadori, Mokhatab, and Towler [12] propose and develop a new method to account for difference in z -factor estimates between natural gases containing sour components and those ones characterized as sweet gases. Their work is based on two correlations for computing pseudo-critical pressure and temperature values as a function of the gas specific gravity. As a result, a simplified calculation method is introduced for quick estimations of z -factor values for sour natural gases.

Borraz-Sánchez and Haugland [21] study the effect caused by the variability of the specific gravity and compressibility of the gas on flow estimates in transmission pipeline systems. They extended previously suggested models by incorporating the variation in pipeline flow capacities with gas specific gravity and compressibility. Their work also applies the principle stated by the

Weymouth equation to compute the resistance of the pipeline, and makes use of the California Natural Gas Association method [36], which depends on gas specific gravity and pressure values, to compute gas compressibility values in each pipeline of the network system. The variability of specific gravity is then estimated at junction points as the weighted average of specific gravities of entering flows. Due to the resulting nonconvex model, they propose a heuristic that iteratively solves a simpler model by means of a global optimizer. Their solution approach turns out to be very promising while providing exact solutions to many test instances and finding deviations less than 12% from optimality in the remaining cases.

5 Compressor Station Modeling

5.1 Introduction to the Fuel Cost Minimization Problem in Natural Gas Pipeline Systems

As natural gas pipeline systems have grown larger and more complex, the importance of optimal operation and planning of these facilities has increased. The investment costs and operation expenses of pipeline networks are so large that even small improvements in system utilization can involve substantial amounts of money.

The natural gas industry services include producing, moving, and selling gas. The main focus in this section is on the transportation of gas through a pipeline network. Moving gas is divided into two classes: transmission and distribution. Transmission of gas means moving a large volume of gas at high pressures over long distances from a gas source to distribution centers. In contrast, gas distribution is the process of routing gas to individual customers. For both transmission and distribution networks, the gas flows through various devices including pipes, regulators, valves, and compressors. In a transmission network, gas pressure is reduced due to friction with the pipe wall as the gas travels through the pipe. Some of this pressure is added back at compressor stations, which raises the pressure of the gas passing through them.

In a gas transmission network, the overall operating cost of the system is highly dependent upon the operating cost of the compressor stations in a network. A compressor station's operating cost, however, is generally measured by the fuel consumed at the compressor station. According to Luongo, Gilmour, and Schroeder [80], the operating cost of running the compressor stations represents between 25% and 50% of the total company's operating budget. Hence, the objective for a transmission network is to minimize the total fuel consumption of the compressor stations while satisfying specified delivery flow rates and minimum pressure requirements at the delivery terminals.

Depending on how the gas flow changes with respect to time, we distinguish between systems in steady state and transient state. A system is said to be in steady state when the values charac-

terizing the flow of gas in the system are independent of time. In this case, the system constraints, particularly the ones describing the gas flow through the pipes, can be described using algebraic nonlinear equations. In contrast, transient analysis requires the use of partial differential equations (PDEs) to describe such relationships. This makes the problem considerably harder to solve from the optimization perspective. In fact, optimization of transient models is one of the most challenging ongoing research areas. In the case of transient optimization, variables of the system, such as pressures and flows, are functions of time.

The issue of how to design a pipeline network involves decisions on diameter and length of pipes, location of compressor stations, and network configuration. For works on optimal design of pipeline systems the reader is referred to the work of Babonneau, Nesterov, and Vial [11], Costa, de Medeiros, and Pessoa [34], El-Shiekh [46], Marcoulaki, Papazoglou, and Pixopoulou [84], Mariani, Ancillai, and Donati [85], Osiadacz and Górecki [104], Sanaye and Mahmoudimehr [112], Tsal et al. [120], and Zhou, Liu, and Li [138]. In this section, we survey the most significant work on both steady-state and transient gas transmission network problems, assuming an existing pipeline system, with the objective of minimizing the operational costs.

Gas transmission network problems differ from traditional network flow problems in some fundamental aspects. First, in addition to the flow variables for each arc, which in this case represent mass flow rates, a pressure variable is defined at every node. Second, besides the mass balance constraints, there exist two other types of constraints: (i) a nonlinear equality constraint on each pipe, which represents the relationship between the pressure drop and the flow; and (ii) a nonlinear nonconvex set which represents the feasible operating limits for pressure and flow within each compressor station. The objective function is given by a nonlinear function of flow rates and pressures. The problem is very difficult due to the presence of a nonconvex objective function and nonconvex feasible region.

5.2 Description of Basic Model

Let $G = (V, A)$ be a directed graph representing a natural gas transmission network, where V is the set of nodes representing interconnection points, and A is the set of arcs representing either pipelines or compressor stations. Let V_s and V_d be the set of supply and demand nodes, respectively. Let $A = A_p \cup A_c$ be partitioned into a set of pipeline arcs A_p and a set of compressor station arcs A_c . That is, $(u, v) \in A_c$ if and only if u and v are the input and output nodes of compressor station (u, v) , respectively.

Two types of decision variables are defined: Let x_{uv} denote the mass flow rate at arc $(u, v) \in A$, and let p_u denote the gas pressure at node $u \in V$. The following parameters are assumed known: B_u is the net mass flow rate in node u , and P_u^L and P_u^U are the pressure limits (lower and upper) at node u . By convention, $B_u > 0$ ($B_u < 0$) if $u \in V_s$ ($u \in V_d$), and $B_u = 0$ otherwise.

The basic mathematical model of the minimum fuel cost problem (MFCP) is given by:

$$\min \quad g(x, p) = \sum_{(u,v) \in A_c} g_{uv}(x_{uv}, p_u, p_v) \quad (7)$$

$$\text{subject to } \sum_{v:(u,v) \in A} x_{uv} - \sum_{v:(v,u) \in A} x_{vu} = B_u \quad u \in V \quad (8)$$

$$(x_{uv}, p_u, p_v) \in D_{uv} \quad (u, v) \in A_c \quad (9)$$

$$x_{uv}^2 = R_{uv}(p_u^2 - p_v^2) \quad (u, v) \in A_p \quad (10)$$

$$p_u \in [P_u^L, P_u^U] \quad u \in V \quad (11)$$

$$x_{uv} \geq 0 \quad (u, v) \in A \quad (12)$$

The objective function (7) measure the total amount of fuel consumed in the system, where $g_{uv}(x_{uv}, p_u, p_v)$ denotes the fuel consumption cost at compressor station $(u, v) \in A_c$. For a single compressor unit the following function is typically used:

$$g^{(1)}(x_{uv}, p_u, p_v) = \frac{\alpha x_{uv}}{\eta} \left\{ \left(\frac{p_v}{p_u} \right)^m - 1 \right\},$$

where α and m are assumed constant and known parameters that depend on the gas physical properties, and η is the adiabatic efficiency coefficient. This adiabatic coefficient is a function of (x_{uv}, p_u, p_v) that is, in general, a complex expression, implicitly defined. A function evaluation of η requires solving a linear system of algebraic equations. In practice, though, polynomial approximation functions that fit the function relatively well and are simpler to evaluate are employed. In other cases, when the fluctuations of η are small enough, η can be assumed to be a constant.

For a compressor station (u, v) with n_{uv} identical compressor units hooked-up in parallel which is very commonly found in industry, the fuel consumption is given by:

$$g_{uv}(x_{uv}, p_u, p_v) = n_{uv} g^{(1)}(x_{uv}/n_{uv}, p_u, p_v).$$

When all n_{uv} units are fixed and operating we have a nonlinear programming (NLP) model. Treating n_{uv} as decision variables, leads to mixed-integer nonlinear programming (MINLP) models.

Constraints (8) establish the mass balance at each node. Constraints (9) denote the compressor operating limits, where D_{uv} denote the feasible operating domain for compressor $(u, v) \in A_c$. Equations (10) express the relationship between the mass flow rate through a pipe and its pressure values at the end points under isothermal and steady-state assumptions, where R_{uv} (also known as the pipeline resistance parameter) is a parameter that depends on both the physical characteristics of the pipeline and gas physical properties. When the steady-state assumption does not hold, this relationship is a time-dependent partial differential equation which leads to transient models. Constraints (11) set the lower and upper limits of the pressure value at every node, and (12) set

the non-negativity condition of the mass flow rate variables. Further details of this model can be found in Wu et al. [133].

5.3 Network Topology

There are three different kinds of network topologies: (a) linear or gun-barrel, (b) tree or branched, and (c) cyclic. Technically, the procedure for making this classification is as follows. In a given network, the compressor arcs are temporarily removed. Then each of the remaining connected components are merged into a big super-node. Finally, the compressor arcs are put back into their place. This new network is called the *associated reduced network*. Figure 12 illustrates the associated reduced network for a 9-node, 8-arc example. As can be seen, the reduced network has 3 supernodes (labeled S1, S2, S3) and 2 arcs (the compressor station arcs from the original network).

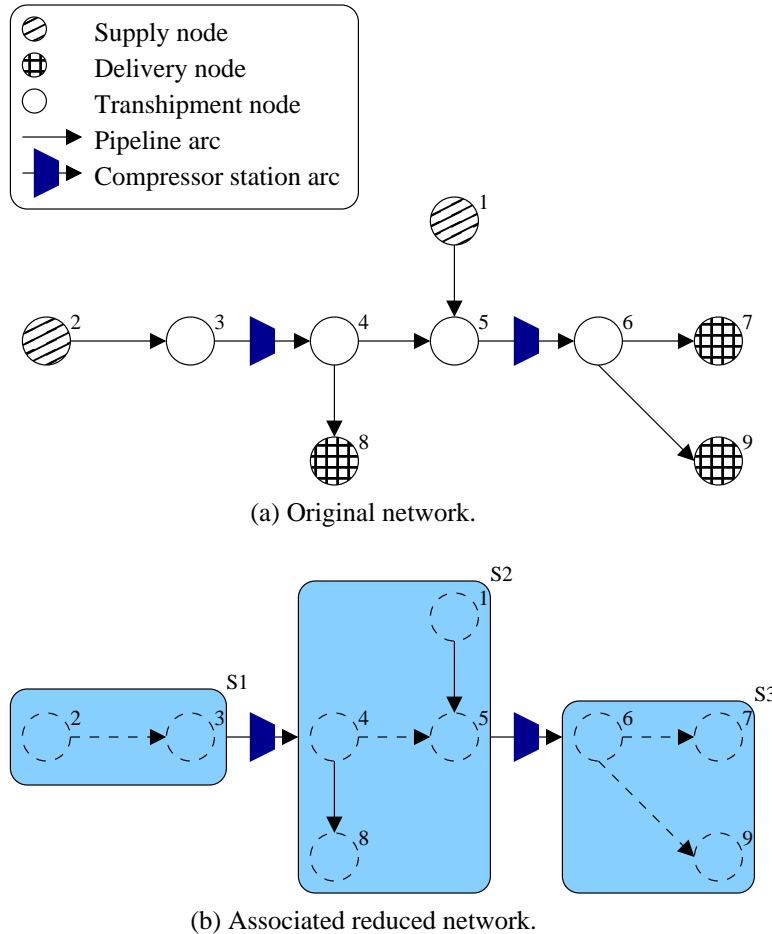


Figure 12: Illustration of a reduced network.

Types of network topologies:

Linear topology: This corresponds to a linear arrangement of the compressor station arcs, that is, when the reduced network is a single path.

Tree topology: This occurs when the compressors are arranged in branches through the system, that is, when the reduced network is a tree.

Cyclic topology: This happens when compressors are arranged forming cycles with other compressor stations. That is, it refers to a cyclic reduced network.

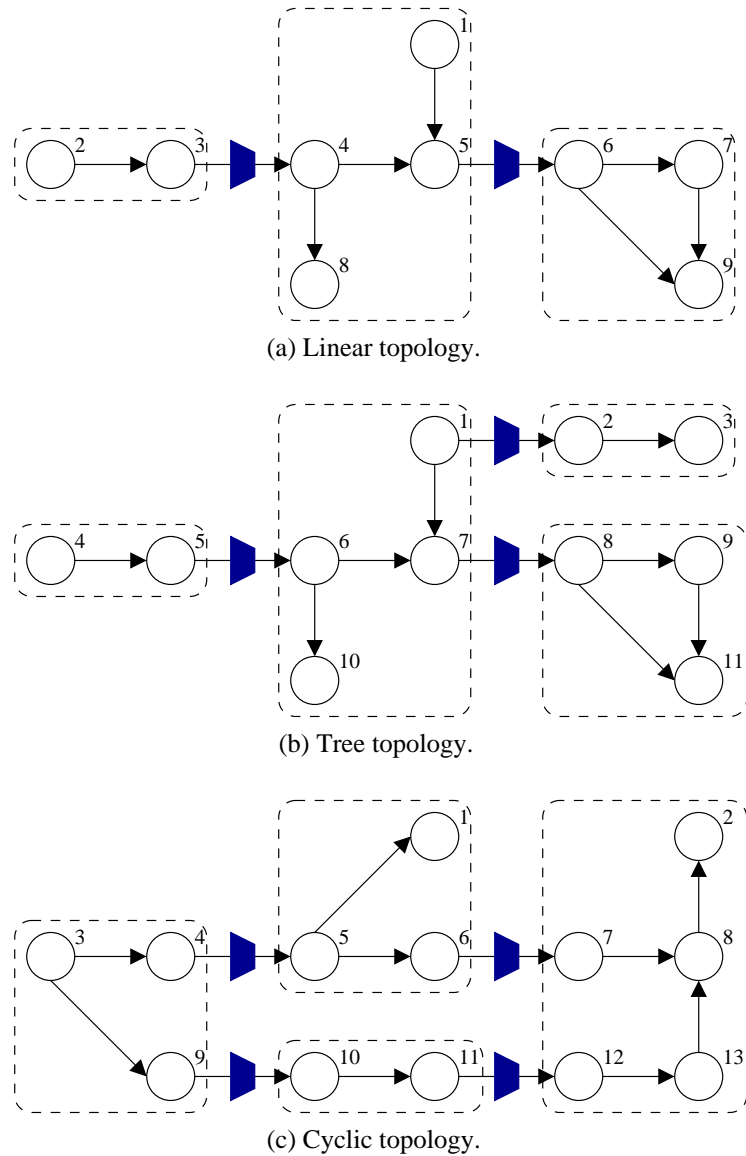


Figure 13: Different kinds of pipeline network topologies.

These different types of network topologies are shown in Figure 13, where the original network is represented by solid line nodes and arcs, and the reduced network by dotted super nodes. Note that even though networks in Figure 13(a) and 13(b) are not acyclic from a strict network definition, they are considered as non-cyclic pipeline network structures.

As it will be seen in the following section, the state of the art on steady-state systems establishes

that linear and tree topologies are more tractable despite the nonconvexity of the problem. Since it has been shown that in this type of topologies, under certain conditions the flow variables can be uniquely determined [111], techniques such as dynamic programming have been successfully applied to solve for the discrete set of pressure variables. Cyclic structures are harder to solve, and, regardless of network topology, transient systems are even more challenging.

5.4 Steady-State Models

This section focuses on reviewing optimization models and approaches. Descriptive simulation models are out of the scope of this work. Nonetheless, for research on simulation of natural gas networks or descriptive simulation models the reader is referred to the seminal work of Osiadacz [99], or the recent work of Zhu, Henson, and Megan [139], Herrán-González et al. [65], and Woldeyohannes and Majid [129].

Optimization of a compressor station has been studied previously by Mahmoudimehr and Sanaye [82] Osiadacz [98], Percell and Van Reet [106], and Wu, Boyd, and Scott [132]. These works focused on a mathematical model in a single compressor unit. Later, Wu et al. [133] completed the analysis for the same problem, but considering several units within compressor stations. Krishnaswami, Chapman, and M. Abbaspour [73] present a systematic approach for operating the units of a compressor station to meet a specified line-pack profile. Nguyen et al. [95] present a comparison of three automation approaches for compressor selection. All these works study the behavior of a compressor station. In this survey we focus on the entire pipeline network optimization.

Model Properties and Lower Bounds

Ríos-Mercado et al. [111] present a study of the properties of gas pipeline networks on steady-state, and exploit them to develop a technique that can be used to significantly reduce problem dimension without disrupting problem structure. This technique has been successfully used in many works on pipeline optimization.

Wu et al. [133] present an in-depth study of the underlying mathematical structure of the compressor stations of the MFCP. Then, based on this study, they propose two model relaxations (one in the compressor domain and another in the fuel cost function) and derive a lower bounding scheme. They present empirical evidence that shows the effectiveness of the lower bounding scheme. For the small problems, where optimal solutions were known, the proposed lower bound yield a relative optimality gap of around 15–20%. For a larger, more complex instance, it was not possible to find optimal solutions, but they were able to compute lower and upper bounds, finding a large relative gap between the two. They show this wide gap is mainly due to the presence of nonconvexity in the set of feasible solutions, since the proposed relaxations did a very good job of

approximating the problem within each individual compressor station. Later, Borraz-Sánchez and Ríos-Mercado [23] compute a lower bound for cyclic instances obtaining relative optimality gaps of less than 16%, and, in most of the cases, less than 10%.

Methods Based on Dynamic Programming

One of the most successful technique for addressing the MFCP is Dynamic Programming (DP). One of the main advantages of DP is that a global optimum is guaranteed to be found and that nonlinearity can be easily handled. Until very recently, its application had been practically limited to noncyclic networks, such as linear (also known as gun-barrel) or tree topologies. It is well known that in DP computation time increases exponentially with the dimension of the problem, commonly referred as the curse of dimensionality.

DP for pipeline optimization was originally applied to gun-barrel systems in the late 1960s. It has been one of the most useful techniques due to both its computational behavior and its versatility for handling nonlinearity on sequential systems. DP was first applied to linear systems by Wong and Larson [130] in 1968, and then applied to tree-structured topologies by Wong and Larson [131]. A similar approach was described by Lall and Percell [76] in 1990, who allow one diverging branch in their system.

In 1989, Luongo, Gilmour, and Schroeder [80] published a hierarchical approach that allowed for both cycles and branches of arbitrary complexity. This represented significant progress in terms of finally addressing the issue of real world pipeline configurations. Their technique was no longer pure DP. Basically, DP was used to optimally describe the pieces of the pipeline that were arranged in a sequential manner. This typically reduced the system to a much smaller combinatorial problem, without any possibility of a recursive DP solution. A sufficiently small instance could be solved exactly via enumeration; otherwise it was solved inexactly using simulated annealing. This hierarchical approach worked very well for some complex pipelines, but for others the computational cost was very high.

One of the most significant works on cyclic networks known to date is due to Carter [26] who developed a non-sequential DP algorithm, but limited to a fixed set of flows. This led to an interesting question of how to find the optimal setting of the flow variables and how to modify the current flow setting to obtain a better objective value. Extensions to work on cyclic systems addressing these issues were developed by Ríos-Mercado, Kim, and Boyd [110]

Ríos-Mercado, Kim, and Boyd [110] propose a heuristic solution procedure for fuel cost minimization on gas transmission systems with a cyclic network topology. Their heuristic solution methodology is based on a two-stage iterative procedure. In a particular iteration, at a first stage, gas flow variables are fixed and optimal pressure variables are found via dynamic programming. At a second stage, pressure variables are fixed and an attempt is made to find a set of flow variables

that improve the objective function by exploiting the underlying network structure. They tested their algorithm in some real-world instances provided by a Houston-based company. Example 1 is a tree-structured system with 16 compressor stations, 56 pipes, and 64 total nodes (that is including supply, demand, and transshipment nodes). Example 2 is single-cycle system with 6 compressor stations, 9 pipes, and 14 nodes. Example 3 is a multi-cycle system with 17 compressor stations, 23 pipes, and 35 nodes. Empirical evidence supports the effectiveness of the proposed procedure by finding relative improvements ranging from 3.34% to 41.77% in the instances tested.

Borraz-Sánchez and Ríos-Mercado [22, 23] propose a hybrid metaheuristic procedure that efficiently exploits the problem structure. This hybrid procedure combines very effectively a non-sequential dynamic programming algorithm for finding an optimal set of pressure variables for a fixed set of mass flow rate variables, and short-term memory tabu search procedure for guiding the search in the flow variable space. Empirical evidence over a number of instances supports the effectiveness of the proposed procedure outperforming a multi-start generalized reduced gradient (GRG) method both in terms of solution quality and feasibility. Furthermore, to assess the quality of the solutions obtained by the algorithm, a lower bound is derived. It is found that the solution quality obtained by the proposed lower bounding procedure is relatively good.

Borraz-Sánchez and Haugland [20] present a two-phase method for the MFCP. As suggested by previous work, they consider a procedure where each iteration consists of a flow improvement step and a pressure optimization step. Alternating between flow and pressure, one set of decision variables is kept fixed in each step. Still in agreement with previously suggested methods, the nonconvex subproblem of optimizing pressure is approximated by a combinatorial one. This is accomplished by discretization of the pressure variables. The contribution of their work is a method for solving the discrete version of the problem in instances where previously suggested methods fail. Unlike methods based on successive network reductions, their method does not make any assumptions concerning the sparsity of the network. By constructing a tree decomposition of the network, and applying dynamic programming to it, they were able to solve the discrete version of the pressure optimization problem without enumerating the whole solution space. By an adaptive discretization scheme, they obtain significant speed-up of the dynamic programming approach in comparison with fixed discretization. They tested their proposed solution method on a set of test instances, and compared the results to those obtained by applying both a global and a local optimizer to the continuous version of the problem. The experiments indicate that a method guaranteeing the global optimum in reasonable time seems unrealistic even for small instances. Further, discretizing the pressure variables and applying dynamic programming to a tree decomposition gives better results than applying a commercially available local optimization package.

The details on the DP formulation can be found in the referred works or in the work by Ríos-Mercado [109].

Methods Based on Gradient Search

In 1987, Percell and Ryan [107] applied a different methodology based on a Generalized Reduced Gradient (GRG) nonlinear optimization technique for noncyclic structures. One of the advantages of GRG, when compared with DP, is that they can handle the dimensionality issue relatively well, and thus, can be applied to cyclic structures. Nevertheless, being a method based on a gradient search, there is no guarantee for a global optimal solution. Villalobos-Morales and Ríos-Mercado [125] evaluated preprocessing techniques for GRG, such as scaling, variable bounding, and choice of starting solution, that resulted in better results for both cyclic and noncyclic structures. Flores-Villarreal and Ríos-Mercado [48] performed an extensive computational evaluation of the GRG method over a large set of instances on cyclic structures with relative success.

Linearization Approaches

De Wolf and Smeers [39] take a different angle to the problem. They present a solution method based on piecewise linear approximations of the nonlinear flow-pressure relations. The approximated problem is solved by an extension of the Simplex method. The solution method is illustrated in an instance of the Belgium gas network, and solved some real-world cases. They compare their approach with other LP-based approach, called Successive Linear Programming (SLP). They found their proposed approach takes less time than SLP. They also found that, as the model is in general nonconvex, the choice of the starting point was crucial if one limits oneself to find only local solutions or upper bounds on the solution in global procedures. Thus, they devised a mechanism for generating the initial solution that was empirically shown to reduce running times by 50%.

Jin and Wojtanowicz [68] present a study aimed at optimizing a very large case study in China. The large size and complex geometry of network required breaking it down into smaller components, optimizing operations of the components locally, re-combining the optimized components into the network and optimizing the network globally. This four-step approach employed four different optimization methods to solve the problem: a penalty function method, pattern search, enumeration, and non-sequential dynamic programming. The results of applying global optimization show that the increase in gas throughput considerably reduces cost savings. For instance, a reduction of operational cost savings from 23% up to 1.2% was observed when increasing the gas rate from 67 to 90 million m^3/d . The study also shows that operation costs approach those found in current practice when compressor stations work at their maximum capacity. Hence, global optimization proves to be more effective when the gas pipeline system works at any mass flow rate other than its maximum rating, a typical case of present operation in Chinese gas networks.

Approaches for MINLP Models

Pratt and Wilson [108] propose a successive mixed-integer linear programming method. Their algorithm solves the nonlinear optimization problem iteratively by linearizing the pressure drop-flow equations (10). Integer variables are included in the formulation for compressor unit selection, and the problem is solved using branch and bound.

Cobos-Zaleta and Ríos-Mercado [31] presented a solution technique based on an outer approximation with equality relaxation and augmented penalty algorithm OA/ER/AP for solving a mixed-integer nonlinear programming model, where an integer decision variable, representing the number of compressor units running within each station, is incorporated. They present satisfactory results as they were able to find local optima for many instances tested.

Martin, Möller, and Moritz [86] incorporate binary decision variables to decide whether to use or not a compressor unit within a compressor station and whether to open or close valves. They describe some techniques for a piece-wise linear approximation of the nonlinearities of the model resulting in a large mixed-integer linear program. They study sub-polyhedra linking these piecewise linear approximations and show that the number of vertices is computationally tractable yielding exact separation algorithms. They also present suitable branching strategies complementing the separation algorithms. They tested their method on three real-world instances provided by their industrial partner, E.ON Ruhrgas AG, a German gas company. The size of the instances range from 11 to 31 pipes and from 3 to 15 compressor stations. They observed that the piece-wise linear approximation is accurate enough to guarantee globally optimal solutions.

Chebouba et al. [30] present an Ant Colony Optimization (ACO) algorithm for the MFCP with a variable number of compressor units within a compressor station. Part of the decision process involves determining the number of operating units in each compressor. The ACO algorithm [42] is a new evolutionary optimization method to solve different combinatorial optimization problems. They tested their method on the Hassi R'mell-Arzew real-world pipeline network in Argelia consisting of 5 pipes, 6 nodes, 5 compressor stations, and 3 units in each compressor. They also built three additional cases with up to 23 compressor stations, and 12 compressor units in each compressor. They compare their method with a DP implementation. Their empirical work shows a good performance of the proposed method in noncyclic systems.

Tabkhi et al. [116] present a computational study of MFCP applied to a case study in the French company Gaz de France. The authors present a MINLP model where binary variables for representing pipeline flow direction are introduced. They used the GAMS/SBB solver for solving the MINLP model, which calls CONOPT for solving the NLP subproblems. The real-world case has 30 pipes and 6 compressor stations. To make the problem more tractable for the solver, the authors consider several different strategies for initializing some or all the binary variables. They also report on a sensitivity analysis discussing one of the particular strategies for initializing the

binary variables.

Wu et al. [134] present a hybrid objective model with compressor switching constraints that aims at maximizing revenue and throughput while considering a weighting value to account for both optimization problems. The model is solved by means of a particle swarm optimization (PSO) algorithm that includes an adaptive inertia weight adjusting procedure to overcome premature convergence issues. A commercially available simulation software is used to provide the initial particles that satisfy the underlying model. The authors present a case study based on a Chinese gas pipeline system with a gun-barrel topological structure and four compressor stations. The proposed algorithm showed a fast convergence speed when compared with other extensions of the PSO algorithm.

Table 1: Summary of research on steady-state models.

Work	Model	N	Approach
Wong and Larson [130]	NLP	L	DP
Percell and Ryan [107]	NLP	L, T	GRG
Villalobos-Morales and Ríos-Mercado [125]	NLP	L, T	GRG
De Wolf and Smeers [39]	NLP	T	Linearization
Lall and Percell [76]	NLP	T	DP
Luongo et al. [80]	NLP	T	Hierarchical, DP, SA
Wong and Larson [131]	NLP	T	DP
Borraz-Sánchez and Haugland [20]	NLP	C	NDP, tree decomposition, adaptive discretization
Borraz-Sánchez and Ríos-Mercado [23]	NLP	C	Tabu search and NDP
Carter [26]	NLP	C	NDP
Flores-Villarreal and Ríos-Mercado [48]	NLP	C	GRG
Jin and Wojtaniowicz [68]	NLP	C	Penalty function, pattern search, NDP
Ríos-Mercado et al. [110]	NLP	C	Decomposition, DP
Chebouba et al. [30]	MINLP	L	ACO
Martin et al. [86]	MINLP	L	Linearization
Cobos-Zaleta and Ríos-Mercado [31]	MINLP	T	OA/ER/AP
Pratt and Wilson [108]	MINLP	C	Successive MILP, B&B
Tabkhi et al. [116]	MINLP	C	SBB/CONOPT
Wu et al. [134]	MINLP	L	PSO

Notation
N := Network topology (L = linear; T = tree; C = cyclic)
Approach: ACO = Ant colony optimization; NDP = Non-sequential DP; SA = Simulated annealing;
SBB = Standard branch and bound; PSO = Particle swarm optimization

Table 1 summarizes the research on steady-state models for pipeline network optimization. Entries are first sorted by model type, and then by network topology.

Other Models

Carter et al. [27] describe a class of noisy optimization problems from the gas transmission industry, and propose an algorithms for their solution. The algorithms they consider are implicit filtering [57], the global optimization algorithm DIRECT [69] and a new hybrid of implicit filtering and DIRECT, which attempts to capture the best features of the two. They consider minimizing

the cost of fuel for the compressor stations in a gas pipeline network. This cost can be reduced by changing both flow patterns through the system and pressure settings throughout the system. In their model the problems have two flows as design variables. The flow variables can be either unknown inlet or outlet flows, or Kirchhoff's law representations of flow splits between different possible alternative paths. Once these boundary and loop flows have been specified, each evaluation of the objective function f involves solving a hierarchy of embedded optimization problems and associated simulation subproblems, and then evaluating the total fuel used at the solutions to the subproblems. The main decision variables in the subproblems are pressure settings throughout the pipeline system, and on/off settings for the large number of individual compressors throughout the system. The value of the objective function f is the total fuel used. The hierarchical optimization that is internal to the function evaluation involves solving a large combinatorial problem using non-sequential dynamic programming [26]. In this formulation, the pressure settings at the discharge side of each compressor station are first discretized into a set of discrete values covering the range of potentially attainable values for the equipment being simulated. Hydraulic analysis is used to propagate each discretized pressure at each station discharge forward in space, which establishes an implicit discretization of potential pressures at the suction side of each station as well. For any station (with specified flow) they pick each possible pair inlet and outlet pressures from the discretization and determine whether station operation is feasible and, if so, what is the fuel cost. This determination in itself is a substantial mixed integer nonlinear optimization and simulation problem [25]. Once the local consequences of operating each station at different combinations of inlet and outlet pressure have been computed, non-sequential dynamic programming is used to select the best possible combination of discretized pressures throughout the system while maintaining hydraulic integrity, satisfying the pressure drop equations between stations, and observing all equipment limitations. They apply the three methods to some instances from the gas pipeline industry and to a suite of test problems from the global optimization literature. They found that the performance of implicit filtering depends strongly on its starting point. When implicit filtering found a feasible point, then it performed much better than either DIRECT or the hybrid. However, for a large percentage of starting points implicit filtering did not find a feasible point. DIRECT did relatively poorly in all problems, needing a larger amount of function evaluations to get comparable results. For the suite of test problems from the literature, implicit filtering was trapped in a local minimum for a significant fraction of the runs. DIRECT was more robust, but not completely successful. They conclude that the hybrid algorithm offers the best compromise between low cost and robustness.

Wu, Lai, and Liu [135] consider a gas transportation problem in a distribution network rather than a transmission network. A pipeline network is generally established either to transmit gas at high pressure from coastal supplies to regional demand points (transmission network) or to distribute gas to consumers at low pressure from the regional demand points (distribution network).

In this study, the distribution network is considered. The distribution network differs from the transmission one in a number of ways. Pipes involved in a distribution network are often much smaller and the network is simpler, having no valves, compressors or nozzles. In that paper, the authors introduce the problem of minimizing the cost of pipelines incurred by driving the gas in a distribute nonlinear network under steady-state assumptions. In particular, the decision variables include the length of the pipes' diameter, pressure drops at each node of the network, and mass flow rate at each pipeline leg. They establish a mathematical optimization model of this problem, and then present a global optimization approach, which is based on the GOP primal-relaxed dual decomposition method by Visweswaran and Floudas [126]. Their method is successfully tested on two real-world instances having 6 nodes and 5 pipes, and 13 nodes and 14 pipes.

Borraz-Sánchez and Haugland [21] extend previously suggested models by incorporating the variation in pipeline flow capacities with gas specific gravity and compressibility for a steady-state isothermal model. Flow capacities are modeled as functions of pressure, compressibility and specific gravity by the commonly-used Weymouth equation. In their work, the California Natural Gas Association method is used to model compressibility as a function of specific gravity and pressure. The sources feeding the transmission network do not necessarily supply gas with equal specific gravity. In their model, they assumed that when different flow streams enter a junction point, the specific gravity of the resulting flow is a weighted average of the specific gravities of entering flows. To handle the nonconvex NLP model, they propose a heuristic method based on an iterative scheme in which a simpler NLP model is solved in each iteration. Computational experiments are conducted in order to assess the computability of the model by applying a global optimizer, and to evaluate the performance of the heuristic approach. When applied to a wide set of test instances, the heuristic method provides solutions with deviations less than 10% from optimality, and in many instances turns out to be exact. They also report several experiments demonstrating that letting the compressibility and the specific gravity be global constants can lead to significant errors in the estimates of the total network capacity.

MohamadiBaghmolaei et al. [90] take a different angle at the MFCP. They argue that there might be some cases where accurate information about the process may not be available or the system may have a nonlinear time variable behavior. In such cases, due to the lack of information and difficulties in prediction of gas turbine and compressor efficiency, techniques which depend on experimental data such as artificial neural networks (ANNs) may be applied. In their work they use ANNs within a genetic algorithm to predict the relationship among the decisive parameters and minimize the fuel consumption in a pipeline network. They apply their approach to a case study in the south of Iran considering a linear system with four compressor stations. The comparison between the efficient total fuel consumption and the final delivery pressure predicted by ANN and conventional numerical models confirms the accuracy of the proposed method.

Gopalakrishnan and Biegler [60] study a Nonlinear Model Predictive Control (NMPC) formu-

lation for optimizing the operational costs of gas pipeline networks. They use an economic NMPC formulation, which directly considers the compressor operating cost as the controller objective. Due to diurnal gas demands, the optimal operation is a cyclic steady state. The controller objective and terminal constraints are suitably defined to ensure asymptotic convergence and closed-loop stability of the cyclic steady state. It is shown through simulations that the performance of the economic NMPC formulation is better than a tracking NMPC. The inherent robustness of the formulation also ensures convergence to a region around the cyclic steady state when demand forecasts are inaccurate. The large scale NLP is also solved within a reasonable CPU time making it practical for online application

To the best of our knowledge, apart from the work of Wu et al. [134] where a hybrid objective model is presented along with a weighting value that accounts for both optimization problems, the only work on the MFCP from a biobjective optimization perspective is due to Hernandez Rodriguez et al. [64] who consider the minimization of fuel consumption and the maximization of gas mass flow delivery simultaneously. They present a computational comparison between a genetic algorithm (GA) coupled with a Newton-Raphson procedure and the well-known ϵ -constraint method for multiobjective programming. Additionally, a study of carbon dioxide (CO₂) emissions is carried out. In their empirical work, it was observed that the two methods obtain overlapping Pareto fronts, however, the one obtained by the GA is considerably larger than the one obtained from the ϵ -constraint method. Along the Pareto front provided by the GA, the CO₂ emissions vary from 1.1% to 1.8% of the natural gas flow delivery. A related study by Garcia-Hernandez and Brun [56] focuses on maximizing flow rate by keeping optimal conditions on the available compression power.

5.5 Transient Models

Transient models are more challenging as the governing PDEs associated to the dynamics of the gas system must be taken into consideration. There has been certainly some research done from a descriptive perspective. Here we survey the most significant work related to the optimization of transient systems.

Hierarchical Control Approaches

Optimization techniques have also been applied for transient (time dependent) models. For instance, Larson and Wismer [77] propose a hierarchical control (HC) approach for a transient operation of a gun-barrel pipeline system. Osiadacz and Bell [102] suggest a simplified algorithm for the optimization of the transient gas transmission network, which is based on a HC approach. The HC approach for transient models can be found in Anglard and David [9], Osiadacz [100], and Osiadacz and Swierczewski [105]. Some degree of success has been reported from these approaches as far

as optimizing the compressor station subproblem. However, these approaches have limitations in globally optimizing the minimum cost.

One of the most significant early efforts to address transient flow in natural gas pipeline systems from an HC standpoint was due to Osiadacz [101] who developed an algorithm based on hierarchical control and network decomposition. Local problems were solved using a gradient search technique. The subsystems are coordinated using a *goal coordination method* to find the global optimum. He formulated discrete state equations for the case in which output pressures are treated as elements of the control vector. The algorithm was tested using part of the National Grid of UK containing 23 nodes, 13 pipelines, 3 compressor stations, 2 storage supply nodes, and 1 source. A time frame of 24 hours with time discretization steps of 2 hours were established. The results were somewhat similar to those obtained by an alternate algorithm based on sequential quadratic programming due to Furey [55]. The maximum discrepancy found was in the order of 15%. The authors indicate that dynamic instances over 24 hours could not be solved exactly in reasonable times. The authors conclude by suggesting their proposed method based on decomposition-coordination is suitable for parallel computing.

Mathematical Programming Approaches

Early work on transient optimization of natural gas pipeline systems is due to Mantri, Preston, and Pringle [83]. They develop a transient gas optimization model that minimizes the cost of transporting natural gas over time periods in which line-pack and throughput are changing due to designated fluctuations in supply and demand. The major component of their optimization engine is based on the GRG method and dynamic programming. Tao and Ti [117] derive a method for transient analysis in a gas pipeline network. Traditionally, the governing equations for transient analysis of a gas pipeline system involve two partial differential equation, which are normally solved by complex numerical methods. The authors extend the electric analogy method by combining resistance and capacitance, which leads to a first order ordinary differential equation and an alternative way to solving the transient problem. The proposed method was found more efficient than previous approaches. Later Ke and Ti [72] use the same analogy to an electrical system to develop a new model. Empirical evidence shows that solutions obtained under this new model are compatible with those using previous models. This new model is found more tractable. Osiadacz and Chaczykowski [103] present a comparison between isothermal and non-isothermal models for transient flow in natural gas pipeline systems.

Ehrhardt and Steinbach [45] address a transient pipeline optimization problem. They present appropriate space and time discretizations to obtain a large-scale nonlinear programming problem (NLP). This large-scale NLP is solved by the general-purpose NLP code SNOPT in combinations with the automatic differentiation add-on SnadiOpt. They tested their approach on a relatively

small network with three compressor stations considering different scenarios.

Aalto [1] present a study on real time optimization of a natural gas pipeline in transient conditions. He points out that many pipeline systems are, however, only mildly nonlinear even in large transients such as compressor station (CS) shutdown or startup. A dynamic, receding horizon optimization problem is defined, where the free response prediction of the pipeline is obtained from a pipeline simulator and the optimal values of the decision variables are obtained solving an approximate Quadratic Programming (QP) problem where the cost function is the energy consumption of the CSs. The problem is extended with discrete decision variables, the shutdown/start-up commands of CSs. A Mixed Logical Dynamical (MLD) system is defined, but the resulting Mixed Integer QP problem is shown to be very high-dimensional. Instead, a sequence of QP problems is defined resulting in an optimization problem with considerably smaller dimension. The receding horizon optimization is tested in a simulation environment and comparison with data from a true natural gas pipeline shows 5 to 8 % savings in compressor energy consumption.

Mahlke, Martin, and Moritz [81] present a simulated annealing metaheuristic for the transient natural gas network optimization problem. For this transient problem, they present a highly complex mixed integer nonlinear program. They relax the equations describing the gas dynamics in pipes by adding these constraints combined with appropriate penalty factors to the objective function. A suitable neighborhood structure is developed for the relaxed problem where time steps as well as pressure and flow of the gas are decoupled. They tested their method on three real-world instances provided by the German gas company E.ON Ruhrgas AG. The range of the size was from 11 to 31 pipes and from 3 to 15 compressor stations. They obtained reasonably good results in very competitive running times.

Domschke et al. [41] apply an implicit box scheme to the isothermal Euler equation to derive a MINLP for the transient MCFP. The model is solved by means of a combination of (i) a novel mixed-integer linear programming approach based on piecewise linearization and (ii) a classical sequential quadratic program applied for given combinatorial constraints. Their empirical work reveals that better approximations to the optimal control problem can be obtained by using solutions of the SQP algorithm to improve the MILP. Moreover, iteratively applying these two techniques improves the results even further.

More recently, Zavala [136] proposes a discretization framework for the PDEs governing the dynamics of the gas system by applying a Finite Difference scheme in space and by using an implicit Euler scheme in time for the continuity and momentum equations. The discretization is used in a two-stage stochastic model that involves recourse actions before the end of the planning horizon. The author claims that the two-stage structure is more restrictive but also computationally more tractable.

6 Research Challenges

The natural gas industry keeps evolving and thus a greater flexibility in the day-to-day gas transport operations is required. While posing huge opportunities for improvements, the natural gas infrastructure has to convey her ideas through analytical methods. With technological breakthrough observed during the last decades, these methods can now become applicable and can be developed by the scientific community.

From the optimization perspective, there are still quite a few areas that pose a wide range of challenges to the scientific community. For example, one of the most significant challenges to the natural gas transport industry is how to integrate and to solve in the analysis transient models, i.e., models depending on time. More precisely, by conducting a steady-state study, we consider the gas flow decision variables in the system to be independent of time. This allows the use of algebraic equations to describe the behavior of natural gas through the pipeline network. A transient analysis requires the use of partial differential equations to describe the continuity, energy, and momentum equations that relate the decision variables, such as gas flow, velocity, density, pressure, and temperature, as a function of time. Due to the challenge imposed by the transient case, while increasing the number of variables, as well as the inherent complexity of the problem, works on this area are still in a developing phase. See Section 5.5.

Second, although in practice, gas transport operations are defined by inherently transient processes, we assume that the problem is in steady-state. That is, the mathematical model provides solutions for pipeline systems that have been operating for a relative large amount of time.

Concerning pipeline capacity, the question of how to handle excess in the maximum capacity in pipelines while meeting strict transportation contracts poses a significant issue for the natural gas industry. Analytical models encompassing the optimization of the pipeline capacity release are required. The volume variability also introduces significant configuration challenges in the natural gas transport, particularly in compressor stations operation.

From the gas quality perspective, the improvements not only concern customer demands satisfaction, but also represent a higher impact on pipeline infrastructure. From the operational perspective, for example, the theory of optimization can become a blunt instrument in gas quality estimates on the acid formation from sulfur compounds that may lead to pipeline corrosion. The difficulty stems from the fact that constituents can vary seasonally or even more frequently. Safety and reliability are also main concerns for the natural gas transport industry.

Moreover, most of the works discussed in this paper are confined to irreversible flows in steady-state, i.e, the gas can flow through a pipeline in only one direction. The authors inherently assume that valves are present to restrict the direction of flow. However, steady-state flow models considering reversible flows may represent a significant contribution to those pipeline networks that connect their major lines with storage facilities, in which the flow in either direction may be allowed.

In steady-state models, the gas flow is considered isothermal at an inlet average effective temperature. This is a common practice in which authors assume that a heat transfer with the surroundings in the pipeline system causes the temperature to remain constant. Works on this area pose a significant challenge due to the inherent complexities associated to the gas temperature. In these works, it is also assumed that the transmission lines are composed of horizontal pipelines. In practice, these systems have frequent changes in their elevation. Hence, a special attention must be paid into the necessary correction factors to compensate the changes in elevation.

The majority of the works discussed here are based on deterministic models, i.e., where each parameter is assumed known in advance. There is an evident need of stochastic programming models and approaches to handle those cases where the variation of the parameters (such as demand or supply) is so high that deterministic assumptions no longer hold. In this paper, we have reviewed some of the few works starting to face this challenge, particularly in compressor station optimization networks and pooling systems. Additionally, we have seen some works in natural gas marketing problems (see [137] for a review on some of these problems); nonetheless, we believe this is a tremendous area of opportunity for problems in natural gas transmission systems.

Final remark: One of the major challenges to efficiently exploit the natural gas supplies arises from the limitation of the optimization techniques, which are already developed in theory, but to less extent applicable in practice due to considerably strong assumptions. For success in the increase in the demand, more than a few successful optimization tools capable of responding to changing conditions in a rational manner are required. This would certainly make more efficient the use of existing natural gas transmission systems, thus resulting in significant economic compensations beyond the expected levels for the natural gas industry. We may expect that this trend will speed up towards a more promising future with the continual contribution of the scientific community.

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