Mixed Integer Programming for the Global Solution of the Economic Load Dispatch Problem With Valve-Point Effect*

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Abstract

Optimal distribution of power among generating units to meet a specific demand subject to system constraints is an ongoing research topic in the power system community. The problem, even in a static setting, turns out to be hard to solve with conventional optimization methods owing to the consideration of *valve-point effects*, which make the cost function nonsmooth and nonconvex. This difficulty gave rise to the proliferation of population-based global heuristics in order to address the multi-extremal and nonsmooth problem. In this paper, we address the economic load dispatch problem (ELDP) with the valve-point effect in its classic formulation, wherein the cost function for each generator is expressed as the sum of a quadratic term and a rectified sine term. We propose an approach that adaptively builds piecewise-quadratic surrogate under-estimations of the ELDP cost function, and the resulting sequence of surrogate mixed-integer quadratic programming (MIQP) problems can therefore be handled by MIQP solvers. We show that any limit point of the sequence of MIQP solutions is a global solution of the ELDP.

Key words: economic load dispatch; global optimization; mixed-integer quadratic programming; valve-point effect

1 Introduction

The objective of solving the economic load dispatch problem (ELDP) is to minimize the cost associated with the power generation by optimally scheduling the load across generating units to meet a certain demand subject to system constraints [SCC03]. In a simpler setting, the cost function associated with a generator may be approximated by a quadratic function. Nevertheless, if the cost function takes highly nonlinear input-output characteristics into account due to *valve-point loadings*, even the static ELDP that ignores the ramp-rate constraints turns out to be difficult to solve. In this respect, the challenges faced by the solvers stem from (i) the nonsmooth cost function and (ii) the multi-extremal nature of the problem. The formal statement of the ELDP is deferred to Section 2.

A popular strategy to address the ELDP is to rely on a (population-based) stochastic search algorithm. Indeed, a myriad of such algorithms have been proposed during recent

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years, including variants of genetic algorithms, evolutionary programming, particle swarm optimization, ant colony optimization, differential evolution, firefly algorithm, bacterial foraging algorithm, and biogeography-based optimization. A list of heuristics applied to the ELDP is presented in [SSV$^+$15]. A shortcoming of those stochastic search algorithms is that convergence to a minimum (even a local one) is not guaranteed. An alternative that recently appeared in the literature—see [YFP13, PKV14, WGCL14]—is to model the ELDP cost function by a surrogate function, yielding an optimization problem that can be solved to global optimality by a mixed-integer programming (MIP) solver.

The purpose of the present paper is to provide a detailed account of the algorithm that made it possible to obtain the global solution of the 13-unit ELDP presented in the technical report [ASLA14] (see table VI therein). The algorithm relies on a technique that, given a set of prescribed breakpoints, builds a surrogate of the ELDP cost function in the form of piecewise-quadratic under-approximation. Such a surrogate function yields a surrogate problem that can be solved with the help of an MIP solver. The algorithm consists of iteratively alternating between the surrogate-building technique and the MIP solver: the minimizer of the surrogate problem found by the MIP solver is used to update the set of breakpoints, yielding a new piecewise-quadratic under-approximation and hence a new surrogate problem. The MIP solver is thus called repeatedly, each time to solve a new surrogate problem. This adaptive piecewise-quadratic under-approximation method ensures that the sequence of surrogate optima converges to the global optimum of the ELDP (Theorem 2), while maintaining the number of quadratic pieces to be small for the sake of efficiency.

The adaptive approach dealt with in this paper differs as follows from the one in [PKV14], regardless of the fact that they share certain aspects. Our scheme introduces fewer breakpoints in the surrogate cost function, which results in a reduced complexity. Moreover, it takes into account that the rectified sine term in (1b) is piecewise-concave while the whole cost function is not; this leads to a piecewise-quadratic under-approximation of (1b), which is effectively handled by the mixed-integer quadratic programming (MIQP).

The remainder of the article is organized as follows. In Section 2, the ELDP with the valve-point effect is briefly presented. In Section 3, the proposed algorithm is presented and analyzed. Conclusions are drawn in Section 4.

2 Problem Statement

In this section, we recall the formulation of the widely investigated ELDP with the valve-point effect as described in [SSV$^+$15].

In the ELDP, the major component of the cost arises from the generator input, i.e., the fuel. Thus, the objective function associated with the problem is related to how we represent the input-output characteristics of each generator. The total cost is then naturally the sum of each generating unit's contribution. The objective function is therefore expressed as

$$f(p) = \sum_{i=1}^{n} f_i(p_i), \tag{1a}$$

where f is the total cost function in \$/h, f_i is the cost due to the i-th generator, and p_i denotes the power assigned to the i-th unit in MW.

A classical, simple, and straightforward approach to model the cost function is to use a quadratic function for each generator, i.e., $f_i(p_i) = a_i p_i^2 + b_i p_i + c_i$, where a_i, b_i, and c_i are

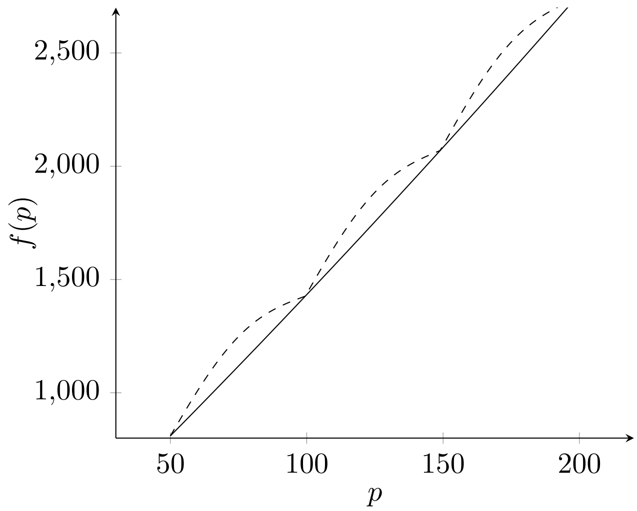

Figure 1: Plots displaying the cost function for a generator without (solid line) or with (dashed line) the valve-point effect.

scalar coefficients.

However, in reality, performance curves do not behave so smoothly; instead, in generating units with multi-valve steam turbines, ripples will be introduced into these curves. Large steam turbine generators usually have a number of steam admission valves that are opened in sequence to meet an increasing demand from a unit. As each steam admission valve starts to open, a sharp increase in losses will occur due to the wire drawing effect [SSV$^+$15], [Rep71]. This is the so-called valve-point effect. In order to mimic this effect in the model, a rectified sine term is usually added to the fuel cost function, as given by

$$f_i(p_i) = a_i p_i^2 + b_i p_i + c_i + d_i |\sin(e_i(p_i - p_i^{min}))| \tag{1b}$$

where d_i and e_i are positive coefficients to account for the valve-point effect [WS93]. The impact of the valve-point effect on the cost function is illustrated in Figure 1.

Unfortunately, the inclusion of the valve-point effect prevents the application of a conventional optimization technique to handle the ELDP, as it gives rise to a nonconvex and nondifferentiable cost function.

Furthermore, the ELDP must also satisfy some constraints, thereby restricting the search space. First, the generated power must meet out the demand, despite some wastage of power in the network. This is termed as the *power balance constraint*, which is formulated as an equality constraint

$$\sum_{i=1}^{n} p_i = p_D + p_L(p) \tag{2}$$

with scalar p_D and function p_L being the power demand and the transmission loss in the network in MW, respectively. As is often followed, we will ignore the transmission losses along the network, i.e., we set $p_L = 0$ in (2).

The other constraint is known as the *generator capacity constraint*, which imposes that the power generated by each unit lies within an allowable range $[p_i^{\min}, p_i^{\max}]$. This box constraint can easily be transcribed as a set of inequality constraints

$$p_i^{\min} \leq p_i \leq p_i^{\max}, \tag{3}$$

where $p_i^{\min}$ and $p_i^{\max}$ are the minimum and maximum power output of the i-th generator in MW, respectively.

The set of points that satisfy the constraints in (2) and (3) is termed the *feasible set* of the ELDP.

In summary, the problem of interest in this paper is the static ELDP without losses, stated as the minimization of $f(p) = \sum_{i=1}^{n} f_i(p_i)$ under the lossless power balance constaint and the generator capacity constraints:

$$\begin{aligned} \min_{p\in\mathbb{R}^n} \quad & \sum_{i=1}^{n} a_i p_i^2 + b_i p_i + c_i + d_i|\sin(e_i(p_i - p_i^{\min}))| \\ \text{subject to} \quad & \sum_{i=1}^{n} p_i = p_D \\ & p_i^{\min} \leq p_i \leq p_i^{\max} \quad \forall i \in \{1, ..., n\}. \end{aligned} \tag{4}$$

3 Proposed Approach

3.1 Surrogate problems

We first describe how to build a piecewise-quadratic under-approximation g_i of the cost function f_i (1b) given breakpoints $0 = X_{i,0} < X_{i,1} < \cdots < X_{i,m_i} = \pi/2$. First consider the linear spline s_i interpolating the rectified sine $|\sin(x)|$ at the points $X_{i,0}, \ldots, X_{i,m_i}$, i.e.,

$$s_i(x) = \alpha_{i,j} x + \beta_{i,j}, \quad x \in [X_{i,j-1}, X_{i,j}],\ j = 1, 2, \ldots, m_i$$

where

$$\alpha_{i,j} = \frac{\sin(X_{i,j}) - \sin(X_{i,j-1})}{X_{i,j} - X_{i,j-1}}$$

and

$$\beta_{i,j} = \sin(X_{i,j-1}) - \alpha_{i,j} X_{i,j-1} = \frac{X_{i,j}\sin(X_{i,j-1}) - X_{i,j-1}\sin(X_{i,j})}{X_{i,j} - X_{i,j-1}}.$$

Note that all the sine terms are positive since they are evaluated at the break points which all belong to $[0, \pi/2]$; this is why the absolute values have been omitted. Next, extend this linear spline by symmetry over the interval $[-\pi/2, \pi/2]$, then extend it periodically over the whole real line to obtain a function $\bar{s}_i$. For example, the function $\bar{s}_i$ for $m_i = 2$ and $X_1 = 1$ is given by the dashed curve in Figure 2. Finally, the piecewise-quadratic under-approximation g_i of f_i is given by

$$g_i(p_i) = a_i p_i^2 + b_i p_i + c_i + d_i \bar{s}_i(e_i(p_i - p_i^{\min})).$$

Obviously, this yields an under-approximation

$$g = \sum_{i=1}^{n} g_i \tag{5}$$

of the ELDP objective function f.

Given breakpoints $0 = X_{i,0} < X_1 < \cdots < X_{i,m_i} = \pi/2$, $i = 1, \ldots, n$, we thus obtain a new optimization problem, termed *surrogate problem*, by replacing f_i with g_i in (4), yielding

$$\begin{aligned} \min_{p\in\mathbb{R}^n} \quad & \sum_{i=1}^{n} a_i p_i^2 + b_i p_i + c_i + d_i \bar{s}_i(e_i(p_i - p_i^{\min})) \\ \text{subject to} \quad & \sum_{i=1}^{n} p_i = p_D \\ & p_i^{\min} \le p_i \le p_i^{\max} \quad \forall i \in \{1, ..., n\}. \end{aligned} \tag{6}$$

3.2 MIQP formulation of the surrogate problem

Since the surrogate problem (6) has a piecewise-quadratic cost function and linear constraints, it can be reformulated as an MIQP problem. We propose the following MIQP formulation, which exploits the symmetry and periodicity of the piecewise-linear terms $\bar{s}_i$:

$$\begin{aligned} \min_{p,k,t,\chi,\eta} & \sum_i \left[a_i p_i^2 + b_i p_i + c_i + d_i \sum_{j=1}^{m_i} (\alpha_{ij}\chi_{ij} + \beta_{ij}\eta_{ij}) \right] \\ s.t. \quad & \sum_i p_i = D \\ & p_i^{\min} \le p_i \le p_i^{\max} \\ & -t_i \le e_i(p_i - p_i^{\min}) - \pi k_i \le t_i \\ & X_{i,j-1}\eta_{i,j} \le \chi_{i,j} \le X_{i,j}\eta_{i,j} \\ & t_i = \sum_{j=1}^{m_i} \chi_{i,j} \\ & \sum_{j=1}^{m_i} \eta_{i,j} = 1 \\ & p_i \in \mathbb{R} \\ & t_i, \chi_{i,j} \in \mathbb{R}^+ \\ & k_i \in \mathbb{N} \\ & \eta_{i,j} \in \{0, 1\}. \end{aligned} \tag{7}$$

Indeed, if we consider p fixed and solve (7) with respect to the other decision variables to get k^*, t^*, χ^*, η^*, then we have $k_i^* = \arg\min_{k_i\in\mathbb{Z}} |e_i(p_i - p_i^{\min}) - \pi k_i|$; $t_i^* = |e_i(p_i - p_i^{\min}) - \pi k_i^*|$; $\chi_{i,j}^* = t_i^*$ for $j = j_i^*$ and 0 otherwise, where j_i^* denotes the j such that $p_i \in [X_{i,j-1}, X_{i,j}]$; and $\eta_{i,j}^* = 1$ for $j = j_i^*$ and 0 otherwise. It can then be seen that $a_i p_i^2 + b_i p_i + c_i + d_i \sum_{j=1}^{m_i} (\alpha_{ij}\chi_{ij}^* + \beta_{ij}\eta_{ij}^*)$ reduces to the above-defined piecewise-quadratic under-approximation $g_i(p_i)$. (For simplicity, we have restriced the reasoning to the case where the p_i's do not coincide with breakpoints. The other case is more intricate to describe since j_i^* is no longer uniquely defined, but it yields the same conclusion.)

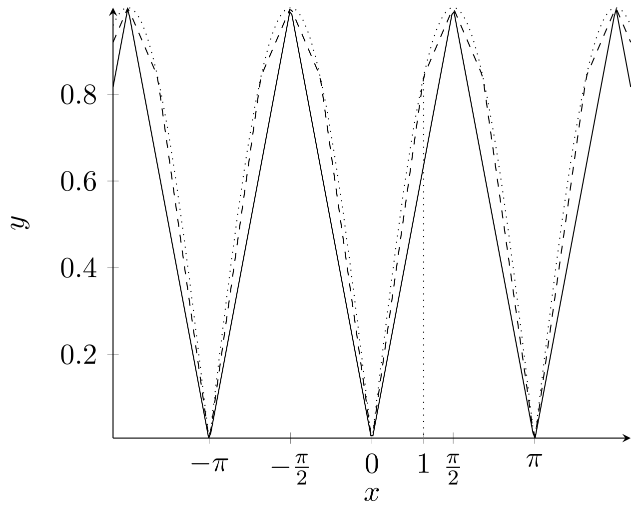

Figure 2: Plot of the two first approximations of the sine term for a given generator. We assume that the solution for this unit in the first iteration was 1.

3.3 Adding breakpoints

Let $\check{g}$ denote the optimal value of the surrogate problem (7) (which can be computed by an MIQP solver) and let $\check{f}$ denote the sought optimal value of the original ELDP (4). In general, $\check{g} \leq \check{f}$. Our aim is now to achieve the equality $\check{g} = \check{f}$ at the limit by adaptively updating the breakpoints. The proposed breakpoint update procedure is (at least vaguely) motivated by the following basic fact.

Proposition 1. *Let $g : X \mapsto \mathbb{R}$ be an under-approximation of $f : X \mapsto \mathbb{R}$, i.e., $g(x) \leq f(x)$ $\forall x \in X$. Let $x^* \in X$ be a global minimizer of g. If f and g coincide at x^*, i.e., $g(x^*) = f(x^*)$, then x^* is a global minimizer of f.*

Proof. Since x^* is a global minimizer of g, we have $g(x^*) \leq g(x)$ for every $x \in X$. Since moreover $g \leq f$, it follows that $g(x^*) \leq f(x)$ for every $x \in X$. If now $f(x^*) = g(x^*)$, then $g(x^*) \leq f(x)$ for every $x \in X$, i.e., x^* is a global minimizer of f. $\square$

Let g^m denote the under-approximation of f stemming from the current choice of the breakpoints through the procedure of Section 3.1, and let $\check{p}^m$ denote the minimizer of g^m (which can be computed by an MIQP solver as shown in Section 3.2). In general, we do not have $f(\check{p}^m) = g^m(\check{p}^m)$ and Proposition 1 is inconclusive. The idea is then to add $\check{p}^m$ as a new breakpoint to get a new under-approximation g^{m+1}. Then we have $f(\check{p}^m) = g^{m+1}(\check{p}^m)$, but $\check{p}^m$ is in general not a minimum of g^{m+1}, hence a new call to the MIQP solver is required to get a new minimizer $\check{p}^{m+1}$, itself subsequently added as a breakpoint, and so on. The process is illustrated in Figure 2. The ambition of this procedure is to achieve $f(\check{p}^m) = g^m(\check{p}^m)$ at the limit $m \to \infty$. Theorem 2 below confirms that this ambition is achieved.

3.4 Algorithm statement and convergence analysis

The procedure outlined above is presented in Algorithm 1, where $\mathcal{X}_i$ denotes a set of $m_i + 1$ breakpoints for generating unit i, whose $j+1$st element in increasing order is denoted by $X_{i,j}$.

Algorithm 1 Adaptive piecewise-quadratic under-approximation

for $i = 1, \dots, n$ **do**
  $\mathcal{X}_i \leftarrow \{0, \pi/2\}$
  $m_i \leftarrow 1$
end for
repeat
  for $i = 1, \dots, n$ **do**
    for $j = 1, \dots, m_i$ **do**
      $\alpha_{i,j} \leftarrow (\sin(X_{i,j}) - \sin(X_{i,j-1}))/(X_{i,j} - X_{i,j-1})$
      $\beta_{i,j} \leftarrow \sin(X_{i,j-1}) - \alpha_{i,j} X_{i,j-1}$
    end for
  end for
  $(\check{p}, \check{g}) \leftarrow$ solve problem (7), where $\check{p}$ denotes the optimal p and $\check{g}$ is the optimal value of the surrogate cost function;
  $\delta \leftarrow f(\check{p}) - \check{g}$
  for $i = 1, \dots, n$ **do**
    $\mathcal{X}_i \leftarrow \mathcal{X}_i \cup \{t_i\}$
    $m_i \leftarrow \#\mathcal{X}_i - 1$
  end for
until $\delta < \epsilon$

Now we set out to provide the convergence analysis of Algorithm 1 in the following theorem. Note that its conclusion $\lim_{m\to\infty} \delta^m = 0$ guarantees that Algorithm 1 terminates with $f(\check{p}^m) - \epsilon \leq \check{f} \leq f(\check{p}^m)$.

Theorem 2 (convergence to the global minimum)**.** *Let $\check{f}$ denote the optimal value of the cost function f in* (1) *of the ELDP defined in* (4)*. For $m = 0, 1, \dots$, let g^m denote the piecewise-quadratic surrogate cost function* (5) *used by Algorithm 1 at iteration m, and let $\check{p}^m \in \mathbb{R}^n$ denote the power allocated to generators by Algorithm 1 at iteration m by solving the MIQP problem* (7)*. Recall that $g^m(\check{p}^m) = \min g^m$ and denote it by $\check{g}^m$. Let $\delta^m = f(\check{p}^m) - \check{g}^m$. Then $\check{g}^m \leq \check{f} \leq f(\check{p}^m)$, i.e., the algorithm provides at each step m a lower bound and an upper bound of $\check{f}$. Moreover, the gap δ^m between those bounds goes to zero as $m \to \infty$; in other words, $\lim_{m\to\infty} \check{g}^m = \lim_{m\to\infty} f(\check{p}^m) = \check{f}$. Finally, any limit point of $(\check{p}^m)^{m\in\mathbb{N}}$ is a global solution of the ELDP* (4)*.*

Proof. The claim $\check{g}^m \leq \check{f} \leq f(\check{p}^m)$ is direct: the first inequality holds because $g^m \leq f$ by construction, and the second inequality holds by definition since $\check{f}$ is the global minimum of f.

As a preparatory result for the other claims, we point out that f and g^m, $m = 0, 1, \dots$, are Lipschitz continuous on the ELDP feasible set with a common Lipschitz constant K. Indeed, for every i, the cost function f_i in (1b) satisfies the Lipschitz property $|f_i(p_i + \Delta) - f_i(p_i)| \leq (2a_i p_i^{\max} + b_i + d_i e_i)\Delta$ for all p_i and $p_i + \Delta$ that satisfy the generator capacity constraints in (3). The ELDP cost function f in (1), being the sum of Lipschitz continuous functions, is thus Lipschitz continuous on the ELDP feasible set, with a constant $K = \sum_{i=1}^n 2a_i p_i^{\max} + b_i + d_i e_i$. Since g^m is obtained by replacing the rectified sines of f by chords, it follows that $2a_i p_i^{\max} + b_i + d_i e_i$ is still a Lipschitz constant for the contribution of generator i to g^m, and hence K is also a Lipschitz constant for g^m.

We show that $\lim_{m\to\infty} \check{g}^m = \check{f}$. Since the construction of g^{m+1} from g^m implies inserting one new breakpoint into the piecewise-linear under-approximation of the piecewise-concave rectified sine for each generator, it follows that, for every m,

$$g^m \leq g^{m+1} \leq f. \tag{8}$$

Therefore $(\check{g}^m)^{m\in\mathbb{N}}$ is a nondecreasing sequence bounded by $\check{f}$. Thus $(\check{g}^m)^{m\in\mathbb{N}}$ converges and $\lim_{m\to\infty} \check{g}^m \leq \check{f}$. It remains to show that the inequality is an equality. By contradiction, assume that $\lim_{m\to\infty} \check{g}^m = \check{f} - \varepsilon$ with $\varepsilon > 0$. Since $(\check{p}^m)^{m\in\mathbb{N}}$ is bounded in view of the generator capacity constraints in (3), there exists a subsequence $(\check{p}^{m_k})_{k\in\mathbb{N}}$ that converges. For k large enough, we then have the following inequalities which we justify in the sequel:

$$\begin{aligned}
\|\check{p}^{m_k} - \check{p}^{m_{k+1}}\| &\geq \frac{1}{K}[g^{m_{k+1}}(\check{p}^{m_k}) - g^{m_{k+1}}(\check{p}^{m_{k+1}})] && (9)\\
&\geq \frac{1}{K}[\check{f} - g^{m_{k+1}}(\check{p}^{m_{k+1}})] && (10)\\
&\geq \frac{1}{K}[\check{f} - (\check{f} - \varepsilon/2)] && (11)\\
&\geq \frac{\varepsilon/2}{K},
\end{aligned}$$

a contradiction since $(\check{p}^{m_k})_{k\in\mathbb{N}}$ converges. Inequality in (9) states the Lipschitz continuity of $g^{m_{k+1}}$ with constant K. Inequality in (10) follows from $g^{m_{k+1}}(\check{p}^{m_k}) = f(\check{p}^{m_k}) \geq \check{f}$; indeed, by construction, $\check{p}^{m_k}$ is a breakpoint of g^{m_k+1} and all subsequent surrogate cost functions. Finally, (11) follows from $g^{m_{k+1}}(\check{p}^{m_{k+1}}) = \check{g}^{m_{k+1}} \leq \check{f} - \varepsilon/2$ which holds for k large enough since we have assumed that $\lim_{m\to\infty} \check{g}^m = \check{f} - \varepsilon$.

We now show that $\lim_{m\to\infty} f(\check{p}^m) = \check{f}$. By contradiction, suppose not. Then there is an infinite subsequence $(\check{p}^{m_k})_{k\in\mathbb{N}}$ and $\varepsilon > 0$ such that $f(\check{p}^{m_k}) \geq \check{f} + \epsilon$. We assume w.l.o.g. that $(\check{p}^{m_k})_{k\in\mathbb{N}}$ converges; if not, we extract a sub-subsequence that does. By the triangle inequality, we have

$$\begin{aligned}
|f(\check{p}^{m_k}) - \check{f}| \leq &|f(\check{p}^{m_k}) - g^{m_{k+1}}(\check{p}^{m_k})|\\
&+|g^{m_{k+1}}(\check{p}^{m_k}) - g^{m_{k+1}}(\check{p}^{m_{k+1}})|\\
&+|g^{m_{k+1}}(\check{p}^{m_{k+1}}) - \check{f}|.
\end{aligned}$$

The first term of the bound is zero by construction of the surrogate functions. The second term goes to zero as $k \to \infty$ in view of the common Lipschitz constant K and the convergence of $(\check{p}^{m_k})_{k\in\mathbb{N}}$. The third term goes to zero as $k \to \infty$ since $g^{m_{k+1}}(\check{p}^{m_{k+1}}) = \check{g}^{m_{k+1}}$ and $\lim_{m\to\infty} \check{g}^m = \check{f}$. Hence $|f(\check{p}^{m_k}) - \check{f}|$ goes to zero, a contradiction.

Finally, let $(\check{p}^{m_k})_{k\in\mathbb{N}}$ be a convergent subsequence and let $\check{p}^{m_\infty}$ denote its limit. By continuity of f, we have that $f(\check{p}^{m_\infty}) = \lim_{k\to\infty} f(\check{p}^{m_k}) = \check{f}$. □

Note that, in view of the breakpoint insertion procedure (step 15 of Algorithm 1), it generally does not hold that g^m converges to f pointwise; otherwise the proof above could have been more direct.

4 Conclusion

We have proposed a new method (Algorithm 1) for the static lossless ELDP with valve-point effect (4). In contrast with most algorithms in the ELDP literature (see [SSV+15]), the new method is deterministic and is shown in Theorem 2 to converge to the global minimum of the ELDP. It proceeds by iteratively building piecewise-quadratic under-approximations of the ELDP cost function, yielding surrogate problems of the form (7) that can be handled by MIQP solvers.

This work opens several avenues of research, e.g., extensions to the lossy case, to the dynamic ELDP, and to more general cost functions. Warm-start techniques for the MIQP solver may also prove valuable.

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