

Modulation Design for Two-Way Amplify-and-Forward Relay HARQ

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Abstract

As a practical technique for enhancing relay and HARQ transmissions, Modulation Diversity (MoDiv) uses distinct constellation mappings for data retransmissions. In this work, we study the MoDiv optimization in a amplify-and-forward (AF) two-way relay channel (TWRC). The design of MoDiv design to minimize the bit-error rate (BER) is formulated into a successive Koopmans-Beckmann Quadratic Assignment Problem (QAP), which is solved sequentially with a robust tabu search method. The performance gain of our MoDiv scheme over retransmission without remapping and a heuristic MoDiv scheme is demonstrated with numerical results.

Index Terms

Modulation diversity, 2-way relay, HARQ, QAP.

I. INTRODUCTION

Hybrid Automatic Repeat reQuest (HARQ) has been effective for improving the robustness of high-rate wireless communications against packet losses due to fading channels and poor link-adaptation. There have been recent applications of HARQ over two-way relay channel (TWRC) [1-4]. In [1], the throughput of a simple Type-I HARQ for both Amplify-and-Forward (AF) and Decode-and-Forward (DF) TWRC schemes have been analyzed. The energy-delay trade-off, and the diversity-multiplexing trade-off of type-II HARQ with Incremental Redundancy (IR), have been studied for AF TWRC scheme in [2] and [3], respectively. Other related works on TWRC with ARQ can also be found in [4] and references therein.

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Also, Type-I HARQ with maximal ratio combining, known as HARQ-Chase Combining (HARQ-CC), is another simple and effective HARQ supported by standards such as HSPA, LTE, among others. As practical transmissions often adopt linear (QAM) modulations of finite-alphabet constellation, HARQ-CC can benefit from Modulation Diversity (MoDiv), which maps a string of $\log_2 Q$ information bits to different symbols in the same constellation for multiple retransmissions. MoDiv has been studied for HARQ [5], relay networks [6], [7] and relay-HARQ systems [8].

In this work, we study MoDiv design for TWRC under a simple AF scheme and based on HARQ-CC protocol. We first derive an approximate bit-error rate (BER) of TWRC-AF channel for Rayleigh fading channels, given M different mapping schemes for multiple transmissions. Accordingly, we formulate a successive BER minimization MoDiv design into a series of Quadratic Assignment Problems (QAPs) in Koopmans-Beckmann (KB) form [9]. Although the QAP is NP-hard, efficient numerical algorithms have been extensively researched [10], some with very good performance on QAPLIB. We adopt a tabu search algorithm [11] to solve each QAP. Moreover, the coefficients of the QAP problems can also be computed efficiently in a successive manner. Our numerical results demonstrate significant BER reduction, even under mismatched design parameters.

II. SYSTEM MODEL

Consider a TWRC with analog network coding (ANC) [2], as shown in Fig. 1. The relay node R is unaware of HARQ and simply performs ANC. Each round of ANC consists of two phases. In the multiple access (MAC) phase, source nodes S_1 and S_2 transmit to R simultaneously. In the broadcast (BC) phase, node R amplifies and broadcasts the signal received during the MAC phase back to both S_1 and S_2 . Denote the uplink channel from S_s to R and downlink channel from R to S_s as h_s and g_s , respectively, where $s = 1, 2$. We assume that the channels follow Rayleigh distribution, i.e. $h_s \sim \mathcal{CN}(0, \beta_{h_s})$ and $g_s \sim \mathcal{CN}(0, \beta_{g_s})$, $s = 1, 2$. Denote the data symbol from S_s as x_s with average power $\mathbb{E}[|x_s|^2] = P_s$. Then the signal received by R during the MAC is

$$y_R = h_1 x_1 + h_2 x_2 + n_R, \quad (1)$$

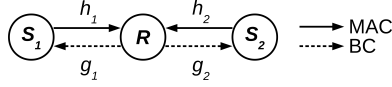


Fig. 1. Two-way relay channel with analog network coding.

where $n_R \sim \mathcal{CN}(0, \sigma_R^2)$ is the noise. Assume that the relay R has a mean power constraint of P_R which requires a power normalization factor

$$\alpha = \sqrt{P_R (|h_1|^2 P_1 + |h_2|^2 P_2 + \sigma_R^2)^{-1}}. \quad (2)$$

The received signal at S_s is

$$y_s = \alpha g_s y_R + n_s, \quad s = 1, 2. \quad (3)$$

Additionally, the HARQ processes on the two directions are not synchronized. Thus, the MoDiv design at S_1 and S_2 can be independently designed. Without loss of generality, we focus on HARQ from S_1 to S_2 here.

Let \mathcal{C} be the constellation of S_1 with cardinality $Q = |\mathcal{C}|$. QAM converts a bit string of length $\log_2 Q$ into QAM symbols. Denote the Q bit string label as $p \in \{0, \dots, Q-1\}$. In MoDiv, we assume that the original transmission still adopts the conventional Gray mapping ψ_0 . However, the retransmissions uses distinct mapping functions $\psi_m : \{1, \dots, Q-1\} \rightarrow \mathcal{C}$ for modulation of each bit string p to achieve diversity gain over conventional HARQ-CC that uses the same ψ_0 repeatedly. Let M be the maximum number of allowed retransmissions. Assume that S_1 and S_2 perform perfect self-interference cancellation (SIC). According to Eqs. (1)(3), the signal received by S_2 after SIC during the m -th (re)transmission of p for $0 \leq m \leq M$ is

$$y_2^{(m)} = \alpha^{(m)} g_2^{(m)} h_1^{(m)} \psi_m[p] + \alpha^{(m)} g_2^{(m)} n_R^{(m)} + n_2^{(m)}. \quad (4)$$

Assume that S_2 acquires perfect channel state information (CSI). Despite the fact that a same label p is retransmitted as in HARQ-CC, MRC does not apply here due to the different remappings. Consequently, we adopt maximum likelihood (ML) detection to demodulate the received symbols after the m -th retransmission until a success or $m > M$:

$$p^* = \arg \min_p \sum_{k=0}^m \frac{|y_2^{(k)} - \alpha^{(k)} g_2^{(k)} h_1^{(k)} \psi_k[p]|^2}{\sigma_2^2 + (\alpha^{(k)})^2 \sigma_R^2 |g_2^{(k)}|^2}. \quad (5)$$

III. SUCCESSIVE CONSTELLATION MAPPING DESIGN FOR MODULATION DIVERSITY

A. A BER approximation

Let label p be uniformly distributed. The BER of the ML receiver after the m -th retransmission can be approximated with the pair-wise error probability (PEP) upperbound [5]:

$$P_{BER}^{(m)} = \sum_{p=0}^{Q-1} \sum_{q=0}^{Q-1} \frac{B[p, q]}{Q} P_{PEP}^{(m)}(q|p), \quad (6)$$

where $B[p, q]$ is the Hamming distance between the binary strings of p and q normalized by $\log_2 Q$, and $P_{PEP}^{(m)}(q|p)$ is the probability that the ML detection favors q over p given the transmission of p . From Eq. (5), we have

$$P_{PEP}^{(m)}(q|p) = \mathbb{E} \left[Q \left(\sqrt{\sum_{k=0}^m \frac{(\alpha^{(k)})^2 \epsilon_k[p, q] \gamma_2^{(k)} \delta_1^{(k)}}{2(\tilde{\sigma}_2^{(k)})^2}} \right) \right] \quad (7)$$

where $\gamma_2^{(k)} = \|g_2^{(k)}\|^2$, $\delta_1^{(k)} = \|h_1^{(k)}\|^2$, $\epsilon_k[p, q] = \|\psi_k[p] - \psi_k[q]\|^2$, and $(\tilde{\sigma}_2^{(k)})^2 = \sigma_2^2 + (\alpha^{(k)})^2 \sigma_R^2 \gamma_2^{(k)}$ is the instantaneous noise variance at S_2 . By adopting Chernoff upper bound $Q(x) \leq e^{-x^2/2}/2$, an approximation to $P_{PEP}^{(m)}(q|p)$ is

$$\tilde{P}_{PEP}^{(m)}(q|p) = \frac{1}{2} \prod_{k=0}^m \mathbb{E} \left[\exp \left(-\frac{(\alpha^{(k)})^2 \epsilon_k[p, q] \gamma_2^{(k)} \delta_1^{(k)}}{4(\tilde{\sigma}_2^{(k)})^2} \right) \right]. \quad (8)$$

Although a coarse approximation, Chernoff bound enables efficient iterative computation of $P_{PEP}^{(m)}(q|p)$ as m increases. Moreover, as shown in Section III-B, this approximation results in a simple KB-form QAP. As will be explained in Section III-B, we can replace the Chernoff bound with a more accurate approximation, which leads to a general-form QAP.

Denote $E_k[p, q]$ as the expectation in Eq.(8). Here it is computed using the prior channel and noise distribution instead of the posterior one conditioned on HARQ feedback. This is supported by our companion work [12]. We have

Proposition 1. *An approximation to $E_k[p, q]$ is*

$$\begin{aligned} \tilde{E}_k[p, q] &= \frac{4\sigma_R^2 + \beta_{h_1} \epsilon_k[p, q] v \exp(v) Ei(v)}{u} \\ u &= 4\sigma_R^2 + \beta_{h_1} \epsilon_k[p, q], \quad v = \frac{4\sigma_2^2}{\tilde{\alpha}^2 \beta_{g_2} u}, \\ \tilde{\alpha} &= \sqrt{\frac{P_R}{\beta_{h_1} P_1 + \beta_{h_2} P_2 + \sigma_R^2}}, \end{aligned} \quad (9)$$

and $Ei(x) = \int_x^\infty e^{-t}/t dt$ as defined in [13].

Proof. Following Eq.(43) of [14], we replace $\alpha^{(k)}$ using the heuristic approximation [15] with constant $\tilde{\alpha}$ to obtain

$$\begin{aligned} E_k[p, q] &\approx \mathbb{E}_{\gamma_2} \left[\mathbb{E}_{\delta_1 | \gamma_2} \left[\exp \left(-\frac{\tilde{\alpha}^2 \epsilon_k[p, q] \gamma_2 \delta_1}{4(\sigma_2^2 + \tilde{\alpha}^2 \sigma_R^2 \gamma_2)} \right) \right] \right] \\ &= \mathbb{E}_{\gamma_2} \left[\left(1 + \frac{\tilde{\alpha}^2 \epsilon_k[p, q] \beta_{h_1} \gamma_2}{4(\sigma_2^2 + \tilde{\alpha}^2 \sigma_R^2 \gamma_2)} \right)^{-1} \right]. \end{aligned} \quad (10)$$

As δ_1, γ_2 both follow exponential distribution, Eq.(9) is derived by evaluating Eq.(10) with Eq.(3.352.4) of [13]. \square

While $E_k[p, q]$ plays a key role in our MoDiv design for minimizing BER, it is also closely related to another performance metric, the ergodic mutual information (EMI):

Proposition 2. *The EMI after the m -th retransmission, denoted as $I^{(m)}$, is lower bounded by*

$$\tilde{I}^{(m)} = 2 \log_2 Q - \log_2 \left[\sum_{p=0}^{Q-1} \sum_{q=0}^{Q-1} \prod_{k=0}^m E_k[p, q] \right]. \quad (11)$$

Proof. Denote the mutual information conditioned on the CSI as $I^{(m)}(\mathbf{h}_1^{(m)}, \mathbf{g}_2^{(m)}, \boldsymbol{\alpha}^{(m)})$, where $\mathbf{h}_1^{(m)} = [h_1^{(0)}, \dots, h_1^{(m)}]^T$, $\mathbf{g}_2^{(m)} = [g_2^{(0)}, \dots, g_2^{(m)}]^T$ and $\boldsymbol{\alpha}^{(m)} = [\alpha^{(0)}, \dots, \alpha^{(m)}]^T$. Assuming uniform constellation symbols, it is lower bounded with [16, Eq.(4.3.37)]:

$$\tilde{I}^{(m)}(\mathbf{h}_1^{(m)}, \mathbf{g}_2^{(m)}, \boldsymbol{\alpha}^{(m)}) = 2 \log_2 Q - \log_2 \left[\sum_{p=0}^{Q-1} \sum_{q=0}^{Q-1} \prod_{k=0}^m \exp \left(-\frac{(\alpha^{(k)})^2 \epsilon_k[p, q] \gamma_2^{(k)} \delta_1^{(k)}}{4(\tilde{\sigma}_2^{(k)})^2} \right) \right]. \quad (12)$$

Since $\log(x)$ is concave and the channels are independent in each (re)transmission, we have $\mathbb{E} \left[\tilde{I}^{(m)}(\mathbf{h}_1^{(m)}, \mathbf{g}_2^{(m)}, \boldsymbol{\alpha}^{(m)}) \right] \geq \tilde{I}^{(m)}$, thus proving Proposition 2. \square

Proposition 2 bridges MoDiv designs based on rate criterion [7] to the BER criterion [5]. Moreover, it gives an efficient KB-form QAP almost identical to that derived from BER minimization, as discussed in Section III-B.

B. Successive Quadratic Assignment Problem (S-QAP)

Our MoDiv design is based on the approximate BER minimization criterion, of which the input is the statistical CSI β_{h_1}, β_{g_2} , the constellation \mathcal{C} , the noise power σ_R^2, σ_2^2 and the power constraints P_1, P_2, P_R . As it is impossible to know the number of actual retransmission m in

advance, we formulate a sequence of M optimization problems [5], in which ψ_m is optimized to minimize the approximated BER given $\psi_0, \dots, \psi_{m-1}$ without expecting future retransmissions:

$$\min_{\psi_m | \psi_k, k=0, \dots, m-1} \tilde{P}_{BER}^{(m)}, m = 1, \dots, M \quad (13)$$

where $\tilde{P}_{BER}^{(m)}$ denotes the approximated BER of Eq.(6) evaluated by replacing $P_{PEP}^{(m)}(q|p)$ with $\tilde{P}_{PEP}^{(m)}(q|p)$ in Eq.(8), and replacing the expectation in Eq.(8) with Eq.(9). Although $\tilde{P}_{PEP}^{(m)}(q|p)$ and $\tilde{E}_k[p, q]$ are rather coarse approximations, our tests suggest that they achieve as good MoDiv design as the more accurate numerical approaches to evaluate $P_{PEP}^{(m)}(q|p)$.

In order to rewrite Eq.(13) into a S-QAP, we denote $\mathbf{x}^{(m)} = \{x_{pi}^{(m)} | p, i = 0, \dots, Q-1\}$ as the permutation matrix representing ψ_m , in which $x_{pi}^{(m)} = 1(\psi_m[p] = \psi_0[i])$ and $1(\cdot)$ is the indicator function. Denote the constraint set

$$\mathcal{S} = \left\{ \mathbf{x} : \sum_{p=0}^{Q-1} x_{pi} = 1, \sum_{i=0}^{Q-1} x_{pi} = 1, x_{pi} \in \{0, 1\} \right\}, \quad (14)$$

Then the MoDiv design of Eq.(13) can be formulated as:

$$\min_{\mathbf{x} \in \mathcal{S}} \sum_{p=0}^{Q-1} \sum_{i=0}^{Q-1} \sum_{q=0}^{Q-1} \sum_{j=0}^{Q-1} f_{pq}^{(m)} d_{ij} x_{pi}^{(m)} x_{qj}^{(m)}, \quad (15)$$

for $m = 1, \dots, M$, where

$$f_{pq}^{(m)} = \frac{B[p, q]}{Q} \tilde{P}_{PEP}^{(m-1)}(q|p), \quad d_{ij} = \tilde{E}_0[i, j] \quad (16)$$

Note that here we assume all channels and noises to be stationary across all retransmissions, such that d_{ij} only needs to be evaluated once. On the other hand, $f_{pq}^{(m)}$ can be computed recursively while solving the S-QAP, since

$$\tilde{P}_{PEP}^{(m)}(q|p) = \sum_{i=0}^{Q-1} \sum_{j=0}^{Q-1} \tilde{P}_{PEP}^{(m-1)}(q|p) d_{ij} \hat{x}_{pi}^{(m)} \hat{x}_{qj}^{(m)} \quad (17)$$

and $\tilde{P}_{PEP}^{(-1)}(q|p) = 1/2$ where $\hat{\mathbf{x}}^{(m)}$ is the solution to Eq.(15).

In this S-QAP formulation, each KB-form QAP is defined by two Q -by- Q matrices, only one of which requires updating. Should we adopt the more accurate approximation in Eq.(14) of [17], each QAP would be in general-form, defined with one Q^4 matrix. Nevertheless, this 4-dimensional matrix can still be updated iteratively using a few Q -by- Q matrices sequentially. On the other hand, if we adopted an EMI-lowerbound maximization design according to Proposition 2, the only change to the KB-form S-QAP would be a new ‘‘flow’’ matrix

$f_{pq}^{*(m)} = \tilde{P}_{PEP}^{(m-1)}(q|p)$ in Eq.(16). Nevertheless, in terms of practical performance such as coded BER, the approximate EMI-lowerbound is generally an inferior criterion.

S-QAP in KB form enables us to handle larger constellations than previous works with an efficient robust iterative tabu search algorithm [11][18], leading to a total complexity of $\mathcal{O}(MNQ^3)$ where N is the number of iterations. The tabu search heuristic yields slight over-estimates of the optimal objective values. As we computed the lower bounds based on semidefinite programming relaxations [19], typically the gap between lower and upper bounds is 10-20% with the exact objective value being much closer to the upper bound. The MoDiv design can be precomputed offline and stored at S_1 and S_2 for easy table lookup.

IV. NUMERICAL RESULTS

We consider a TWRC where the distances between S_1, S_2 and R are $d_1 = d_2 = 0.5$, thus $\beta_{h_s} = \beta_{g_s} = d_s^\nu$ where $\nu = 3$ is the path-loss coefficient, $s = 1, 2$. We also fix $P_1 = P_2 = 0.5P_R = 1$ and assume that $\sigma_1^2 = \sigma_2^2 = \sigma_R^2 = \sigma^2$. The strong AF relay is not necessary but is reasonable for linking two wireless nodes. In our HARQ, we set $M = 4$. We compare three MoDiv strategies: simple repeats in retransmission without MoDiv, our QAP-optimized solution, and a heuristic constellation rearrangement (CoRe) proposed for HSDPA [20] for 64-QAM constellation. In our results, the three schemes after the m -th retransmission are labeled as NM_m , QAP_m and CR_m , respectively.

First we demonstrate the average approximate BER and the Monte-Carlo uncoded BER of S_1 and S_2 in Fig. 2 and Fig. 3, respectively. In the approximate BER results, for comparison purpose we also give the Gilmore-Lawler bound of the QAP, denoted as GLB_m . These results show that the QAP solution offers a substantial performance gain over non-MoDiv and the heuristic CoRe. For instance, 3 retransmissions of QAP MoDiv achieves lower BER than 4 heuristic CoRe retransmissions (at high SNR), and 2 MoDiv retransmissions outperform 4 retransmissions without MoDiv.

To further illustrate the performance and robustness of the QAP-optimized MoDiv, we compare the coded-BER of the three MoDiv schemes in a LDPC-coded transmission based on [21]. We use a LDPC code of length $L = 2400$ of code rate $3/4$ and a Monte-Carlo of up to 2000 LDPC frames. Since an important motivation of HARQ is to cope with link adaptation inadequacy, we deliberately optimize the remappings at $\sigma^2 = 4.5\text{dB}$ and test their performances for mismatched σ^2 , though our test results (not shown here) indicate only a slight coded BER variation for a

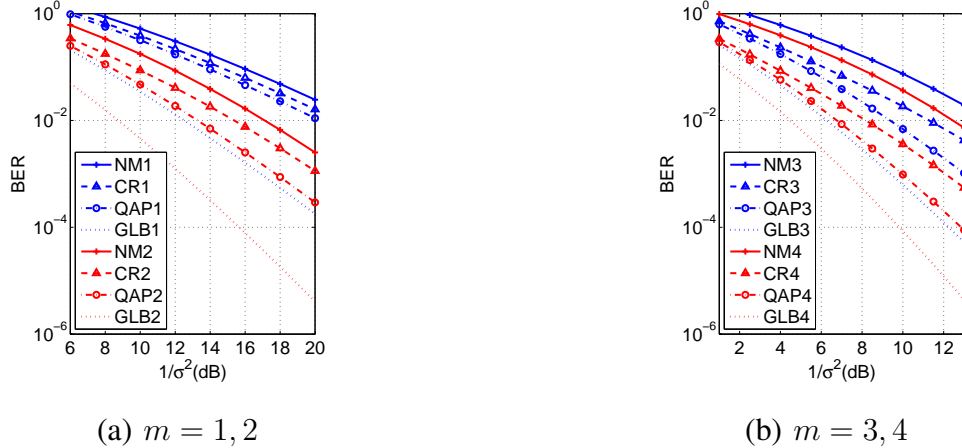


Fig. 2. Analytical approximation results of uncoded BER.

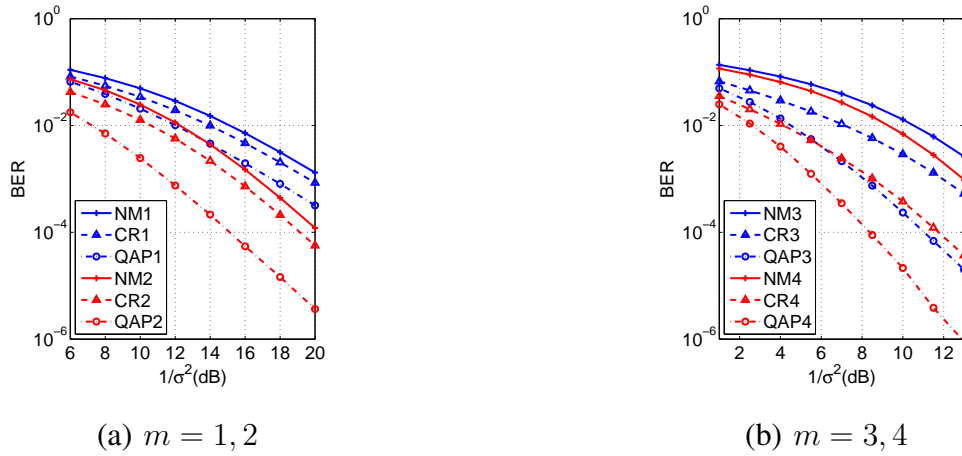


Fig. 3. Monte-Carlo simulation results of uncoded BER.

wide range of design parameter σ^2 from -7dB to 3dB. From the results in Fig. 4, the advantage of QAP solution is robust despite of the mismatch. Specifically, QAP2 still performs better than NM4, while CR4 outperforms QAP3 by less than 1dB. We also plot the average HARQ throughput of this LDPC coded system in Fig. 5. It is noted that the QAP MoDiv design enjoys a performance gain over the heuristic scheme almost as the latter over non-MoDiv design.

V. CONCLUSION

This work studies the modulation diversity design based on Chase Combining HARQ in two-way AF relay channels. To minimize an approximate BER, we formulated the MoDiv design

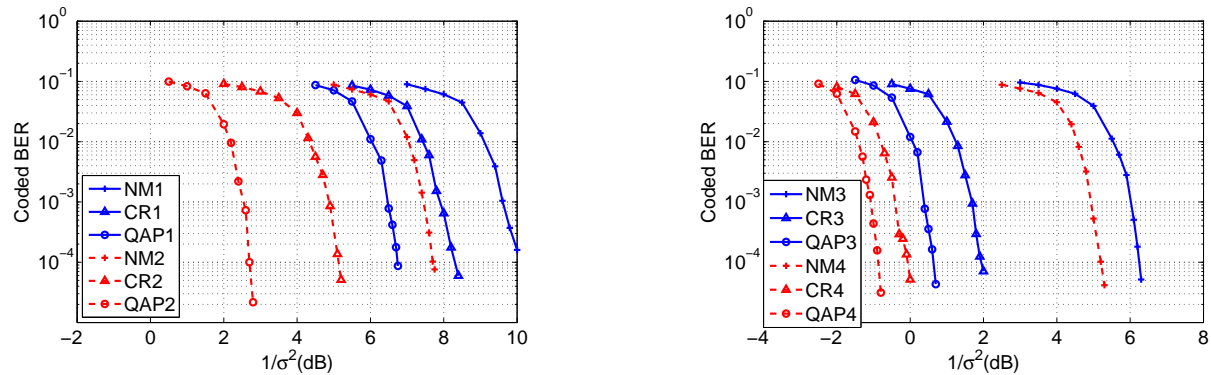


Fig. 4. Coded BER. $m = 1, 2$ (left) and $m = 3, 4$ (right).

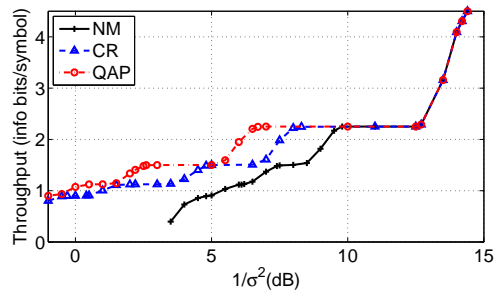


Fig. 5. Average throughput.

into a successive Koopmans-Beckmann Quadratic Assignment Problem solved with a robust tabu search algorithm. Our numerical tests demonstrated that the QAP-optimized MoDiv outperforms repeated use of Gray retransmission and a heuristic constellation rearrangement scheme under various settings and is robust against mismatched design parameters.

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