MIP Formulation Improvement for Large Scale Security Constrained Unit Commitment

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Abstract – As a part of the day-ahead market clearing process, Midcontinent Independent System Operator (MISO) solves one of the largest and most challenging Security Constrained Unit Commitment (SCUC) models. Better computational performance of SCUC models not only improves the market efficiency but also facilitates future market development. This paper introduces two developments on Mixed Integer Programming (MIP) SCUC formulation that results in significant performance improvement on existing MISO day-ahead SCUC: piecewise linear incremental energy curve (PWL) and aggregating variables (AGG). The two developments are also extend to configuration based combined cycle modeling. Other constraints for configuration based combined cycle modeling are also studied. To illustrate the effectiveness of the proposed methods in this paper, numerical results based on MISO system are presented.

Index Terms – Combined cycle units, day-ahead electricity market, locally ideal formulation, mixed integer programming, security constrained unit commitment

NOMENCLATURE

A. Indices

\( j \) Index of generation resources, including both non-CC units and CC units.

\( k \) Index of other dispatchable variables (virtuals, dispatchable demand and dispatchable transaction).

\( i \) Index of transmission constraints.

\( t, t' \) Index of interval.

\( n \) Index of nodes in the network.

\( y \) Index of configuration of a combined cycle unit.

\( w \) Transition type.

B. Set

\( I \) Set of transmission constraints.

\( J \) Set of resources.

\( J^c \) Set of CC units.

\( T \) Set of intervals.

\( Y_j \) Set of configuration of combined cycle \( j \).

\( Y^\text{from}_j \) Set of configurations that have feasible transition to configuration \( y \in Y^\text{from}_j \).

\( Y^\text{to}_j \) Set of configurations that have feasible transition from configuration \( y \in Y^\text{to}_j \).

\( W \) Set of transition cost type {hot start-up, intermediate start-up, cold start-up}.

C. Parameters

\( A \) Resource level constraint binary variables matrix.

\( B_{in,t} \) Sensitivity of the flow on transmission constraint to injection at node and withdrawal at the reference bus.

\( C^\text{NL}_j \) No-load cost of resource \( j \) configuration \( y \) at time interval \( t \).

\( C^T_{j,y,y',w} \) Transition cost of resource \( j \) from configuration \( y \) to configuration \( y' \) in transition type \( w \).

\( F_{i,t} \) Transmission limit.

\( G_{j,t} \) Upper bound for other dispatchable resources.

\( O_t \) Vector of offer cost for other dispatchable resources.

\( P_r \) Fixed injection.

\( P^y_{j,t}, P^\text{up}_j \) Generation resource minimum and maximum limits for configuration \( y \).

\( RD^y_j \) Ramp down rate of resource \( j \) configuration \( y \).

\( RD^y_{j,y'} \) Transition ramp rate from configuration \( y' \) to \( y \) of resource \( j \).

\( RU^y_j \) Ramp up rate of resource \( j \) configuration \( y \).

\( RU^y_{j,y'} \) Ramp up rate from configuration \( y' \) to \( y \) of resource \( j \).

\( TD^y_j \) Minimum downtime of resource \( j \) configuration \( y \).

\( TU^y_j \) Minimum uptime of resource \( j \) configuration \( y \).

D. Functions

\( C^P_{j,t}(\cdot) \) Incremental energy cost function.

\( C^P_{j,y}(\cdot) \) Incremental energy cost function for combined cycle \( j \) configuration \( y \) at time interval \( t \).

\( F^P_{j,t}(\cdot) \) Transmission flow.

E. Variables

\( x \) Binary variables (commitment, start-up, shut-down).

\( u_{j,t} \) Commitment variable for resource \( j \) at interval \( t \).

\( u^y_{j,t} \) Commitment variable for resource \( j \) configuration \( y \) at interval \( t \).

\( p_{j,t} \) Cleared energy on generation resource \( j \) at interval \( t \).

\( g_{k,t} \) Cleared energy on other dispatchable resources.

\( p_t, g_t \) Vectors of cleared energy on generation and on other dispatchable resources.

\( v^\text{yy'}_{j,t} \) Transition variable from \( y \) to \( y' \) for resource \( j \) at interval \( t \) in transition type \( w \), where \( y \neq y' \).

Disclaimer: The views expressed in this paper are solely those of the authors and do not necessarily represent those of MISO.
I. INTRODUCTION

MIDCONTINENT Independent System Operator (MISO) manages one of the largest electricity markets in the world. Following a significant expansion of its market footprint and an uptick in virtual trading volumes, MISO encountered computational performance challenges in solving its day-ahead security constrained unit commitment (SCUC) in early 2014, which is formulated by mixed integer programming (MIP). In [1], three heuristic approaches are proposed as alternative solving methods of commercial MIP solvers. The incremental solving and marginal resource identification algorithm is further developed from these heuristic approaches. The developed approaches can improve the efficiency of the iterative solution process and may perform better than MIP solvers when starting from a good initial solution. However, the heuristic methods usually cannot provide global lower bound and hence, has challenges to justify the global optimality gap. MIP solver is still used as the primary tool for solving SCUC.

A lot of research has been done to improve the tightness and compactness of MIP formulation [2]-[4]. Even though some of the resource level constraints are facet defining on energy and startup variables projection, system-wide constraints such as transmission constraints and reserve constraints can invalidate the convexity of the model. Practices at MISO show that large number of transmission constraints especially with large number of continuous variables such as virtual bids can create significant computational challenges [1].

In this paper, two formulations that lead to significant improvement on SCUC formulation are introduced. The first improvement is the re-formulation of piecewise linear incremental energy curve (PWL Enhancement). The PWL Enhancement is related to the locally ideal formulations for piecewise linear functions with indicator variables in [5]. Reference [6] discussed the application of similar formulation to improve the performance of network constrained unit commitment problem.

The second improvement is the re-formulation of transmission constraints (AGG Enhancement) to group variables with the same impact on all transmission constraints together. It can significantly reduce the number of nonzeros in the optimization matrix. In many cases, similar generators at one location may cause extra-long solving time, especially when those generators impact transmission constraints. Aggregating those generators together can improve SCUC performance.

The installed capacity of combined-cycle units is increasing worldwide because of improved operational efficiency and flexibility. Combined-cycle units are also critical to system operation as more renewable energy such as wind and solar due to their fast ramping capability, quick response, and relatively low cost. Better modelling of combined-cycle units in the market clearing engine can result in production cost saving and incentivize participants to offer the true cost. It has been studied by researchers and with real world implementations of different modeling approaches. There are mainly four different approaches used to represent the combined-cycle units. 1) Aggregate model. This model simplifies the combined cycle unit as a pseudo conventional unit by allowing the combine-cycle owner to provide an aggregate offer. This model sacrifices the accuracy of combined-cycle units because the aggregate offer cannot accurately represent the operating cost of the combined cycle units underneath. Several ISOs including MISO are using this model [7]-[9]. 2) Separate modelling. Each component (sub-unit) of combined cycle is modelled separately and the combined cycle units production cost is reflected in the objective function. This model preserves the generation characteristic for each sub-unit but it does not reflect the relationship between ST and CT. [10] 3) Configuration-based combined-cycle model. This model can better represent the operational modes, transition characteristics, and operating cost of combined-cycle units. However, it doesn’t distinguish sub-units under each configuration, and thus may undermine the accuracy of operating constraints such as min up/down time constraints [11]-[12]. 4) Graph-based combined-cycle model [13]. This model can describe the operating constraints for each sub-unit but this model may incur significant computational complexities. As the number of combined-cycle units increases, modelling of combined-cycle units could impose sizable impacts on the SCUC computational performance. MISO has been evaluating the configuration based combined cycle modeling and the prototype software performance hasn’t been satisfactory enough for production implementation [11]. With the PWL and AGG enhancement, the solving time with the configuration based combined cycle modeling has improved significantly. We also worked on developing tighter binary constraints to further improve the performance of the prototype software.

Major contributions of the paper include:

1) Applied locally ideal formulation to large scale real world SCUC problems and showed significant improvement on solution time on both the challenging cases and the normal cases.

2) Proposed approaches to group resources with the same impact on transmission constraints. The approach can significantly reduce the non-zeros for cases with large number of distributed variables such as virtual biddings. It can also help improve MIP convergence for cases with symmetric or near symmetric generators.

3) The two formulation improvement above are applied to configuration based combined cycle model and also showed significant improvement on MIP solution time. Furthermore, additional improvement has been made to the configuration based combined cycle MIP formulation to better represent the relationship between the transition variables and the commitment variables. It results in much tighter binary constraints and further improved performance.

The rest of the paper is organized as follows. Section II presents the locally ideal piecewise linear function formulation. Section III introduces the formulation to group variables with the same impact on all transmission constraints. Section IV presents the MIP formulation to tighten configuration based combined cycle modeling to improve the computational performance. Section V shows computational results on MISO day-ahead SCUC cases. Section VI concludes this paper.
II. PIECEWISE LINEAR INCREMENTAL ENERGY CURVE (PWL ENHANCEMENT)

The following SCUC model does not include reserves for simplification of notation. The real implementation is on the full MISO SCUC model. \[ \min N^T x + \sum_{j \in P}(C_{fj}(p_{j,t})) + \sum_{t \in T}(O_t^f g_t) \] s.t.:

Resource level constraints:

Limit constraints:
\[ u_{j,t} \leq p_{j,t} \leq u_{j,t} \cdot p_{j,t} \quad \forall j \in J \]  
\[ 0 \leq g_{k,t} \leq g_{k,t} \cdot u_{j,t} \quad \forall k \in K \]  

Resource binary constraints
\[ Ax \leq U \]  

Other Resource constraints

Transmission constraints:
\[ F^P_{i}(p_{i}, g_{i}, P_{i}) \leq E_{i} \quad \forall i \in I \]  
\[ F^T_{i}(p_{i}, g_{i}, P_{i}) = \sum_{j \in J}(p_{j,t} \cdot B_{j,n,j}) + \sum_{k \in K}(g_{k,t} \cdot B_{k,n,k}) + \sum_{n \in N}(P_{n,t} \cdot B_{n,n,t}) \quad \forall i \in I \]  

Power balance constraints

System wide reserve constraints and zonal reserve constraints

The incremental energy function is modeled as monotonically non-decrease piece-wise linear function. The modeling of the piecewise linear function can have significant impact on the performance. In [14], a special order type 2 set (SOS2) model was introduced to model piecewise linear approximation of nonlinear functions. For incremental energy cost function \( C_{fj}(p_{j,t}) \) approximated between pre-determined fixed points \( 0 = p_{j,t} \leq P_{j,t} \leq \cdots \leq P_{j,n,t} \), define a set of nonnegative continuous variables \( y_{j_0}, y_{j_1}, \ldots, y_{j_n} \) and the constraints:

\[ y_{j_0} + y_{j_1} + \cdots + y_{j_n} = 1 \]  
\[ p_{j,t} = y_{j_0} \cdot P_{j,t} + y_{j_1} \cdot P_{j,t} + \cdots + y_{j_n} \cdot P_{j,n,t} \]  

The incremental energy cost function is represented by:

\[ C_{fj}^{C}(p_{j,t}) = y_{j_0} \cdot C_{fj}(p_{j,t}) + \cdots + y_{j_m} \cdot C_{fj}(p_{j,m,t}) \]  

The incremental energy offer is convex and satisfies:

\[ 0 = C_{fj}^{C}(p_{j,t}) \leq C_{fj}^{P}(p_{j,t}) \leq \cdots \leq C_{fj}^{C}(p_{j,m,n}) \]  

In this model, at most two consecutive variables in (7) can be nonzero for convex cost functions. The piecewise linear incremental energy offer formulation in MISO production SCUC is different but equivalent. By revising the model to constraints (7)-(9), the performance doesn’t change very much.

However, considering constraint (2) and (8), and also the cost curves always start at \( C_{fj}(p_{j,n,t}) = 0 \) with \( P_{j,n,t} = 0 \), constraint (7)-(9) can be reformulated as:

\[ y_{j_0} + y_{j_1} + \cdots + y_{j_n} \leq u_{j,t} \]  
\[ p_{j,t} = y_{j_0} \cdot P_{j,t} + y_{j_1} \cdot P_{j,t} + \cdots + y_{j_n} \cdot P_{j,n,t} \]  
\[ C_{fj}^{P}(p_{j,t}) = y_{j_0} \cdot C_{fj}(p_{j,t}) + \cdots + y_{j_m} \cdot C_{fj}(p_{j,m,t}) \]  

Constraint (7a) links generation dispatch variable to the commitment variable. The test results in Table I show significant improvement (under column “With only PWL”) for the hard cases compared to the original solution (under “Original”).

We later found out that similar formulations were discussed in [5] and [6]. In [5] and [6], \( y_{j_0} \) was kept and the binary variable is directly introduced into constraint (7) as:

\[ y_{j_0} + y_{j_1} + \cdots + y_{j_n} = u_{j,t} \]  

It was proved that by introducing the binary variable to (7), the formulation with (7b) becomes locally ideal, i.e., the vertices of its corresponding LP relaxation satisfy all required integrality conditions.

Since \( 0 \leq y_{j_0} \leq 1 \), (7a) is equivalent to (7b). With \( P_{j,t} = 0 \) and \( C_{fj}(p_{j,t}) = 0 \), \( y_{j_0} \) can be removed from the formulation.

The formulation (7a)-(9a) is locally ideal and with less number of continuous variables. For MISO cases, it’s observed that root node relaxation time can be much faster (over 20% for some cases) by reducing \( y_{j_0} \).

III. REDUCE NON-ZERO AND SYMMETRY IMPACT ON TRANSMISSION CONSTRAINTS (AGG ENHANCEMENT)

The challenging cases usually have large number of virtuals coupled with large number of transmission constraints. It can introduce large number of non-zeros in constraint (6). Inspired by discussions following the presentation at the FERC technical conference [15], a reformulation of transmission constraint was developed by introducing a variable \( y_{n,t} \) for each node with multiple generators and/or virtuals connected. Constraint (6) can be reformulated by constraint (10a) and (6a) to aggregate the variables on the same nodes:

\[ \sum_{j \in J} \{n \in n \in N \{p_{j,t} \} \} + \sum_{k \in K} \{g_{k,t} \} \quad \forall n \in N, t \in T \]  

\[ F^P(y_t, P_t) = \sum_{n \in N} \{y_{n,t} \cdot B_{n,t} \} + \sum_{n \in N} \{P_{n,t} \cdot B_{n,t} \} \quad \forall i \in I, t \in T \]  

For cases with multiple virtuals offered to the same node, it can significantly reduce the number of non-zeros (e.g., from over 10 million to about 3 million for some of the hard cases). Assume the number of transmission constraints is \( C_i \) and the number of continuous variables on one node \( n \) is \( C_n \).

- With constraint (6), the number of non-zeros introduced by those variables on all the transmission constraints is \( C_i \cdot C_n \).
- With revised constraint (10a) and (6a), the number of non-zeros introduced by those variables on all the transmission constraints becomes: \( C_n + C_i \). However, it requires adding continuous variables \( y_{n,t} \).

The improvement from the reformulation is not consistent. For the most difficult cases with large number of virtuals and transmission constraints, even if the non-zeros can be reduced by over 70% with the aggregation under (10a) and (6a), the performance improvement is not obvious.

After further investigation on case H-3 in Table I, it was identified that the 10.81% MIP gap under PWL Enhancement was caused by a transmission constraint violation. Three generators at one station have over –60% impacts on the constraint (Fig. 1). All three of them are committed under the 1.36% solution. Apparently they should all be committed.

However, it took very long time for CPLEX MIP to search around 10.81% gap.

The issue can partly attribute to symmetry described in [16]. However, in real world problems, the generators may not be perfectly symmetry and can have slight differences, e.g. different initial on time, slightly different offers, etc. The ap-
proaches proposed to break symmetry are general applicable only to generators that are perfectly symmetric.

In MISO commercial model, one commercial pricing node (Cpnode) is created for each of the generator. There are several virtual bids and offers on each of the three Cpnodes in Fig. 1. The aggregation above can merge the virtual clearing and generator dispatch variables at the same Cpnode. MISO usually doesn’t monitor step up transformers. Hence, for all transmission constraints in (5) and (6), the three Cpnodes in Fig. 1 should have the same impact and can be further combined on the LHS of transmission constraints. However, the three generators are on different nodes. Under certain breaker contingency, the high KV bus may split and cause the Cpnodes to have different impact on transmission constraints. Therefore, a sensitivity based approach is developed to identify Cpnodes that can be combined.

![Fig. 1 Generators at the same plant](image)

In this approach, the set of nodes $N$ is partitioned into subsets $N^t_a$, $N^t_b$, ..., $N^t_k$ for each $t \in T$. Each set can include one or more nodes. For any two nodes $n_a$ and $n_b$ within one subset $N^t_i$, $i \in \{1, 2, ..., s_i\}$, we have: $B_n(\text{nt}_{a,t}) = B_n(\text{nt}_{b,t}) \quad \forall i \in I$. In other words, all nodes within the same set have the same impact on transmission. We use $B_l^t$ to represent all these equal sensitivities within the subset. Then further modify (10a) and (6a) to (10b) and (6b):

$$z_{l,t} = \sum_{j \in J} n_{j} t \{P_{j,t}\} + \sum_{k \in K} n_k t \{g_{l,t}\} \quad \forall l \in \{1, 2, ..., s_l\}, t \in T \quad (10b)$$

$$F_l(t, P_t) = \sum_{i \in [1, 2, ..., s_i]} \{z_{l,t} \cdot B_l^t\} + \sum_{n \in N} \{P_{\text{nt}} \cdot B_{\text{nt},t}\} \quad \forall i \in I, t \in T \quad (6b)$$

In (10b), all continuous dispatch variables with the same impact on transmission are grouped together. With these revised constraints, the three Cpnodes in Fig. 1 are grouped into one subset since they have the same impact on all monitored transmission constraints. The number of non-zeros can be further reduced. More importantly, there is only one variable to represent them all on the LHS of transmission constraints. Even if there is symmetry issue with those generators, the section of the matrix with symmetry can be significantly reduced since the generator dispatch variables are not directly shown in the transmission constraints.

The SCUC performance can be improved significantly as shown in Section V.A.

IV. TIGHTEN FORMULATION FOR CONFIGURATION BASE COMBINED CYCLE MODELING

A combined cycle power plant has a combination of gas and steam turbine units and the waste heat from gas turbine is reused by steam turbine to generate extra power. Combine cycle units have lower levelized cost of electricity, high efficiency, lower CO2 emission, better operation flexibility and faster response. Therefore, there is an increasing trend of installed combined cycle units [17]-[19].

MISO has been evaluating configuration based combined cycle modeling for several years [11]. The performance hasn’t been ideal, especially for the hard cases. The proposed formulation improvement in Section II and III was applied to the combined cycle modeling and has brought significant improvement. With configuration based combined cycle model, each configuration is modeled as a generator and the transition matrix represent the relationship between the configurations. It’s equivalent to adding many generators at the same location. Most of them have the same impact on transmission constraints. The AGG Enhancement can significantly reduce the non-zeros introduced by those configurations.

A. Characteristics of Combined-Cycle Unit

A combined cycle unit has multiple operating configurations and each configuration has different combinations of CT and ST commitment status. In Fig. 2, a two CTs and one ST example demonstrates the feasible transitions between each configuration. In one transition, only one unit is allowed to start up or shut down and ST cannot operate independently. In this section, the combined cycle mathematical model will be discussed.

In general, conventional units have four transitions: one shut down and three types of start-ups, namely, hot start-up, intermediate start-up, and cold start-up. Similarly, there are four possible types of transition between combined-cycle configurations, hot start-up, intermediate start-up, cold start-up, and shut-down. Different types of transitions may incur different transition cost.

![Configuration 2 Feasible transition diagram for a combine cycle unit](image)

B. Mathematical Model of Configuration-based Combined Cycle Units

To realize the transition and states of each configuration, the proposed formulation for configuration-based combined cycle is shown as below.

1) Operating Cost:

The operating cost of combined cycle units in equation (11) includes three components, which are incremental energy cost $C_{jct}^{y', y''} (\text{pt})$, no load cost $C_{jct}^{\text{NL,y}} (\text{ut})$, and transition cost $\sum_{y \in Y} C_{jct}^{y'y''} (\text{pt})$. The combined cycle need to provide incremental energy cost, no load cost for each configuration and transition cost for each feasible transition.
\[
\begin{align*}
\sum_{j \in J} \sum_{y \in Y} \sum_{t \in T} C^y_{jt}(p_{jt}) + C^{NLy}_{jt} u^y_{jt} + \\
\sum_{w \in W} \sum_{y' \in Y, y'' \neq y} C_{jyt}^{y'y''} v^{y'y''}_{jt} \tag{11}
\end{align*}
\]

2) **Unique Transition Constraints:**

\( v^{y'y''}_{jt} \) is defined as a binary variables that represent the transition between configurations. In each combined cycle unit, at most one transition is allowed at time interval \( t \).

\[
\sum_{y \in Y} \sum_{t \in T} v^{y'y'}_{jt} + \sum_{t \in T} v^{y'y''}_{jt} \leq 1 \tag{12}
\]

This constraint is one of the most effective constraints. It significantly reduces the possible combinations of the transition variables. For some early test case, it reduced the solving time from over 1300s to ~500s.

3) **Unique Configuration State Constraints:**

Variable \( u^y_{jt} \) is defined as the commitment status for each configuration. At each interval \( t \), each combined cycle can only have one configuration commitment status. Here all off is also a valid configuration.

\[
\sum_{y \in Y} u^y_{jt} = 1 \tag{13}
\]

4) **Exclusive Type of Transition**

Even though there are three types of transitions, some transitions may only involve shut down. For example, from configuration 3 (2CT+OST) to configuration 2 (1CT+OST), only shut-down is involved in the transition. Since MISO only accounts for startup cost, the transition cost for this example should be 0. Furthermore, the startup type variables for traditional generator or transition variables for the same pair of configurations for a combined cycle can be combined for units that have at least two startup costs or transition cost equal. As a result, we can reduce the number of startup type or transition variables.

For transitions with intermediate startup cost equal to hot start-up cost,

\[
v^{y'y',\text{hot}}_{jt} = 0 \quad \forall t, j, y, y' \text{ with } y \neq y' \tag{14}
\]

For transitions with cold start-up cost equal to intermediate start-up cost,

\[
v^{y'y',\text{intermediate}}_{jt} = 0 \quad \forall t, j, y, y' \text{ with } y \neq y' \tag{15}
\]

Note that in this case, the variables that in equations (14) and (15) can be removed from the formulation to reduce the number of transition variables. This technique can also be applied to conventional units to reduce the number of startup type variables.

5) **Configuration Transition Constraints:**

Term \( \sum_{w \in W} \sum_{y' \in Y} v^{y'y'}_{jt} \) represents the summation of all feasible transitions to configuration \( y \). \( \sum_{w \in W} \sum_{y' \in Y, y'' \neq y} v^{y'y''}_{jt} \) represents the summation of all feasible transitions from configuration \( y \). Therefore, \( \sum_{w \in W} \sum_{y' \in Y} v^{y'y'}_{jt} \) and \( \sum_{w \in W} \sum_{y' \in Y} v^{y'y''}_{jt} \) can be viewed as start-up and shut-down variables for configuration \( y \) respectively and similar to conventional units [20], the transition relationship between two consecutive hours commitments for configuration \( y \) can be expressed as equation (16).

\[
u^y_{jt} - u^y_{jt-1} = \sum_{w \in W} \sum_{y' \in Y} v^{y'y'}_{jt} - \sum_{w \in W} \sum_{y' \in Y} v^{y'y''}_{jt} \quad \forall t, j, y \tag{16}
\]

6) **Minimum Up/Down Time Constraints:**

Since the \( \sum_{w \in W} \sum_{y' \in Y} v^{y'y'}_{jt} \) and \( \sum_{w \in W} \sum_{y' \in Y} v^{y'y''}_{jt} \) can be viewed as start-up and shut-down variables for configuration \( y \) respectively, similar to conventional units, the minimum up and down constraints for configuration \( y \) can be written as (17) and (18) respectively.

\[
\sum_{t=t+1}^{t} \sum_{y' \in Y} v^{y'y'}_{jt} \leq u^y_{jt} \quad \forall t \in [TU^y, T], j, y \tag{17}
\]

\[
\sum_{t=t+1}^{t+1} \sum_{y' \in Y} v^{y'y''}_{jt} \leq 1 - u^y_{jt} \quad \forall t \in [TD^y, T], j, y \tag{18}
\]

Reference [21]-[22] showed that conventional units version of equations (17) and (18) and is \( k \) metric for the projection of commitment, start-up, and shut-down variables. Similarly, (17) and (18) are tight CC transition Minimum Up/Down time constraints.

7) **Minimum and Maximum Output:**

For each configuration, a combined cycle may have different minimum output \( P^y_{jt} \) and maximum output \( P^y_{jt} \).

\[
u^y_{jt} \cdot P^y_{jt} \leq p^y_{jt} \leq P^y_{jt} \cdot \bar{P}^y_{jt} \quad \forall j, t, y \tag{19}
\]

8) **Ramping Constraints**

Different combinations of CTs and STs may have different ramp up and ramp down rates. For each configuration, the generators need to provide ramp up and ramp down rates. The ramp up and ramp down constraints are enforced by equation (20) and (21) respectively. In equation (20), if there is no transition to configuration \( y \), i.e. \( \sum_{w \in W} \sum_{y' \in Y} v^{y'y'}_{jt} = 0 \), then ramp up constraints are ruled by \( p^y_{jt} - p^y_{jt-1} \leq RU^y_{jt} u^y_{jt} \).

If there is transition to configuration \( y \), i.e. \( \sum_{w \in W} \sum_{y' \in Y} v^{y'y'}_{jt} = 1 \), this implies \( u^y_{jt} = 1 \) based on equation (16). Equation (20) will be \( p^y_{jt} - p^y_{jt-1} \leq \sum_{w \in W} v^{y'y'}_{jt} \cdot P^y_{jt} \cdot RU^y_{jt} \). This is similar to start up ramp up constraints of conventional units [23]. Same implications can also apply to ramp down constraints, i.e., equation (21).

\[
p^y_{jt} - p^y_{jt-1} \leq RU^y_{jt} \left( u^y_{jt} - \sum_{w \in W} v^{y'y'}_{jt} \cdot P^y_{jt} \right) + (\sum_{w \in W} v^{y'y'}_{jt} \cdot P^y_{jt} \cdot RU^y_{jt}) \quad \forall t, j, y \tag{20}
\]

\[
p^y_{jt} - p^y_{jt-1} \leq RD^y_{jt} \left( u^y_{jt} - \sum_{w \in W} v^{y'y'}_{jt} \cdot P^y_{jt} \right) + (\sum_{w \in W} v^{y'y'}_{jt} \cdot P^y_{jt} \cdot RD^y_{jt}) \quad \forall t, j, y \tag{21}
\]

Another way of modelling the ramping constraints is equations (20a) and (21a) and they describe the same start-up and shut-down ramp constraints. Because \( u^y_{jt} - \sum_{w \in W} v^{y'y'}_{jt} \cdot P^y_{jt} \leq u^y_{jt-1} \), (20) dominates
Similarly, because $\sum_{w=1}^{W} \sum_{y=1}^{Y} x_{t, w}^{y, t} \geq u_{t, t-1}^{y} - u_{t, t}^{y}$, we have $u_{t, t-1}^{y} - \sum_{w=1}^{W} \sum_{y=1}^{Y} x_{t, w}^{y, t} \leq u_{t, t}^{y}$ and $\sum_{w=1}^{W} \sum_{y=1}^{Y} x_{t, w}^{y, t} \leq 1 - u_{t, t}^{y}$, hence (21) dominates (21a).

However, equations (20) and (21) incur a lot more non-zeros compared to (20a) and (21a). Therefore, equations (20) and (21) are tighter than equations (20a) and (21a) while equations (20a) and (21a) are more compact than equations (20) and (21). The computational study on MISO cases shows that equations (20a) and (21a) work better.

The computational study on MISO cases shows that compared to (20a) and (21a). Therefore, equations (20) and (21) dominate (21a).

The rest of the cases in Table II can reach the MIP relative gap tolerance before 1200s. For these cases, the new PWL and AGG formulations can reduce the solving time significantly. In Fig. 3, we further studied additional 23 production day-ahead SCUC cases. All of the cases can be solved within 900s CPU time in the original scenario. With the new PWL and AGG formulation, the solving time has been reduced by over 30% with similar MIP gap results.

B. PWL and AGG Enhancement on cases with configuration based combined cycle model

Tables III shows the effectiveness of the PWL and AGG improvements on configuration based combined cycle modeling. MISO currently has about 40-50 combined cycle groups. We collected about 20 configuration-based combined cycle modeling data from participants to replace the aggregated combined cycle models (i.e., 20-CC case). In this section, three sets of results are compared:

The first set of results: the columns under “Configuration based CC Model - Without PWL and AGG” show the solving time and MIP gap under the formulation without PWL and AGG improvement.

The second set of results: the columns under “Configuration based CC Model - With PWL and AGG” show the solving time and MIP gap under the formulation with PWL and AGG improvement. For the five tested cases, it shows ~50% reduced solving time for four cases and 30% reduced solving time for one case.

The third set of results: the last two columns show the aggregated combined cycle model used in current production system with the PWL and AGG improvement.

With the PWL and AGG improvement, replacing aggregated combined cycle model with configuration based combined cycle model results in reasonable increase of solving time for the 20-CC case. The overall solving time is within 1200s time limit. Without PWL and AGG improvement, it requires increasing time limit up to 2790s to reach reasonable solution.

C. Tighten configuration based combined cycle modeling

The performance of the configuration based combined cycle modelling can be greatly improved with the formulation proposed in section IV. In this section, the numerical results of the proposed configuration-based CC formulation are compared with those of the formulation in [9]. The numerical results based on reference [9] are labeled as “Ref” in the tables IV, V, and VI. In the hard day-ahead cases, there are usually a large volume of virtual bids, which generate a large number of non-zeros. More non-zeros usually require longer time to solve. The AGG formulation can help reduce the non-zeros. However, the optimization solver still needs to handle larger number of variables. Tables IV and V compares the optimality gap with and without virtual bids in 1200s time limit to show the performance of Ref and the proposed methods and the impacts of virtual bids on computational performance as well. The proposed formulation is tighter and more compact than the Ref. Improved tightness can benefit the root node relaxation solution quality and heuristics. Improved compactness can reduce the solution time for each node. It can be seen that the proposed CC formulation is consistently better than the original formulation in Ref.
Table VI and Table VII present the time needed to first reach <3% optimality gap for cases with and without virtual bids respectively. The proposed formulation still has consistent shorter solution time compared to the results from Ref. It can be observed that larger number of CCs increases the computational complexity of the problem and 80 CCs case and 120 CCs case require much longer time to reach 3% optimality gap. The proposed CC modeling has much better computational performance than the formulation in [9]. From Table VI, for the 120 CCs case, Ref can only solve the problem to 63.444% after 12000s while the proposed can reach 3% gap at 5756s.

![Fig.3 Results on normal cases (with and w/o PWL and AGG, w/o configuration based combined cycle model)](image)

<table>
<thead>
<tr>
<th>Table I Performance Comparison - Hard Cases</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Hard cases (1200s time limit)</strong></td>
</tr>
<tr>
<td>---------------------------------</td>
</tr>
<tr>
<td>H-1</td>
</tr>
<tr>
<td>H-2</td>
</tr>
<tr>
<td>H-3</td>
</tr>
<tr>
<td>H-4</td>
</tr>
<tr>
<td>H-5</td>
</tr>
<tr>
<td>H-6</td>
</tr>
<tr>
<td>H-7</td>
</tr>
<tr>
<td>H-8</td>
</tr>
<tr>
<td>H-9</td>
</tr>
<tr>
<td><strong>Average</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table II Performance Comparison - Normal Cases</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Normal Cases</strong></td>
</tr>
<tr>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td><strong>CPU (s)</strong></td>
</tr>
<tr>
<td>N-1</td>
</tr>
<tr>
<td>N-2</td>
</tr>
<tr>
<td>N-3</td>
</tr>
<tr>
<td>N-4</td>
</tr>
<tr>
<td>N-5</td>
</tr>
<tr>
<td>N-6</td>
</tr>
<tr>
<td>N-7</td>
</tr>
<tr>
<td>N-8</td>
</tr>
<tr>
<td><strong>Average</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table III PWL and AGG Results on configuration based combined cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Configuration Based CC Model</strong></td>
</tr>
<tr>
<td><strong>Cases</strong></td>
</tr>
<tr>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>CC-1</td>
</tr>
<tr>
<td>CC-2</td>
</tr>
<tr>
<td>CC-3</td>
</tr>
<tr>
<td>CC-4</td>
</tr>
<tr>
<td>CC-5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table IV Optimality gap with a time limit 1200 seconds without virtual bids for one hard case</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number of CCs</strong></td>
</tr>
<tr>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>20</td>
</tr>
<tr>
<td>40</td>
</tr>
<tr>
<td>80</td>
</tr>
<tr>
<td>120</td>
</tr>
<tr>
<td><strong>Average</strong></td>
</tr>
</tbody>
</table>
This paper proposes two modifications on MIP formulation that lead to significant improvement on SCUC performance. The improvements are also extended to configuration based combined cycle cases and lead to significant improvement in SCUC solution time. In addition, further improvement is made to the configuration based combined cycle formulation. The proposed CC model shows significant improvement on cases with large number of configuration based combined cycles.

**VI. CONCLUSIONS**

**Table V Optimality gap with a time limit 1200 seconds with virtual bids for one hard case**

<table>
<thead>
<tr>
<th>Number of CCs</th>
<th>Ref</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.485%</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>12.28%</td>
<td>0.889%</td>
</tr>
<tr>
<td>40</td>
<td>8.01%</td>
<td>0.6%</td>
</tr>
<tr>
<td>80</td>
<td>67.29%</td>
<td>49.37%</td>
</tr>
<tr>
<td>120</td>
<td>73.51%</td>
<td>54.68%</td>
</tr>
<tr>
<td>Average</td>
<td>40.27%</td>
<td>26.38%</td>
</tr>
</tbody>
</table>

**Table VI Time needed to first reach <3% optimality gap without virtual bids for one hard case**

<table>
<thead>
<tr>
<th>Number of CCs</th>
<th>Ref</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>596s</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>930s</td>
<td>888s</td>
</tr>
<tr>
<td>40</td>
<td>541s</td>
<td>378s</td>
</tr>
<tr>
<td>80</td>
<td>3288s</td>
<td>1843s</td>
</tr>
<tr>
<td>120</td>
<td>6702s</td>
<td>2635s</td>
</tr>
</tbody>
</table>

**Table VII Time needed to first reach <3% optimality gap with virtual bids for one hard case**

<table>
<thead>
<tr>
<th>Number of CCs</th>
<th>Ref</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>292s</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>1598s</td>
<td>435s</td>
</tr>
<tr>
<td>40</td>
<td>1746s</td>
<td>821s</td>
</tr>
<tr>
<td>80</td>
<td>4487s</td>
<td>3854s</td>
</tr>
<tr>
<td>120</td>
<td>12000s</td>
<td>(63.444%)</td>
</tr>
</tbody>
</table>

**VII. REFERENCES**


**VIII. BIOGRAPHIES**

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