MIP Formulation Improvement for Large Scale Security Constrained Unit Commitment with Configuration based Combined Cycle Modeling

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Abstract – As a part of the day-ahead market clearing process, Midcontinent Independent System Operator (MISO) solves one of the largest and most challenging Security Constrained Unit Commitment (SCUC) models. Better computational performance of SCUC models not only improves the market efficiency but also facilitates future market developments. This paper introduces recent developments in SCUC formulation with configuration based combined cycle modeling, which include the improvement on constraints associated with binary variables, the improvement of formulation on piecewise linear incremental energy curve (PWL) and reduction of non-zeros by aggregating variables (AGG). Furthermore, mathematical proof shows that the proposed enhanced locally ideal PWL model is the convex envelope of the PWL cost function. To illustrate the effectiveness of the proposed formulation in this paper, numerical results and analysis based on MISO system are presented.

Index Terms – Combined cycle units, convex envelope, day-ahead electricity market, locally ideal formulation, mixed integer programming, security constrained unit commitment

NOMENCLATURE

A. Indices

\( j \) Index of generation resources, including both non-CC units and CC units.

\( k \) Index of other dispatchable variables (virtuals, dispatchable demand and dispatchable transaction).

\( i \) Index of transmission constraints.

\( t, t' \) Index of interval.

\( n \) Index of nodes in the network.

\( y, y' \) Index of configuration of a combined cycle unit.

\( w \) Transition type.

B. Set

\( I \) Set of transmission constraints.

\( J \) Set of resources.

\( J_y \) Set of CC units.

\( T \) Set of intervals.

\( Y_j \) Set of configuration of combined cycle unit \( j \).

\( Y_{from,y} \) Set of configurations that have feasible transition to configuration \( y \), \( y \notin Y_{from,y} \).

\( Y_{y,to} \) Set of configurations that have feasible transition from configuration \( y \), \( y \notin Y_{y,to} \).

\( W \) Set of transition cost type \{hot-start-up, intermediate start-up, cold start-up\}.

C. Parameters

\( A \) Resource level constraint binary variables matrix.

\( B_{ln,t} \) Sensitivity of the flow on transmission constraint to injection at node and withdrawal at the reference bus.

\( C_{NL,y}^{j,t} \) No-load cost of resource \( j \) configuration \( y \) at time interval \( t \).

\( C_{j,ty,y''}^{trans} \) Transition cost of resource \( j \) from configuration \( y \) to configuration \( y'' \) in transition type \( w \).

\( F_{j,t} \) Transmission limit.

\( G_{j,t} \) Upper bound for other dispatchable resources.

\( N \) Vector of cost for binary variables.

\( O_t \) Vector of offer cost for other dispatchable resources.

\( P_t \) Fixed injection.

\( P_{min,t}, P_{max,t} \) Generation resource minimum and maximum limits.

\( P_{ymin,t}, P_{ymax,t} \) Generation resource minimum and maximum limits for configuration \( y \).

\( RD_{j,y}^{trans} \) Ramp down rate of resource \( j \) configuration \( y \) at non-shutdown intervals.

\( RU_{j,y}^{trans} \) Ramp up rate of resource \( j \) configuration \( y \) at non-startup intervals.

\( RU0_{j,y} \) Ramp up rate of resource \( j \) configuration \( y \) at startup intervals.

\( TD_{j,y}^{trans} \) Minimum downtime of resource \( j \) configuration \( y \).

\( TU_{j,y}^{trans} \) Minimum uptime of resource \( j \) configuration \( y \).

D. Functions

\( C_{j,t}^P(\cdot) \) Incremental energy cost function.

\( C_{j,t}^{P,trans}(\cdot) \) Incremental energy cost function for combined cycle \( j \) configuration \( y \) at time interval \( t \).

\( F_{j,t}^P(\cdot) \) Transmission flow.

E. Variables

\( x \) Binary variables (commitment, start-up, shut-down).

\( u_{j,t} \) Commitment variable for resource \( j \) at interval \( t \).

\( u_{j,t}^y \) Commitment variable for resource \( j \) configuration \( y \) at interval \( t \).

\( p_{j,t} \) Cleared energy on generation resource \( j \) at interval \( t \).

\( g_{k,t} \) Cleared energy on other dispatchable resources.

\( p_t, g_t \) Vectors of cleared energy on generation and on other dispatchable resources.

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Transition variable from \(y\) to \(y'\) for resource \(j\) at interval \(t\) in transition type \(w\), where \(y \neq y'\).

### I. INTRODUCTION

**MIDCONTINENT** Independent System Operator (MISO) manages one of the largest electricity markets in the world. Following a significant expansion of its market footprint and an uptick in virtual trading volumes, MISO encountered computational performance challenges in solving its day-ahead security constrained unit commitment (SCUC) in early 2014, which is formulated as a mixed integer programming (MIP). SCUC is a NP-hard problem and commercial solver may not always guarantee a good quality of solution in required time limit. In [1], three heuristic approaches are proposed as alternative solving methods of commercial MIP solvers. The incremental solving and marginal resource identification algorithm is further developed from these heuristic approaches. The developed approaches can improve the efficiency of the iterative solution process and may perform better than MIP solvers when starting from a good initial solution. However, the heuristic methods usually cannot provide global lower bound and hence, are unable to justify the global optimality gap unless a global lower bound is provided by other methods. MIP solver is still used as the primary tool for solving SCUC.

Many studies aim to improve the tightness and compactness of SCUC formulation [2]-[4]. Some constraints are facet defining on energy and startup variables projection, which is the tightest possible for a single resource, however, ramp rate constraints and system-wide constraints such as transmission constraints and reserve constraints can invalidate the convexity of these resource level facet defining constraints. Practices at MISO show that large number of transmission constraints especially with large number of continuous variables such as virtual bids can create significant computational challenges to SCUC [1].

Introducing new features in electricity market can further add computational complexity to existing difficult SCUC problem and one of such new features is combined cycle power plants. A combined cycle power plant has a combination of gas and steam turbine units and the waste heat from gas turbine is re-used by steam turbine to generate extra power. Combine cycle units have lower levelized cost of electricity, high efficiency, lower CO2 emission, better operation flexibility and faster response. Therefore, there is an upward trend of installing combined cycle units [17]-[19]. MISO has been evaluating configuration based combined cycle modeling for several years [11]. The performance hasn’t been ideal, especially for some challenging cases. With configuration based combined cycle model, each configuration is modeled as a generator and the transition matrix represent the relationship between the configurations. Combined-cycle units are also critical to system operation with more integration of renewable energy due to their fast ramping capability, quick response, and relatively low cost. Better modelling of combined-cycle units in the market clearing engine can result in production cost saving and incentivize participants to offer the true cost. MISO study show that better combined cycle modeling can potentially bring over $16 million annual production cost savings.

It has been studied by researches and with real world implementations of different modeling approaches. There are mainly four different approaches used to represent the combined-cycle units. 1) **Aggregate model**. This model simplifies the combined cycle unit as a pseudo conventional unit by allowing the combine-cycle owner to provide an aggregate offer. This model sacrifices the accuracy of combined-cycle units because the aggregate offer cannot accurately represent the operating cost of the combined cycle units underneath. Several ISOs including MISO are using this model [7]-[9]. 2) **Separate modelling**. Each component (sub-unit) of combined cycle is modelled separately and the production costs of the individual units are reflected in the objective function. This model preserves the generation characteristic for each sub-unit but it does not reflect the relationship between steam turbine (ST) and combustion turbine (CT). [10] 3) **Configuration-based combined-cycle model**. This model can better represent the operational modes, transition characteristics, and operating cost of combined-cycle units. However, it doesn’t distinguish sub-units under each configuration, and thus may undermine the accuracy of operating constraints such as min up/down time constraints [11]-[12]. 4) **Graph-based combined-cycle model** [13]. This model can describe the operating constraints for each sub-unit but this model may incur significant computational complexities. As the number of combined-cycle units increases, modelling of combined-cycle units could impose sizable impacts on the SCUC computational performance. MISO has been evaluating the configuration based combined cycle modeling and the prototype software performance hasn’t been satisfactory enough for production implementation [11].

This paper introduces recent work from MISO to improve the SCUC formulation with configuration based combined cycle modeling. Improvements in three areas have led to significant performance improvement and the possibility for production implementation.

The first area is tightening constraints related to binary variables associated with configuration based combined cycle modeling. The binary variables include the commitment variables and transition variables. The second improvement is the re-formulation of piecewise linear incremental energy curve (PWL Enhancement). The PWL Enhancement is related to the locally ideal formulations for piecewise linear functions with indicator variables [5]. Reference [6] discussed the application of similar formulation to improve the performance of network constrained unit commitment problem. Using similar approach as shown in [24], we also prove that this formulation forms the convex envelope of the PWL cost function. Hence it’s the tightest modeling of the PWL cost function. The third improvement is the re-formulation of transmission constraints (AGG Enhancement) to group variables with the same impact on all transmission constraints together. It can significantly reduce the number of nonzeros in the optimization matrix. In many cases, similar generators at one location may cause extra-long solving time, especially when those generators impact transmission constraints. Aggregating those generators together can improve SCUC performance.
The rest of the paper is organized as follows. Section II presents the MIP formulation to tighten configuration based combined cycle modeling to improve the computational performance. Section III presents the locally ideal piecewise linear function formulation. Section IV introduces the formulation to group variables with the same impact on all transmission constraints. Section V shows computational results on MISO day-ahead SCUC cases. Section VI concludes this paper.

II. TIGHTEN FORMULATION FOR CONFIGURATION BASED COMBINED CYCLE MODELING

A. Characteristics of Combined-Cycle Unit

A combined cycle unit has multiple operating configurations and each configuration has different combinations of CT and ST commitment status. In Fig. 1, a two CTs and one ST example demonstrates the feasible transitions between each configuration. In one transition, only one unit is allowed to start up or shut down and ST cannot operate independently. In this section, the combined cycle mathematical model will be discussed.

In general, conventional units have four transitions: one shut down and three types of start-ups, namely, hot start-up, intermediate start-up, and cold start-up. Similarly, there are four possible types of transition between combined-cycle configurations, hot start-up, intermediate start-up, cold start-up, and shut-down. Different types of transitions may incur different transition cost.

![Feasible transition diagram for a combine cycle unit](image)

B. Mathematical Model of Configuration-based Combined Cycle Units

To mathematically model the transition and states of each configuration, the proposed formulation for configuration-based combined cycle is shown as below.

1) Operating Cost:

The operating cost of combined cycle units in equation (1) includes three components, which are incremental energy cost \( c_{jt}^{P} \), no load cost \( c_{jt}^{NL} \), and transition cost \( \sum_{y' \in Y_{jt}} c_{jt}^{T} v_{jt}^{yy',w} \). The combined cycle need to provide incremental energy cost, no load cost for each configuration and transition cost for each feasible transition.

\[
\sum_{t \in T} \sum_{y \in Y} \sum_{t \in T} \left[ C_{jt}^{P}(P_{jt}) + C_{jt}^{NL} y_{jt}^{y'} + \sum_{w \in W} C_{jt}^{T} v_{jt}^{yy',w} \right]
\]

2) Unique Transition Constraints:

\( v_{jt}^{yy',w} \) is defined as a binary variables that represent the transition between configurations. In each combined cycle unit, at most one transition is allowed at time interval \( t \).

\[
\sum_{y \in Y} \sum_{w \in W} \sum_{y' \in Y^{y}} v_{jt}^{yy',w} \leq 1
\]  

Constraint (3) and (6) should dominate constraint (2). Constraints (2) won’t tighten the bound on the best possible integer solution, but in practice, constraints (2) can prevent unnecessary search branches. In some cases, constraints (2) can significantly improve the computational solving time.

3) Unique Configuration State Constraints:

Variable \( u_{jt}^{y} \) is defined as the commitment status for each configuration. At each interval \( t \), each combined cycle can only have one configuration commitment status. Here all off is also a valid configuration.

\[
\sum_{y \in Y} u_{jt}^{y} = 1
\]

4) Exclusive Type of Transition

Even though there are three types of transitions, some transitions may only involve shut down. For example, from configuration 3 (2CT+0ST) to configuration 2 (1CT+0ST), only shut-down is involved in the transition. Since MISO only accounts for startup cost, the transition cost for this example should be 0. Furthermore, the start-up type variables for traditional generator or transition variables for the same pair of configurations for a combined cycle can be combined for units that have at least two startup costs or transition cost equal. As a result, we can reduce the number of startup type or transition variables.

For transitions with intermediate startup cost equal to hot start-up cost,

\[
v_{jt}^{y' \text{hot}} = 0 \quad \forall t, j, y, y' \text{ with } y \neq y'
\]

For transitions with cold start-up cost equal to intermediate start-up cost,

\[
v_{jt}^{y' \text{intermediate}} = 0 \quad \forall t, j, y, y' \text{ with } y \neq y'
\]

Note that in this case, the variables in equations (4) and (5) can be removed from the formulation to reduce the number of transition variables. This technique can also be applied to conventional units to reduce the number of startup type variables.

5) Configuration Transition Constraints:

Term \( \sum_{w \in W} \sum_{y' \in Y} v_{jt}^{y',w} \) represents the summation of all feasible transitions to configuration \( y \).

\( \sum_{w \in W} \sum_{y' \in Y} v_{jt}^{y',w} \) represents the summation of all feasible transitions from configuration \( y \). Therefore, \( \sum_{w \in W} \sum_{y' \in Y} \sum_{y \in Y} v_{jt}^{y',w} \) and \( \sum_{w \in W} \sum_{y' \in Y} v_{jt}^{y',w} \) can be viewed as start-up and shut-down variables for configuration \( y \) respectively and similar to conventional units in [20], the transition relationship between two consecutive hours commitments for configuration \( y \) can be expressed as equation (6).
\[
\begin{align*}
    u_{ft}^y - u_{ft-1}^y &= \sum_{w \in W} \sum_{y \in Y^w} v_{ft}^{y,w} - \sum_{w \in W} \sum_{y \in Y^{w,t}} v_{ft}^{y,w,0} \forall t, j, y \\
6) \quad \text{Minimum Up/Down Time Constraints:} & \\
    \quad & \text{Since } \sum_{w \in W} \sum_{y \in Y^w} v_{ft}^{y,w} \text{ and } \sum_{w \in W} \sum_{y \in Y^{w,t}} v_{ft}^{y,w,0} \text{ can be viewed as start-up and shut-down variables for configuration } y \text{ respectively, similar to conventional units, the minimum up and down constraints for configuration } y \text{ can be written as (7) and (8) respectively.} \\
    \sum_{t' = t - T_U}^{t} \sum_{w \in W} \sum_{y \in Y^w} v_{ft'}^{y,w} - u_{ft}^y &\leq 0 \forall t, j, y, T_U \text{ (7)} \\
    \sum_{t' = t - T_D}^{t} \sum_{w \in W} \sum_{y \in Y^w} v_{ft'}^{y,w,0} - u_{ft}^y &\leq 1 - u_{ft}^y \forall t \in [T_D, T], j, y \quad (8)
\end{align*}
\]

Reference [21]-[22] showed that conventional units version of equations (7) and (8) are tight CC transition Minimum Up/Down time commitment, start-up, and shut-down variables. Similarly, (7) and (8) are tight CC transition Minimum Up/Down time constraints.

7) \quad \text{Minimum and Maximum Output:} \\
For each configuration, a combined cycle may have different minimum output \( p_{f,t}^{y,\text{min}} \) and maximum output \( p_{f,t}^{y,\text{max}} \).
\[
    u_{ft}^y, p_{ft}^{y} \leq p_{ft}^{y,\text{min}} \leq u_{ft}^y, p_{ft}^{y,\text{max}} \forall j, t, y \quad (9)
\]

8) \quad \text{Ramping Constraints:} \\
Different combinations of CTs and STs may have different ramp up and ramp down rates. Under configuration based combined cycle model, each configuration has its own ramp up and ramp down rates. Ramp up and ramp down constraints are enforced by equation (10) and (11) respectively. In equation (10), if there is no transition to configuration \( y \), i.e., \( \sum_{w \in W} \sum_{y' \in Y^w} v_{ft}^{y',w} = 0 \), then ramp up constraints are ruled by \( p_{ft}^{y} - p_{ft-1}^{y} \leq RU_{i}^{y} u_{ft}^{y} \). If there is transition to configuration \( y \), i.e. \( \sum_{w \in W} \sum_{y' \in Y^w} v_{ft}^{y',w} = 1 \), this implies \( u_{ft}^{y} = 1 \) based on equation (6). Equation (10) becomes \( p_{ft}^{y} - p_{ft-1}^{y} \leq \sum_{w \in W} \sum_{y' \in Y^w} v_{ft}^{y',w} \cdot RU_{i}^{y} \). This is similar to start up ramp up constraints of conventional units [23]. Some implications can also apply to ramp down constraints, i.e., equation (11). Note, in shut down hours, MISO does not enforce ramp constraints. Constraint (11) ensures that the resource can move from any \( p_{ft-1}^{y} \) to \( p_{ft}^{y} = 0 \).
\[
    p_{ft}^{y} - p_{ft-1}^{y} \leq RU_{i}^{y} \cdot u_{ft}^{y} + (\sum_{w \in W} \sum_{y' \in Y^w} v_{ft}^{y',w})(RU_{i}^{y} - RU_{i}^{y}') \forall t, j, y \quad (10)
\]
\[
    p_{ft}^{y} - p_{ft-1}^{y} \leq RD_{i}^{y} \cdot u_{ft-1}^{y} + (\sum_{w \in W} \sum_{y' \in Y^{w,t}} v_{ft}^{y',w,0})(P_{ft-1}^{y} - RD_{i}^{y}) \forall t, j, y \quad (11)
\]

Another way of modelling the ramping constraints is equations (10a) and (11a) and they describe the same start-up and shut-down ramp constraints. Apparently, (10) and (11) dominates (10a) and (11a). However, equations (10) and (11) incur a lot more non-zeros compared to (10a) and (11a). Therefore, equations (10) and (11) are tighter than equations (10a) and (11a) while equations (10a) and (11a) are more compact than equations (10) and (11). The computational study on MISO cases results in section V.B show that equations (10a) and (11a) work better.

III. PIECEWISE LINEAR INCREMENTAL ENERGY CURVE (PWL ENHANCEMENT)

The following SCUC model does not include reserves for simplification. The numerical results are based on full MISO formulation with energy and reserve co-optimization.
\[
\begin{align*}
    \text{Min } N^T x + \sum_{j \in J, t} \left[ C_{j,t}^p(p_{j,t}) \right] + \sum_{t \in T} \left[ O_{g,t}^l g_{l,t} \right] \\
    \text{s.t.:} \\
    \text{Resource level constraints:} \\
    \text{Limit constraints:} & \quad u_{j,t} \cdot p_{j,t} - p_{j,t}^l \leq u_{j,t} \cdot p_{j,t} \forall j \in J \quad (13) \\
    & \quad 0 \leq g_{k,t} \leq g_{k,t}^u \forall k \in K \quad (14) \\
    \text{Resource binary constraints} & \quad A x \leq U \quad (15) \\
    \text{Other Resource constraints} & \\
    \text{Transmission constraints:} & \quad F_{g,i}(p_{i,t}, g_{i,t}, p_{r,i}) \leq F_{g,i,t} \quad \forall i \in I \quad (16) \\
    & \quad F_{p}^p(p_{i,t}, g_{i,t}, p_{r,i}) = \sum_{j \in J} \left[ p_{j,t} \cdot B_{l,j,t} \right] + \sum_{k \in K} \left[ g_{k,t} \cdot B_{l,k,t} \right] + \sum_{n \in N} \left[ p_{n,t} \cdot B_{l,n,t} \right] \quad \forall i \in I \quad (17) \\
    \text{Power balance constraints:} & \\
    \text{System wide reserve constraints and zonal reserve constraints} & \\
\end{align*}
\]

The incremental energy function is modeled as monotonically non-decrease piece-wise linear function (i.e. convex PWL function). The modeling of the piecewise linear function can have significant impact on the performance. In [14], a special order type 2 set (SOS2) model was introduced to model piecewise linear approximation of nonlinear functions. For incremental energy cost function \( C_{j,t}^p(p_{j,t}) \) approximated between pre-determined fixed points \( 0 \leq p_{j,t} \leq \cdots \leq p_{j,t}^l \), define a set of nonnegative continuous variables \( \gamma_{j,0}, \gamma_{j,1}, \cdots, \gamma_{j,n} \) and the constraints:
\[
\gamma_{j,0} + \gamma_{j,1} + \cdots + \gamma_{j,n} = 1 \quad (18) \\
\gamma_{j,0} = p_{j,t} \leq p_{j,t} + \gamma_{j,1} \cdot p_{j,t} + \cdots + \gamma_{j,n} \cdot p_{j,t} \quad (19)
\]

The incremental energy cost function is represented by:
\[
C_{j,t}^p(p_{j,t}) = \gamma_{j,0} \cdot C_{j,t}^p(p_{j,t}) + \cdots + \gamma_{j,n} \cdot C_{j,t}^p(p_{j,t}) \quad (20)
\]

The incremental energy offer is convex and satisfies:
\[
0 = C_{j,t}^p(P_{j,t}) \leq C_{j,t}^p(p_{j,t}) \leq \cdots \leq C_{j,t}^p(P_{j,t}) \quad (21)
\]

In this model, at most two consecutive variables in (18) can be nonzero for convex cost functions. The piecewise linear incremental energy offer formulation in MISO day-ahead SCUC is formulated differently but mathematically
equivalent. By revising the energy offer cost functions to constraints (18)-(21), the computational performance of SCUC is very similar.

However, considering constraints (13) and (19), and also the cost curves always start at $C^P_t(P_{j,t}) = 0$ with $P_{j,t} = 0$, constraint (18)-(20) can be reformulated as:

$$\gamma_j + \cdots + \gamma_j = u_{j,t}$$  \hspace{1cm} (18a)

$$p_{j,t} = \gamma_j \cdot p_t + \cdots + \gamma_j \cdot p_t$$  \hspace{1cm} (19a)

$$C^P_t(P_{j,t}) = \gamma_j \cdot C^P_t(P_{j,t}) + \cdots + \gamma_j \cdot p_t$$  \hspace{1cm} (20a)

Note for some systems, $C^P_t(0)$ may be allowed to be non-zero. It can be easily adjusted by moving $C^P_t(0)$ to no-load cost.

Constraint (18a) links generation dispatch variables to commitment variables. The test results in Section V show significant improvement for some of the hard cases.

We later found out that similar formulations were discussed in [5] and [6]. It was proved that by introducing the binary variable to (18), the formulation $y_j + y_j + \cdots + y_j = u_{j,t}$ becomes locally ideal, i.e., the vertices of its corresponding LP relaxation satisfy all required integrality conditions.

Since $0 \leq y_j \leq 1$, with $P_{j,t} = 0$ and $C^P_t(P_{j,t}) = 0$, $y_j$ can be removed from the formulation. The formulation (18a)-(20a) is locally ideal and with less number of continuous variables. For MISO cases, it’s observed that root node relaxation time can be much faster (over 20% for some cases) by reducing $y_j$.

For SCUC, most resources have $P_{j,t} > 0$ and also have startup and no load cost, these make the PWL cost function non-convex. In [24], a convex primal formulation for convex hull pricing was introduced. The convex envelope of the quadratic cost function was explicitly proposed and proved. The author Bowen Hua of [24] pointed out to us that the locally ideal SOS2 PWL modeling in (18a)-(20a) is closely related to the convex envelope of the PWL cost function for each interval. We are able to prove that the least cost function from relaxed MIP under constraints (18a)-(20a) is the convex envelop of the original PWL cost function when minimizing the cost.

Define the set $X^w_{j,t} = \{ p_{j,t} \in R, u_{j,t} \in [0,1] \}$

$$u_{j,t} \cdot P_{j,t} \leq p_{j,t} \leq u_{j,t} \cdot P_{j,t}$$

$$p_{j,t} = \gamma_j \cdot P_{j,t} + \cdots + \gamma_j \cdot P_{j,t}$$

$$y_j = \gamma_j \cdot P_{j,t} + \cdots + \gamma_j \cdot P_{j,t}$$

$$y_j = \gamma_j \cdot P_{j,t} + \cdots + \gamma_j \cdot P_{j,t}$$

$$y_j = \gamma_j \cdot P_{j,t} + \cdots + \gamma_j \cdot P_{j,t}$$

The cost function with PWL energy offer $C^P_t(X^w_{j,t} \rightarrow R)$ is then defined as:

$$C^P_t(X^w_{j,t}) = \{ u_{j,t} \cdot C^P_t(p_{j,t}) + C^{NL}_t \cdot u_{j,t} \}$$  \hspace{1cm} (1)

$$u_{j,t} \geq 0$$

$$u_{j,t} = 0$$

**Proposition 1:** When minimizing cost, $u_{j,t} \cdot C^P_t(p_{j,t})$ defined on $X^w_{j,t}$ can be represented by no more than two consecutive non-zero $y_{j,k}$ for $k = 0, 1, \ldots, m$.

**Proof:** For $y_{j,k} = 1$ and $p_{j,k,t} \leq P_{j,t} \leq p_{j,k+1} \leq 1$, the piece-wise linear function $C^P_t(p_{j,t})$ can be represented as follows when minimizing cost:

$$C^P_t(p_{j,t}) = C^P_t(P_{j,t}) + C^P_t(p_{j,k+1} - p_{j,k}) \cdot (p_{j,t} - P_{j,k,t})$$

$$C^P_t(p_{j,t}') = C^P_t(P_{j,t}) \cdot \frac{P_{j,k+1} - P_{j,k}}{p_{j,k+1} - p_{j,k}} + C^P_t(P_{j,k+1}) \cdot \frac{p_{j,t} - P_{j,k,t}}{p_{j,k+1} - p_{j,k}}$$

Hence, it can be represented by:

$$y_{j,k+1} = \frac{[p_{j,t} - P_{j,k}]}{p_{j,k+1} - p_{j,k}}$$

$$y_{j,k+1} = \frac{p_{j,t} - P_{j,k}}{p_{j,k+1} - p_{j,k}}$$

Since $p_{j,t} \leq 0$ for $n \neq j$ and $n \neq j + 1$.

Hence, $y_{j,k} + y_{j,k+1} = 1$ and

$$P_{j,k,t} = y_{j,k} \cdot P_{j,k} + y_{j,k+1} \cdot P_{j,k+1}$$

Thus, for $u_{j,t} \in (0,1)$, similar SOS2 property can be prove for $u_{j,t} \cdot C^P_t(p_{j,t}/u_{j,t})$.

For $u_{j,t} \cdot P_{j,t} \leq p_{j,t} \leq u_{j,t} \cdot P_{j,t}$, when minimizing cost, we have:

$$C^P_t(p_{j,t}/u_{j,t}) = \alpha_{j,k} C^P_t(p_{j,t}) + \alpha_{j,k+1} C^P_t(p_{j,k+1})$$

$$p_{j,t} = \gamma_j \cdot P_{j,t} + \cdots + \gamma_j \cdot P_{j,t}$$

Assume $y_{j,k} = \alpha_{j,k} u_{j,t}$ and $y_{j,k+1} = \alpha_{j,k+1} u_{j,t}$.

We have:

$$x_{j,t} = \gamma_j \cdot P_{j,t} + \cdots + \gamma_j \cdot P_{j,t}$$

When solving relaxed MIP under the locally ideal SOS2 formulation with $u_{j,t} \in (0,1)$, the resulting cost $y_{j,k} C^P_t(p_{j,t}) + y_{j,k+1} C^P_t(p_{j,k+1})$ equals to $u_{j,t} \cdot C^P_t(p_{j,t})$. This can be illustrated by the example in the Appendix.

**Proposition 2:** $C^P_t(X^w_{j,t})$ defined above is the convex envelope of the single interval least cost function $C_t(p_{j,t}, u_{j,t}) = C^P_t(p_{j,t}) + C^{NL}_t \cdot u_{j,t}$ defined over $X_{j,t} = \{ p_{j,t} \in R, u_{j,t} \in [0,1] \}$ such that $C^P_t(p_{j,t}, u_{j,t})$ is the largest one.

If not, there exists a point $(p_{j,t}', u_{j,t}')$ with $u_{j,t}' \in (0,1)$ such that $C^P_t(p_{j,t}', u_{j,t}') > C^P_t(p_{j,t}', u_{j,t})$.

From Proposition 1, there exists $y_{j,k}$ and $y_{j,k+1}$ with $y_{j,k} + y_{j,k+1} = u_{j,t}'$, for $P_{j,k,t} \leq p_{j,t}' \leq P_{j,k+1}$ and $p_{j,k,t}' = y_{j,k} \cdot P_{j,k,t} + y_{j,k+1} \cdot P_{j,k+1}$.

Hence,

$$p_{j,t}' / u_{j,t}' = y_{j,k} \cdot P_{j,k,t} + y_{j,k+1} \cdot P_{j,k+1}$$

$$u_{j,t}' = 1$$

For convex PWL function,$$C^P_t(p_{j,t}' / u_{j,t}') \leq \gamma_j \cdot P_{j,t} + \gamma_{j+1} \cdot u_{j,t}' \cdot p_{j,k,t}$$

Hence,

$$C^P_t(P_{j,k,t} / u_{j,t}) + C^P_t(P_{j,k+1} / u_{j,t}' \cdot u_{j,t}' \cdot p_{j,k,t}$$

Hence,

$$C^P_t(p_{j,t} / u_{j,t}) \geq u_{j,t} \cdot C^P_t(p_{j,t} / u_{j,t}) + C^{NL}_t \cdot u_{j,t}$$

Therefore, $C^P_t(p_{j,t} / u_{j,t})$ and $C^{NL}_t \cdot (u_{j,t}')$.
\[ \Gamma_{i,t}^{b_0} \left( p_{i,t}^{b_0}, u_{i,t}^{b_0} \right) > 0 + u_{i,t}^{b_0} \cdot \left( C_{i,t}^{P}(p_{i,t}^{b_0} / u_{i,t}^{b_0}) + C_{i,t}^{NL} \cdot u_{i,t}^{b_0} \right) \]

\[ C_{i,t}^{NL} \] is not convex over the line interval \((0,0)\) and \((p_{i,t}^{b_0} \cdot u_{i,t}^{b_0})\) and hence contradiction.

With this approach, the Extended LMP implemented by MISO can also be improved as shown in the appendix.

IV. REDUCE NON-ZERO AND SYMMETRY IMPACT ON TRANSMISSION CONSTRAINTS (AGG ENHANCEMENT)

Transmission constraints significantly increase computational complexity of SCUC problem and a large number of virtuals further complicates the transmission constraints by adding a large number of non-zeros in constraint (17). Inspired by discussions following the presentation at the FERC technical conference [15], a reformulation of transmission constraint was developed by introducing a variable \(y_{n,t}\) for each node with multiple generators and/or virtuals connected. Constraint (6) can be reformulated by constraint (22a) and (17a) to aggregate the variables on the same nodes:

\[ y_{n,t} = \sum_{j \in J} \text{and } n_j = n \left[ p_{j,t} + \sum_{k \in K} \sum_{n_k = n} \left[ g_{k,t} \right] \right] \]

\[ \forall n \in N, t \in T \] (22a)

\[ F_{i}^{P} \left( y_{t}, P_{t} \right) = \sum_{n \in N} \left( y_{n,t} \cdot B_{i,n,t} \right) + \sum_{n \in N} \left( p_{n,t} \cdot B_{i,n,t} \right) \]

\[ \forall \ i \in I, t \in T \] (17a)

For cases with significant amount of virtuals offered at the same node, aggregating virtuals offers on the same node can significantly reduce the number of non-zeros (e.g., from over 10 million to about 3 million for some of the hard cases). Assume the number of transmission constraints is \(C_{i}\) and the number of continuous variables on one node \(n\) is \(C_{n}\).

- With constraint (17), the number of non-zeros introduced by those variables on all the transmission constraints is \(C_{i} \cdot C_{n}\).
- With revised constraint (22a) and (17a), the number of non-zeros introduced by those variables on all the transmission constraints becomes: \(C_{n} + C_{i}\). However, it requires adding continuous variables \(y_{n,t}\).

However, aggregation of virtuals offers may not always guarantee significant improvement as SCUC problem is NP-hard. For most difficult cases with a large number of virtuals and transmission constraints, even if the non-zeros can be reduced by over 70% with the aggregation under (22a) and (17a), the performance improvement is not significant.

After further investigation on case H-3 in Table I, it was identified that the 10.81% MIP gap under PWL Enhancement was caused by a transmission constraint violation. Three generators at one station have over -60% impacts on the constraint (Fig. 2). All three of them are committed under the 1.36% solution. Apparently they should all be committed. However, it took very long time for CPLEX to search around 10.81% gap.

The issue can partly attribute to symmetry described in [16]. However, in real world problems, the generators may not be perfectly symmetric and can have slight differences, e.g., different initial on time, slightly different offers, etc. The approaches proposed to break symmetry are general applicable only to generators that are perfectly symmetric.

In MISO commercial model, one commercial pricing node (Cnode) is created for each of the generator. There are several virtual bids and offers on each of the three Cnodes in Fig. 2. The aggregation above can merge the virtual clearing and generator dispatch variables at the same Cnode. MISO usually doesn’t monitor step up transformers. Hence, for all transmission constraints in (16) and (17), the three Cnodes in Fig. 2 should have the same impact and can be further combined on the LHS of transmission constraints. However, the three generators are on different nodes. Under certain breaker contingency, the high KV bus may split and cause the Cnodes to have different impact on transmission constraints.

To cover this scenario, a sensitivity based approach is developed to identify Cnodes that can be combined.

Fig. 2 Generators at the same plant

In this approach, the set of nodes \(N\) is partitioned into subsets \(N_{1}^{t}, N_{2}^{t}, \ldots, N_{S_{t}}^{t}\) for each \(t \in T\). Each set can include one or more nodes. For any two nodes \(n_{a}\) and \(n_{b}\) within one subset \(N_{l}^{t}\), \(l \in \{1,2,\ldots, S_{t}\}\), we have: \(B_{i,n_{a}t} = B_{i,n_{b}t} \) \(\forall i \in I\). Specifically, all nodes within the same set have the same impact on transmission lines. We use \(B_{i,t}^{l}\) to represent all these equal sensitivities within the subset. Then further modify (22a) and (17a) to (22b) and (17b):

\[ z_{l,t} = \sum_{j \in J} \text{and } n_j = n \left[ p_{j,t} + \sum_{k \in K} \sum_{n_k = n} \left[ g_{k,t} \right] \right] \]

\[ \forall l \in \{1,2,\ldots, S_{t}\}, t \in T \] (22b)

\[ F_{i}^{P} \left( y_{t}, P_{t} \right) = \sum_{l \in \{1,2,\ldots, S_{t}\}} \left[ z_{l,t} \cdot B_{i}^{l} \right] + \sum_{n \in N} \left( p_{n,t} \cdot B_{i,n,t} \right) \]

\[ \forall \ i \in I, t \in T \] (17b)

In (22b), all continuous dispatch variables with the same impact on transmission are grouped together. With these revised constraints, the three Cnodes in Fig. 2 are grouped into one subset since they have the same impact on all monitored transmission constraints. The number of non-zeros can be further reduced. It is worth noting that only one variable is needed to represent the subset dispatch on the LHS of transmission constraints. Even if there is symmetry issue with those generators, the section of the matrix with symmetry can be significantly reduced since the generator dispatch variables are not directly shown in the transmission constraints. The SCUC performance can be improved significantly for some cases as shown in Section V.A. The proposed approach is equivalent to adding many generators at the same location. Most of them have the same impact on transmission constraints. The AGG Enhancement can significantly reduce the non-zeros introduced by those configurations.

V. CASE STUDY

All results in this section are performed on Intel® Xeon® CPU E5-2690 0 @ 2.90 GHz RAM 64 GB with AIMMS4.2 and CPLEX12.6.
A. PWL and AGG Enhancement on cases without configuration based combined cycle model

We show the comparison of the original cases with the PWL and AGG improvements in Table I and Table II. The column “Original” corresponds to the original production SCUC formulation. The column “With only PWL” shows the results with the implementation of PWL enhancement. The column “With both PWL and AGG” shows the results with the implementation of both PWL and AGG enhancement.

Table I shows the results of the hardest cases with a large number of transmission constraints and virtual bids. With 1200s time limit, many of the cases are solved with large MIP gap under the original formulation. With the locally ideal SOS2 PWL improvement, most of the cases can reach much better MIP gaps. With both the PWL and AGG enhancements, the improvement is more robust and dramatic.

Table II demonstrates the results with samples of normal cases. In the day-ahead market clearing process, there is certain percentage of cases solved at 1200s time limit. Those cases cannot reach 0.1% MIP relative gap tolerance or $2400 MIP absolute gap tolerance. However, MIP relative gap below 3% is usually considered as acceptable. The incremental polishing method described in [1] can improve the local optimality, i.e., optimizing the commitment of out-of-money resources while fixing the commitment decisions of all other resources. In Table II, most of the cases solved around 1200s time in the original scenario also need similar amount of time under the new formulations. The MIP gaps are similar. It indicates that the new PWL and AGG formulation can meet the requirement for this set of cases.

The rest of the cases in Table II can reach the MIP relative gap tolerance before 1200s. For these cases, the new PWL and AGG formulations can reduce the solving time significantly. In Fig. 3, we further studied additional 23 production day-ahead SCUC cases. All of the cases can be solved within 900s CPU time in the original scenario. With the new PWL and AGG formulation, the solving time has been reduced by over 30% with similar MIP gap results.

B. Tighten configuration based combined cycle modeling and PWL and AGG Enhancement

MISO currently has about 40–50 combined cycle groups. We collected about 20 configuration-based combined cycle modeling data from participants to replace the aggregated combined cycle models (i.e. 20-CC case). To compare the resulting impact from different formulations, we list the following results in Table III:

C-1: Initial formulation from [11] without PWL and AGG with MIP Start from previous day solution. Note previous day solution from existing production case doesn’t have the configuration based combined cycle model. Hence, the initial commitments for CC configurations are all off.

C-2: Results from the formulation in Section II with ramp constraints (10) and (11), without PWL and without AGG, and with MIP start from previous day solution. These are similar formulation from reference [12] except that single transition type is replaced by three transition types in this paper.

C-3: Results from the formulation in Section II with ramp constraints (10a) and (11a), with PWL and AGG, and with MIP start from previous day solution. This incorporates all the improvement suggested in this paper and results in the best performance.

C-4: From C-3, replace ramp constraints by (10) and (11).

C-5: Without unique transition constraint (2) in C-3.

C-6: Without PWL in C-3.

C-7: Without AGG in C-3.

C-8: Without using MIP start in C-3.

It can be observed by comparing C-7 and C-3 that AGG significantly improves the MIP gap at 1200s for 20 CCs and 40 CCs and reduce the time to reach below 3% optimality gap for 80 CCs and 120 CCs. This is because AGG helps to reduce the number of non-zeros. Improved compactness can reduce the solution time for each node.

The difference between C-6 and C-3 shows that PWL significantly reduces the MIP gap at 1200s for 20CCs and it offers great help in reducing the time to reach below 3% optimality gap. The proposed PWL is a tighter representation of generators cost functions and improves the quality of root node relaxation solution. Better root node relaxation solution benefits the lower bound, rounding heuristics, and branch and bound efficiency.

From C-5 and C-3, unique transition constraint (2) is extremely helpful for case 20CC and 40CC cases. It’s not very helpful for 80CC and 120CC cases.

The difference between C-8 and C3 shows that MIP-Start can play a very important role in computational performance, especially for difficult cases. From Case 8, without MIP start, the MIP gap stays at 95% at 1200s for both 80 CCs and 120 CCs. Even though the initial solution used in this study is not very good with missing CC configuration commitment, MIP-start from previous day commitment for other resources can still accelerate MIP solver to obtain a better solution.

Ramp rate constraints also have significant impacts on the computational performance. C-4 is used to represent the ramp formulation in [12], i.e., constraints (10) and (11). Ramp rate constraints in [12] can only reach 35.02% and 41.83% optimality gap at 1200s for 20 CCs and 40 CCs respectively while the proposed ramp rate constraints (10a) and (11a) can reach 0.89% and 0.60% optimality gap at 1200s.

C-2 represents the CC formulation proposed in this paper without AGG and PWL improvement and with ramp constraints [10] and [11]. It’s similar to the formulation proposed in [12] except for three transition types instead of one transition type.

C-1 represents the formulation proposed in [11] without AGG and PWL improvement. It can be observed that the proposed tightened CC formulation can significantly improve the computational performance in both the optimality gap at 1200s and time to reach below 3% optimality gap for different number of CCs.

From Table III, C-3 has the best computational performance overall and it integrates both the PWL and AGG improvement and tightened CC formulation.
Fig. 3: Results on normal cases (with and w/o PWL and AGG, w/o configuration based combined cycle model).

### Table I: Performance Comparison - Hard Cases

<table>
<thead>
<tr>
<th>Hard cases (1200s time limit)</th>
<th>MIP Optimality Gap</th>
<th>With only PWL</th>
<th>With Both PWL and AGG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H-1</td>
<td>1.63%</td>
<td>0.47%</td>
<td>0.25%</td>
</tr>
<tr>
<td>H-2</td>
<td>0.97%</td>
<td>0.83%</td>
<td>0.49%</td>
</tr>
<tr>
<td>H-3</td>
<td>99.99%</td>
<td>10.81%</td>
<td>1.36%</td>
</tr>
<tr>
<td>H-4</td>
<td>54.70%</td>
<td>12.45%</td>
<td>2.56%</td>
</tr>
<tr>
<td>H-5</td>
<td>54.76%</td>
<td>4.21%</td>
<td>3.20%</td>
</tr>
<tr>
<td>H-6</td>
<td>110.72%</td>
<td>14.46%</td>
<td>3.32%</td>
</tr>
<tr>
<td>H-7</td>
<td>27.58%</td>
<td>8.41%</td>
<td>8.62%</td>
</tr>
<tr>
<td>H-8</td>
<td>0.60%</td>
<td>0.60%</td>
<td>0.74%</td>
</tr>
<tr>
<td>H-9</td>
<td>10.07%</td>
<td>0.25%</td>
<td>0.70%</td>
</tr>
<tr>
<td>Average</td>
<td>40.11%</td>
<td>5.83%</td>
<td>2.36%</td>
</tr>
</tbody>
</table>

### Table II: Performance Comparison - Normal Cases

<table>
<thead>
<tr>
<th>Normal Cases</th>
<th>Original</th>
<th>With both PWL and AGG</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CPU (s)</td>
<td>Gap</td>
</tr>
<tr>
<td>N-1</td>
<td>863</td>
<td>0.06%</td>
</tr>
<tr>
<td>N-2</td>
<td>1137</td>
<td>0.08%</td>
</tr>
<tr>
<td>N-3</td>
<td>949</td>
<td>0.03%</td>
</tr>
<tr>
<td>N-4</td>
<td>726</td>
<td>0.03%</td>
</tr>
<tr>
<td>N-5</td>
<td>1231</td>
<td>0.26%</td>
</tr>
<tr>
<td>N-6</td>
<td>1232</td>
<td>1.06%</td>
</tr>
<tr>
<td>N-7</td>
<td>969</td>
<td>0.06%</td>
</tr>
<tr>
<td>N-8</td>
<td>640</td>
<td>0.01%</td>
</tr>
<tr>
<td>Average</td>
<td>968</td>
<td>0.20%</td>
</tr>
</tbody>
</table>

### Tables III: Performance Comparison - Combined Cycle Cases

<table>
<thead>
<tr>
<th># of CC</th>
<th>20</th>
<th>40</th>
<th>80</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MIP gap at 1200s</td>
<td>Time to &lt;3% gap (s)</td>
<td>MIP gap at 1200s</td>
<td>Time to &lt;3% gap (s)</td>
</tr>
<tr>
<td>C-1</td>
<td>64%</td>
<td>3561</td>
<td>65%</td>
<td>2458</td>
</tr>
<tr>
<td>C-2</td>
<td>64.76%</td>
<td>2387</td>
<td>48.65%</td>
<td>2026</td>
</tr>
<tr>
<td>C-3</td>
<td><strong>0.89%</strong></td>
<td><strong>738</strong></td>
<td><strong>0.60%</strong></td>
<td><strong>835</strong></td>
</tr>
<tr>
<td>C-4</td>
<td>35.03%</td>
<td>4212</td>
<td>41.83%</td>
<td>2127</td>
</tr>
<tr>
<td>C-5</td>
<td>41%</td>
<td>2277</td>
<td>0.46%</td>
<td>1056</td>
</tr>
<tr>
<td>C-6</td>
<td>30%</td>
<td>1439</td>
<td>1.02%</td>
<td>847</td>
</tr>
<tr>
<td>C-7</td>
<td>41%</td>
<td>2882</td>
<td>48%</td>
<td>1990</td>
</tr>
<tr>
<td>C-8</td>
<td>12%</td>
<td>1908</td>
<td>0.69%</td>
<td>944</td>
</tr>
</tbody>
</table>
VI. CONCLUSIONS

This paper proposes two modifications on MIP formulation that lead to significant improvement on SCUC performance. The improvements are also extended to configuration based combined cycle cases and lead to significant improvement in SCUC solution time. In addition, further improvement is made to the configuration based combined cycle formulation. The proposed CC model shows significant improvement on cases with large number of configuration based combined cycles.

VII. APPENDIX

In this section, we use a simple example to illustrate the difference between the locally ideal and non-locally ideal PWL formulation under relaxed MIP. We also show the relationship of the resulting prices to extended LMP [25].

For a unit G1 with three segments on its incremental energy offer:
- $1/MWh between [0, 30MW],
- $5/MWh between (30MW, 50MW],
- $9/MWh between (50MW, 65MW).

Assume the no load cost is $100/h, the minimum limit is 35MW and maximum limit is 65MW.

Assume another unit G2 has zero offers with dispatch range between 0MW and 60MW.

Table IV shows the relaxed MIP solution under locally ideal SOS2 PWL formulation (18a)-(20a). The price below minimum limit 35MW is the cost $155 at 35MW average over the range below 35MW. It’s the red line in Fig.4. Above 35MW, it’s overlap with the original cost curve. Together it forms the convex envelope of the original cost function between 35MW and 65MW.

Table V shows the relaxed MIP solution under non-locally ideal SOS2 PWL formulation (18)-(20). It’s the green line in Fig. 4. Apparently it’s not as tight as the locally ideal formulation. When considering startup cost, this is equivalent to the Extended LMP currently implemented at MISO for quick start resources [25].

When load is between 60MW and 90MW, the ELMP price from relaxed MIP is $1/MWh under current MISO ELMP. It can’t cover the cost for G1 at minimum limit 35MW. Hence it still requires make whole payment. The price from the relaxed MIP under the locally ideal formulation is $4.43 and it can cover the cost at 35MW for G1.

When load is above 95MW, u1 is solved below 1 under current ELMP. It’s equivalent to average no load cost over maximum limit of G1. ELMP price is higher than the LMP from true marginal cost of G1 even though LMP can cover the total cost. The price can incentivize G1 to deviate and move up from the dispatch target. With the locally ideal formulation, u1 is solved at 1 when load is above 95MW. ELMP equals to LMP. There is no incentive for G1 to deviate from its dispatch target.

MISO is considering changing the PWL formulation in the pricing engine to be the locally ideal formulation (i.e., the same as (18a)-(20a) implemented in DA-SCUC).

VIII. ACKNOWLEDGMENT

The authors thank for the valuable discussions with Bowen Hua, Dr. Paul Gribik and Dr. Ross Baldick.

IX. REFERENCES


Schedule 29A, MISO tariff, online available: https://www.misoenergy.org/Library/Tariff/Pages/Tariff.aspx

X. BIOGRAPHIES

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