Risk management for forestry planning under uncertainty in demand and prices

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Abstract

The forest-harvesting and road-construction planning problem basically consists of managing land designated for timber production and divided into harvest cells. For each time period in the given time horizon one must decide which cells to cut and what access roads to build in order to maximize expected net profit under a risk manageable scheme to control the negative impact of the solutions in the objective function value of the unwanted scenarios (i.e., the so-called black swans) on the objective function value. We have previously developed deterministic and risk neutral stochastic mixed 0-1 linear optimization models for similar problems [2]. The stochastic version of the problem that we presented there enables the planner to make more robust decisions based on a range of timber price scenarios over time, maximizing the expected value instead of merely analyzing a single (e.g., average) scenario as performed in the deterministic version of the problem. The main contribution of the current work consists of introducing the so-called time consistent and time inconsistent Conditional Value-at-Risk (CVaR) risk-averse measures in forestry planning. They avoid the risk neutral optimal (expected profit) solutions with low probability high variability in the profit scenario. In particular, those risk-averse measures are compared computationally with the risk neutral one under different price, demand and probability of the scenarios under consideration, as well as with a risk-averse measure that is a mixture of both.

Keywords: OR in natural resources, forestry planning, optimization under uncertainty, risk management, consistency in risk-averse measures.

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1. **Introduction**

Forest companies must plan over a given time horizon the sustainable harvest of their resources, which are then sold in specific local and international markets. They have to meet demand, primarily from pulp plants and sawmills. The main aim of the companies is to maximize profits while complying with environmental regulations. In previous studies we formulated and solved a specific problem addressing various issues that arise in forestry planning, namely, planning the harvest of forest land designated for timber production and the construction of access roads needed to transport the timber. Good surveys of forest-based supply chain planning cover such aspects as planting, cutting, construction of access roads for transportation, etc. See for instance [35, 37]. See also [9, 12, 27], among others. Starting in the 70s, environmental and wildlife issues were increasingly considered in forest management models at different planning levels; see [10], among others.

In the last 30 years the twin problems of planning harvesting and access road construction have been addressed jointly using mathematical optimization models and computational tools. The advantage of integrating the two processes in a single mixed 0-1 model was demonstrated in [44], whose solutions range from 15% to 45% better than models that optimized the processes separately, as in [24]. The logistics needed for developing efficient forestry planning are highly complex and must be firmly based on efficient mathematical models that can support decision-making, see [13], among others.

There exist relevant studies on the different phases of forestry planning, especially regarding access road construction and harvesting. In essence, the problem may be formulated in terms of a partition of the forest into harvesting units so-called cells. For a chosen time horizon one must determine which cells will be cut in each period, which roads need to be constructed to access those cells and when, and what quantity of wood will be transported from one point to another. These decisions are made in relation to an optimization criterion, typically profit maximization.

Selling prices of forest products are a key element in forestry planning. Price fluctuations have a direct impact on profits from sales and figure prominently in the planners’ decision-making. The role played by randomness in forestry planning is closely related to the length of the chosen time horizon. Planners who must make tactical decisions are therefore concerned about price variations during a time horizon of two to five years. (For operational planners, whose planning horizons are measured in days or weeks, uncertainty is not a key issue). Although future wood prices are the most relevant source of uncertainty, uncertainty in tree growth or in timber losses due to fires is also significant. The approach developed in this paper analyzes decision-making under uncertainty in wood selling prices. We assume that they can be modeled over time by means of a set of scenarios with different associated probabilities.

In mathematical terms, the deterministic version of the problem, which assumes that all parameters are known, may be formulated as a mixed 0-1 linear optimization model. Even that (not very realistic) case is difficult to solve, due to its size and the presence of thousands of binary variables. Approaches for solving this problem have been described in [4, 16, 20, 44] and references therein. Our approach will benefit from these earlier reports. Some of them use robustness and decomposition techniques such as Lagrangean...
relaxation to obtain very good solutions in reasonable computation times with low residual gaps.

We should point out that the forest planning problems studied in the above cited papers consider either the expected scenario or a single scenario. There are very few papers on forestry planning where the information about the uncertainty (mainly on wood prices given their high volatility) is specifically included in the model. We did present elsewhere [2] a risk neutral multiperiod stochastic mixed 0-1 model based on a finite set of representative scenarios for the price uncertainty. The computational results outperformed those obtained by considering the expected scenario approach. That model type enables the planner to make more robust decisions (by taking into account the stochastic behavior of the selling price of timber), by considering a representative range of timber price scenarios over time, maximizing the expected value instead of merely analyzing a single (e.g., average) scenario as performed in the deterministic version of the problem.

The value added by this work over what was previously published is mainly the consideration of different time-consistent variations of the popular inconsistent risk-averse measure so-called Conditional Value-at-Risk (CVaR), see [31, 38]. There the risk reduction is performed in the last period of the time horizon for preventing risk-neutral optimal (expected profit) solutions with high variability in per-scenario profit of the forestry planning problem.

To our knowledge, [34] is the first study where forest plantation planning with uncertain timber price is addressed by considering risk management, as opposed to a risk-neutral approach. A stochastic dynamic programming approach is used and CVaR is considered as the risk measure for determining the best harvesting policy, but without considering the logistic aspect of the problem (road construction, transportation, etc.). In any case, the extension to time-inconsistent CVaR for risk reduction at intermediate time periods (the so-called TCVaR) could be very interesting, particularly for long time horizons. The consistent expected CVaR (so-called ECVaR) performs risk reduction for groups of scenarios (instead of considering the whole set of scenarios); it is obviously very beneficial. See [23, 31, 42], among others.

In this work we do consider mixing both types of CVaR measures: so-called MCVaR (in particular, the inconsistent version for intermediate periods and the last one in the time horizon, and the consistent version for intermediate periods). Our pilot study is performed on a real-life problem of timber harvesting and road building under uncertainty in Chile. The forest industry is Chile’s second largest source of exports, surpassed only by copper mining. According to data from INFOR (Instituto de Investigación Forestal de Chile - the Chilean Institute of Forest Research), the forestry exports in 2014 exceeded for the first time the barrier of US$6 billions, registering a sum of US$ 6,094.3 millions, which represents an increase of 6.7% over 2013. Such a figure confirms the magnitude of the industry and underlines the importance of providing its planners with efficient decision-making tools. Forest companies must plan the sustainable harvest of their resources over a given time horizon. Cut timber is then sold in a specific market to meet demand. The main objective of the companies is to maximize profit while complying with environmental regulations. One of the main difficulties encountered in planning harvesting operations is the stochasticity of future timber demand and sale prices.

The objective of the tactical forestry planning addressed in our study is to determine a policy for forest
harvesting and access road construction that will maximize the expected profit over the scenarios. Given the uncertainty in wood demand and price along the time horizon, the constraints should be satisfied at each node of the scenario tree and one will consider the time-consistent and -inconsistent versions of the risk-averse CVaR measure. The company is assumed to own its own timber land, which is subdivided using geographic information systems into units or cells for harvesting purposes.

The remainder of the paper is organized as follows. Section 2 describes the forest harvest planning problem as a pilot case to test the behaviour of the risk-averse measure examined in this work. In Section 3 the multiperiod mixed 0-1 optimization problem and its risk neutral model considering uncertainty is presented for the detailed description of the deterministic case. Section 4 studies different variants of the risk-averse CVaR-based measures. Section 5 reports the main computational results comparing the different approaches including risk aversion in the model and its impact in the final solution. Finally, we draw some conclusions in Section 6. Appendix A shows the mathematical formulation for the deterministic version of the problem we are dealing with in this work.

2. The forestry problem

Consider the following management planning problem in the timber industry with a time horizon of two to five years. The firm under consideration owns plantation lands that are divided into areas. Within each area there are different stands, considered homogeneous as defined by age of tees, soil quality (site index), and volume available per hectare (see Fig. 1). All areas are planted with pine trees, which mature at age 22 to 28. The stands that can be harvested during the time horizon are therefore known. Growth-simulator models developed by the forest firms are used to estimate timber yields in future periods. This is for the supply side.

On the demand side, timber production goes to export, to sawmills, and to pulp plants, as logs. While in reality there are many different products, defined mainly by log length and diameter, at this level of planning we define only a few basic aggregate products, referred to as export, sawmill, and pulp. Usually a higher-level quality can be used for lower-level purposes, at a loss in sale price. For example, the pulp mill
takes any type of timber, while only export quality can be exported. The main goal of the planning process is to match the supply of standing timber with demand for timber product of specific grades, lengths, and diameters, and, thus, reducing losses in revenues due to down-grading and nonprofitable additional cutting.

The problem also considers the logistics of producing and delivering those timber products. Most timber areas are near paved public roads, but in order to get access to the different stands in each area, inside the areas private roads are needed. At the beginning of the time horizon there are potential roads, i.e., roads that can be built, as well as existing roads. In any later period there are roads already built and projected ones. In addition to taking into account the existence or nonexistence of roads, one also has to consider their surface quality. First, private roads can be built of either dirt or gravel, and this has an impact on operations. Gravel roads are more expensive to build, but lead to lower transportation costs and can be used year-round, while dirt roads are only useful in summer. Next, road building and upgrading should be carried out in proper sequence so as to be consistent, timed with stand harvesting, as well as to avoid excessive road building. In addition, road building can only be carried out in summer.

Harvested timber can be stocked from summer to winter in stocking yards; this allows timber harvested in summer to be hauled, still during the summer, to stocking yards near final destinations, and then sent to final destinations in winter using gravel roads. Due to deterioration of timber and cost of maintenance, no stocking from one summer to the next is considered.

Finally, consider the production and delivery of timber demand. Aggregate demand are projected to future periods, often as lower and upper bounds and so are the expected prices. Cable logging (or towers) carry out harvesting for steep areas, while skidders harvest flat terrain. Timber hauling is carried out by truck to such destinations as ports, pulp plants, sawmills, or stocking yards. Harvesting machinery and crews are usually subcontracted with yearly contracts. There is a fixed cost associated with installing a harvesting operation, but this cost is very difficult to evaluate a priori. In order to avoid paying for it, one tends not to harvest areas that are too small. A typical policy is, in the case of larger stands, to harvest 10 or 15 hectares at least, and for smaller stands, to harvest the whole stand, if at all.

To summarize, the basic decisions to be considered in each period are as follows:

- stands to be harvested;
- amount of timber production, by aggregate product for harvesting to satisfy demand;
- roads to be built, in gravel or dirt;
- roads to be upgraded, from dirt to gravel;
- amount of timber transported to destinations;
- amount of timber stocked from summer to winter;
- choice of harvesting machinery and trucks.
Fig. 1 represents a possible logistic infrastructure for the problem presented above. In this example, there are public roads for transporting the material from the two harvesting areas and two stocking yards. Each harvesting area is accessible through an existing gravel road, but there are other possible access roads. Not all stands are accessible through the existing roads and additional roads are needed to be able to access them.

The logistic structure can be modeled as a network $G = (I, L)$, where $I$ is the set of nodes and $L$ is the set of links. The set $I$ can be defined as follows:

- Stands: each stand can be associated to a node in the network that represents the access point to the stand.
- Origins: each access point to a stand is linked to a origin node, such that, from each origin point one or more stands are accessible but each stand is accessible from only one origin.
- Stocking yards.
- Final destinations.
- Intermediate points: road junctions (linking different pieces of roads, public or private).

Notice that products are sent to the markets from the final destination nodes or directly from the stocking yards.

The set of links $L$ includes all roads in the model (public and private, the latter, existing or potential) and the links between origins and stands. Fig. 2 shows the network associated with the logistic structure in Fig. 1.

See Appendix A for the detailed mathematical formulation of the deterministic version of the problem, in which all parameters are considered known at the beginning of the time horizon. This formulation is an improved version of the model in [4].
### Year Exports Prices

<table>
<thead>
<tr>
<th>Year</th>
<th>Exports</th>
<th>Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>2001</td>
<td>112.4</td>
<td>70.9</td>
</tr>
<tr>
<td>2002</td>
<td>122.6</td>
<td>67.3</td>
</tr>
<tr>
<td>2003</td>
<td>155.4</td>
<td>82.2</td>
</tr>
<tr>
<td>2004</td>
<td>157.2</td>
<td>80.2</td>
</tr>
<tr>
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<td>161.3</td>
<td>89.5</td>
</tr>
<tr>
<td>2006</td>
<td>184.4</td>
<td>102.4</td>
</tr>
<tr>
<td>2007</td>
<td>188.0</td>
<td>108.9</td>
</tr>
<tr>
<td>2008</td>
<td>176.3</td>
<td>81.2</td>
</tr>
<tr>
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<td>2010</td>
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<tr>
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<tr>
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<td>185.2</td>
<td>107.6</td>
</tr>
<tr>
<td>2013</td>
<td>194.6</td>
<td>109.4</td>
</tr>
</tbody>
</table>

![Figure 3: Chilean forest exports index of wood quantity and prices. (base: Avr. year 2000=100)](image)

### 3. Uncertainty in wood prices and demand

The deterministic model assumes that prices and demand are known in advance of the planning decision. However, as can be seen in Fig. 3, wood demand and prices can vary along the planning horizon. Notice the volatility of the uncertain parameters which are therefore very difficult to predict.

To represent the uncertainty in wood demand and prices, we will use a scenario analysis approach, where the scenario set can be visualized as a tree. Let $T$ denote the set of time periods in the planning horizon, where $T = |T|$ is the number of periods and $\Omega$ be the finite set of representative scenarios. A scenario $\omega \in \Omega$ is a particular realization of the uncertain parameters during the time horizon represented in the tree as a root-to-leaf path. A node of the tree represents the event where the realization of uncertain parameters and decision variables for a given period takes place. Notice that the group of scenarios that have the same realization of the uncertain parameters up to any given period have the same value for the decision variables up to the period and thus satisfy the well-known nonanticipativity principle. Notice that there is a 1-to-1 correspondence between nodes and scenario groups for the same period. Let $n$ and $\mathcal{N}$ denote a node and the lexicographically numbered set of nodes $\{1, \ldots, |\mathcal{N}|\}$ in the tree, res., and let $\mathcal{N}^t$ denote the subset of nodes that belong to period $t$, such that $\mathcal{N} = \cup_{t \in T} \mathcal{N}^t$. To facilitate the presentation of our scheme for the scenario tree generation given in section 3.1, let $\mathcal{N}^0 = \{0\}, \mathcal{N}^1 = \{1\}, \ldots \mathcal{N}^{T+1} = \{|\mathcal{N}^T| + 1\}$. Let also $\Omega_n \subseteq \Omega$ denote the subset of scenarios with a 1-1 correspondence with node $n$ in the tree. For scenario $\omega \in \Omega$ the weight $w^n$ represents the probability of its occurrence. Let $\hat{A}^n$ and $\hat{S}^n$ denote the sets of ancestor and successor nodes to node $n$ (including itself in both of them), res., for $n \in \mathcal{N}$. Let $\mathcal{S}^n \subseteq \hat{S}^n$ denote the set of immediate successors of node $n \in \mathcal{N}$. Note: $\mathcal{N}^1$ is singleton, $\hat{A}^1 = \{1\}$ and $\hat{S}^n = \emptyset$ for $n \in \mathcal{N}^T$. Finally, let $\sigma(n)$ denote the immediate ancestor node of node $n$, for $n \in \mathcal{N} \setminus \{1\}$, where $n = 1$ is the root node of the scenario tree.
As an example, let us consider the decision tree in Fig. 4. Each node, say $n$, represents a point in time where a decision can be made. Once a decision is made, some contingencies may occur (in this example the number of contingencies varies from two to three for periods 1 to 3), and information related to those contingencies is available at the beginning of the next period.

Without loss of generality, let us consider the following mixed 0-1 formulation that gives a compact view of the deterministic multiperiod mixed 0-1 model (6)–(13) shown in Appendix A for this forestry planning problem,

$$
\begin{align*}
\mathbf{z}_{EV} &= \max \sum_{t \in T} a_t^1 x_t + b_t^1 y_t \\
\text{s.t.} \quad & \sum_{t' \in T, t' \leq t} A_{t'}^t x_{t'} + B_{t'}^t y_{t'} = h_t \quad \forall t \in T \\
x_t^1 & \in \{0, 1\}^{n_x(t)}, y_t \in \mathbb{R}^{n_y(t)} \quad \forall t \in T,
\end{align*}
$$

(1)

where $x^t$ and $y^t$ are the $n_x(t)$ and $n_y(t)$ dimensional vectors of the 0-1 and continuous variables, res., $a^t_1$ and $b^t_1$ are the vectors of the coefficients of the objective function, $A^t_{t'}$ and $B^t_{t'}$ are the constraint matrices of period $t$ for the 0-1 and continuous variables in its ancestor period $t'$ (including itself), res., and $h^t$ is the right-hand-side vector (rhs) for period $t$. Note: $a^t_1$ and $b^t_1$ in (1) can be considered as the unit profit related to the variables in vectors $x^t$ and $y^t$, res.

Without loss of generality as well, let us consider the compact representation of the multiperiod mixed 0-1 model for maximizing the expected value of the objective function over the set of scenarios $\Omega$ in the scenario tree such that the so-called Risk Neutral (RN) measure can be expressed

$$
\begin{align*}
\mathbf{z}_{RN} &= \max \sum_{n \in \mathcal{N}} w^n (a^n_1 x^n + b^n_1 y^n) \\
\text{s.t.} \quad & \sum_{q \in \mathcal{A}^n} (A^n_q x^q + B^n_q y^q) = h^n \quad \forall n \in \mathcal{N} \\
x^n & \in \{0, 1\}^{n_x(n)}, y^n \in \mathbb{R}^{n_y(n)} \quad \forall n \in \mathcal{N},
\end{align*}
$$

(2)

where $w^n$ is the weight or probability of node $n$ in the scenario tree and is computed as $\sum_{\omega \in \Omega^n} w^n$; $\mathcal{A}^n \subseteq
\( \tilde{A}^n \) is the set of ancestor nodes to node \( n \) with nonzero elements in the constraints matrices of node \( n \); \( x^n \) and \( y^n \) are the vectors of the 0-1 and continuous variables for node \( n \), res.; \( a^n \) and \( b^n \) are the vectors of the objective function coefficients for the 0-1 and continuous variables, res.; \( A^n_q \) and \( B^n_q \) are the constraint matrices of node \( n \) for the variables in \( x^q \) and \( y^q \) in ancestor node \( q \) of node \( n \), res.; \( h^n \) is the rhs for node \( n \); and \( n_x(n) \) and \( n_y(n) \) are the numbers of 0-1 and continuous variables, for \( n \in \mathcal{N} \), res.

Additionally, let \( t(n) \) denote the period from set \( T \) to which node \( n \) belong to. See e.g., [8, 29] for the main concepts on stochastic optimization via scenario tree analysis. Notice that \( \tilde{A}^n \) for \( n \in \mathcal{N}^T \) is the set of nodes for scenario \( n \in \Omega \), so for convenience, let \( n = \omega \) for \( n \in \mathcal{N}^T \).

3.1. Scenario tree generation scheme

It is beyond the scope of this work to present a methodology for multiperiod scenario tree generation and reduction; see e.g., [22, 26, 30] for different alternative ways of performing it. A rigorous development of scenario trees for future wood prices is extremely complex (see the recent paper [36] for a novel proposal).

At any rate, let us consider that the tree structure represented in the set \( \mathcal{N}^t, \forall t \in T \) and that the wood prices are mean-reverting as other commodities. Their stochastic behaviour can then be modeled through the stochastic differential equation,

\[
dp(t) = \mu(\nu - p(t))dt + \sigma p(t)dw(t), \quad p(0) = p_0, \quad t \geq 0,
\]

where \( p(t) \) is the price at time \( t \), \( p(0) \) is the present price (or an estimate) at time 0, \( \mu \) is the speed of price convergence towards its long-term value \( \nu \), \( \sigma \) is the standard deviation and \( w(t) \) is a Wiener process. The solution of the equation can be expressed as follows (see [36] for more details)

\[
p(0) = \nu(1 - e^{-\mu t}) + p(0) \exp\left[-\left(\mu + \frac{\sigma^2}{2}t + \sigma w(t)\right)\right]
\]

which corresponds to a displaced log-Gaussian process with mean and variance

\[
E[p(t)] = \nu(1 - e^{-\mu t}) + p_0 e^{-\mu t} \\
V[p(t)] = (p(0)e^{-\mu t})^2(e^{\sigma^2t} - 1)
\]

Using data from 1988 to 2009, [36] proposes the following estimation for the coefficients of the process associated to sawtimber:

\[
p_0 = 1.0749, \quad \nu = 1.1998, \quad \mu = 0.0462, \quad \sigma^2 = 0.0319.
\]

For pulp-wood, again on the 1988-2009 database, the coefficients are as follows:

\[
p_0 = 0.5501, \quad \nu = 0.5623, \quad \mu = 0.0979, \quad \sigma = 0.0086.
\]

Modelling export quality timber is not presented in [36]; thus we considered, based on historical data, that export quality is 20% more expensive than sawtimber. Additionally, in order to consider some variability in
the prices, a random perturbation of 10% at most has been introduced in the timber price for each market.

By following the ideas in [36] for building robust scenario trees, we use available information about price to conduct an analysis of the cumulative distribution functions (CDF) associated to the densities. The main step of the methodology applied to our case from period \( t = 1 \) to period \( t = |T| \) consists of obtaining a finite set of points that summarize the CDF, giving flexibility for considering specific segments of the domain, e.g. tail events. For that purpose, the CDF is split in a finite number of segments with a 1-to-1 correspondence with the nodes in set \( \mathcal{N}^t \), \( \forall t \in T \); then the information of each segment is assigned to a representative point. Formally, let \( \xi_t \) be a random variable with support \( \Xi_t \), probability density function \( f_{\xi}(t) \) and cumulative distribution function \( F_{\xi}(t) \). In addition, for the nodes \( i \equiv |\mathcal{N}^{t-1}| + 1 \) and \( j \equiv |\mathcal{N}^t| \) i.e., the lexicographically ordered first and last nodes of period \( t \), res., let \( C^t = \{c_i, \ldots, c_{j+1}\} \) be a partition of the interval \([0, 1]\),

\[
c_i = 0, \quad c_{j+1} = 1, \quad c_n < c_{n+1}, \quad \forall n \in \{i, \ldots, j\}, \quad \text{and} \quad \bigcup_{n=i}^j [c_n, c_{n+1}] = [0, 1].
\]

Considering the value of \( F_{\xi}^{-1} \) evaluated in \( C^t \) we obtain the \( c_n \)-quantiles of \( \xi_t \), say \( Q_{c_n} \), for \( c_n \in C^t \), \( \forall n \in \mathcal{N}_t \), which define the partition of the support of \( \xi_t \). Note: Depending on the set \( C^t \), the approach allows the number of scenarios to be changed, as well as the segments of the support of \( \xi_t \). Once the segments are defined, the expected value of the random variable for each one can be expressed as follows,

\[
\xi^n = E(\xi_t \mid Q_{c_{n}} \leq \xi_t \leq Q_{c_{n+1}}) \quad \forall n \in \{i, \ldots, j\}
\]

To generate the scenarios for the product demands, we considered a similar scheme by assuming that the demands are normally distributed. As in [4], the model considers lower and upper bounds on the demand of each wood product to be offered in different markets. These bounds are different for each product \( q \) and market \( m \) at any node of the scenario tree. Therefore, for a given product \( q \) and market \( m \), let us consider a node \( n \in \mathcal{N} \) with its set \( S^n \) of the immediate lexicographically ordered successors in the tree, and let \( o_s \) denote the order in the set related to node \( s \in S^n \) from 1 to \( k \equiv |S^n| \) denote the last node in that set. The demand, say \( \zeta \) in node \( s \in S^n \) can be approximated by a random variable normally distributed with mean \( \mu^n \) and standard deviation \( \sigma^n \), lower and upper bounds, say \( Q_{o_s-1}^{\zeta} \) and \( Q_{o_s}^{\zeta} \), being \( Q_{o_s}^{\zeta} \) the \( o_s \)-quantile of the normal distribution \( N(\mu^n, \sigma^n) \). Note: Instead of the 0- and 1-quantiles of the normal distribution (which, in fact, are \( -\infty \) and \( +\infty \), res.), we consider the 0.001- and 0.999-quantiles.

For computing the demand bounds of any node, say \( s \in S^n \), the parameters of the normal distribution to be used \( N(\mu^n, \sigma^n) \), are computed as follows: The mean \( \mu^n \) is the conditional expectation of the random variable \( \zeta^n \) in the interval defined by the bounds \( \{Q_{o_s-1}^{\zeta}, Q_{o_s}^{\zeta}\} \), i.e.,

\[
\mu^n = E(\zeta^n \mid Q_{o_s-1}^{\zeta} \leq \zeta^n \leq Q_{o_s}^{\zeta}),
\]

and the standard deviation is set to 30% of the mean. Note 1: For the computational experience reported in
Section 5, $\sigma^s = 0.3\mu^s$. Note 2: The parameters for the distribution function of the demand at the root node is a decision maker-driven data.

As an example, let us consider the three-period scenario tree depicted in Fig. 5 where $T = \{1, 2, 3\}$, the root node is $n = 1$ and its immediate successor set $S^1$ is $\{s = 2, \ldots, 5\}$. For a decision maker-driven data with $\mu = 2000$ and $\sigma = 600$, the lower and upper bounds for the demand at the nodes in period 2 are as follows:

- Node 2: $Q_{0.001} = 146$ and $Q_{1.000} = 1505$.
- Node 3: $Q_{1.000} = 1505$ and $Q_{2.000} = 2000$.
- Node 4: $Q_{2.000} = 2000$ and $Q_{3.000} = 2495$.
- Node 5: $Q_{2.000} = 2495$ and $Q_{0.999} = 3854$.

The normal distribution associated with the random variable $\xi^3$ used for computing the bounds on the demand for the immediate successor node set $S^3$ has a mean equal to:

$$\mu^3 = E(\xi^1 | 1505 \leq \xi^1 \leq 2000) = 1811.$$
scenarios and, in particular, the left tail of the undesirable scenarios. There are, however, other approaches that, additionally, deal with risk management; see in [3] a comprehensive computational comparison of the most popular time-inconsistent risk-averse measures. Well-known theoretical research suggests that the measures based on quantiles are good functions for risk management. Among them, the Value-at-Risk (VaR) and Conditional VaR (CVaR) have become a benchmark for many applications in the financial, transportation and productions planning sectors, among others; see [21, 23, 28, 31, 38, 43].

**Definition 1.** VaR$_\beta(X, \Omega)$ of a solution $X$ is the highest value, say $\alpha$, such that the sum of the weights of scenarios with a profit smaller than $\alpha$ is lower than $\beta$, where $\beta \in (0, 1)$ is a modeler-driven parameter.

Notice that the advantage of the VaR measure over the traditional maxmin measure is obvious, since it specifies the bound $\beta$ on the probability of the occurrence of a scenario whose profit is below $\alpha$. It does not however consider how bad the scenarios with a profit below $\alpha$ can be. As an alternative, [38] introduce the CVaR$_\beta(X, \Omega)$ measure for linear models:

**Definition 2.** CVaR$_\beta(X, \Omega)$ of a solution $X$ is the conditional expectation of the profit below $\alpha = \text{VaR}_\beta(X, \Omega)$.

CVaR takes into account the profit for those non-desirable scenarios. In this section we present two modifications of model RN (2) that allow risk management to be considered via the Conditional-Value-at-Risk (CVaR).

Classical CVaR and many other risk averse approaches in the literature, reduce the probability of a negative input of the solution of the model in the unwanted scenarios, but they do not address scenarios with higher profit. On the contrary, decision makers usually look for a trade-off between risk minimization and profit maximization. For this reason, the risk measures are usually combined with the optimization of the expected value of the objective function, leading to mean-risk models that combine Expected objective function Value and CVaR, resulting in the following model, see [41]:

$$z_{\text{CVaR}} = \max \left[ \gamma \sum_{n \in N} w^n (a^n x^n + b^n y^n) + \rho \left( \alpha - \frac{1}{\beta} \sum_{\omega \in \Omega} w^\omega v^\omega \right) \right]$$

subject to:

$$\sum_{q \in A^n} (A^n q x^n + B^n q y^n) = h^n \quad \forall n \in N$$

$$\alpha - \sum_{q \in A^\omega} (a^n q x^n + b^n q y^n) \leq v^\omega \quad \forall \omega \in \Omega$$

$$x^n \in \{0, 1\}^{n x(n)}, y^n \in \mathbb{R}^{n y(n)} \quad \forall n \in N$$

$$v^\omega \in \mathbb{R}_+ \quad \forall \omega \in \Omega$$

$$\alpha \in \mathbb{R}$$

where $v^\omega$ is a non-negative variable equal to the difference (if it is positive) between $\alpha$ and the profit for scenario $\omega$, and $\gamma \in \{0, 1\}$ and $\rho > 0$ are weight factors. Therefore, in the objective function, the weighted sum of the $v$-variables is minimized.
4.1. Multiperiod TCVaR measure

CVaR model (3) only performs risk management for the profit, but in many contexts, the decision maker wants to manage the risk for other functions such as the one that measures the environmental impact. Let \( F \) denote the set of functions where risk management is to be performed in the value \( \sum_{n\in\tilde{A}^\omega}(a^n_fx^n + b^n_fy^n) \), \( \forall f \in F \) for the set of scenarios \( \omega \in \Omega \). This model also measures and controls the risk at the end of the time horizon. However, since the value of the function \( f \) under risk-control of each scenario is calculated as the sum of the values of the function at the nodes \( \tilde{A}^\omega \), this approach does not prevent very bad results at any of the stages. Notwithstanding, it could eventually be compensated by good results in the others stages.

It may thus happen that bad results at intermediate nodes drive the decision maker to a situation of no-return. To avoid such situations, risk management can be performed at some intermediate time periods; let us define \( \tilde{T}^f \) as the subset of time periods where function \( f \) is under risk-control (note that for singleton set \( \tilde{T}^f = \{ T \} \), the classical CVaR is obtained). Model (3) can be extended to consider multi-period and multi-function risk management, resulting in the following model related to the TCVaR measure,

\[
z_{TCVaR} = \max \left[ \gamma \sum_{n\in N} w^n(a^n_1x^n + b^n_1y^n) + \sum_{f\in F} \sum_{t\in \tilde{T}^f} \rho^f_t \left( \alpha^t_f - \frac{1}{\beta^t_f} \sum_{n\in N_t} w^n v^n_f \right) \right]
\]

s.t.
\[
\begin{align*}
\sum_{q\in A^n} (A^n_qe^q + B^n_qy^q) &= h^n \quad \forall n \in N \\
\alpha^t_f - \sum_{q\in A^n} (a^n_qx^q + b^n_qy^q) &\leq v^n_f \quad \forall n \in N^t, t \in \tilde{T}^f, f \in F \\
x^n &\in \{0,1\}^{n_x(n)}, y^n \in \mathbb{R}^{n_y(n)} \quad \forall n \in N \\
v^n &\in \mathbb{R}_+ \quad \forall n \in N^t, t \in \tilde{T}^f, f \in F \\
\alpha^t_f &\in \mathbb{R} \quad \forall t \in \tilde{T}^f, f \in F,
\end{align*}
\]

where \( \alpha^t_f \) is \( \text{VaR}_{\beta^t_f} \) for function \( f \in F \) up to period \( t \in \tilde{T}^f \) in the time horizon for the whole set of scenarios, i.e., set \( N^t \), and \( \rho^t_f \) for \( t \in \tilde{T}^f \) is a modeler-driven parameter, such that \( \rho^t_f \) weights the risk reduction importance for the pair \( (t,f) \). Notice that the bigger \( \beta^t_f \) is, the smaller the importance given to the expected shortfall on reaching \( \alpha^t_f \) over the scenarios. As an illustration, consider the four-period scenario tree in Fig. 6 and consider that \( \tilde{T}^f = \{3,4\} \), then TCVaR model (4) performs risk management by minimizing the CVaR associated to the two depicted scenario subtrees.

It was shown in [28] that CVaR (and, then, TCVaR) is a coherent risk measure, according to the standards setup in [5], see also [6], since it satisfies the properties of translation invariance, positive homogeneity, monotonicity and convexity.

4.2. Time consistent ECVaR measure

One desirable property for a solution of a multiperiod model is time consistency. The rationale behind a time consistent risk measure is that the solution to be obtained for any node, say \( n \), of the scenario tree and its successor node set \( \tilde{S}^n \) in the related submodel 'solved' at period \( t(n) \) should have the same value as the solution obtained for that node and its successors in the original model 'solved' at period \( t=1 \). It is
well known that the Risk Neutral approach is time consistent, while risk measures CVaR and TCVaR are not. It was shown in [23] that the time consistency property of CVaR depends on parameter $\beta_f^t$; the smaller it is, the higher the consistency probability of CVaR, in the sense that the difference between the solution obtained for the original problem and the solution for the problem 'solved' at any other period decreases.

As a time-consistent alternative to TCVaR model (4), the risk management can be performed at any node $n \in N^t : t(n) < T$, considering the scenarios in the associated subtree with root in node $n$ of the original scenario tree. Let $\tilde{T}_f$ denote the period subset where the risk reduction in the value of the function indexed with $f$ is to be performed, for $f \in F$. Observe that any group of scenarios, say $\Omega^n$, has a 1-to-1 correspondence with node $n$ in the tree. The model related to the Expected Conditional VaR (for short, ECVaR) measure for performing the required risk reduction can be expressed as

$$z_{ECVaR} = \max \left[ \gamma \sum_{n \in N} w^n (a^n_1 x^n + b^n_1 y^n) + \sum_{f \in F} \sum_{t \in \tilde{T}_f} \rho^t_f \sum_{n \in N^t} \left( w^n \alpha^n_f - \frac{1}{\beta_f^t} \sum_{\omega \in \Omega^n} w^\omega v^n_\omega \right) \right]$$

s.t.

$$\sum_{q \in A^n} (A^n_q x^n + B^n_q y^n) = h^n \quad \forall n \in N$$

$$\alpha^n_f - \sum_{g \in A^n} (a^n_g x^n + b^n_g y^n) \leq v^n_f \quad \forall \omega \in \Omega^n, n \in N^t, t \in \tilde{T}_f, f \in F$$

$$x^n \in \{0, 1\}^{n_x(n)}, y^n \in \mathbb{R}^{n_y(n)} \quad \forall n \in N$$

$$v^n_f \in \mathbb{R}_+, \alpha^n_f \in \mathbb{R} \quad \forall \omega \in \Omega^n, n \in N^t, t \in \tilde{T}_f, f \in F$$

where $\alpha^n_f$ is $VaR_{\beta_f^t}$ for function $f \in F$ in the whole time horizon for the scenarios that belong to group $n$, and $v^n_f$ is a non-negative variable that gives the shortfall of scenario $\omega$ for reaching VaR $\alpha^n_f$, for $\omega \in \Omega^n$. As an illustration, for the scenario tree in Fig. 7, consider that $\tilde{T}_f = \{1, 2\}$, then ECVaR model (5) performs the risk management by minimizing the CVaR associated to the full set of scenarios for $t = 1$, and the set of scenarios $\Omega^n$ in the $|N^t| = 3$ depicted subtrees, each rooted with node $n$, for $n \in N^t$, for $t = 2$. It is worth pointing out that model (5) is close to the average VaR proposals introduced in [19, 31, 32].

The ECVaR measure as presented in model (5) belongs to the family of Expected Conditional Risk
Measures (ECRMs) considered in [23], where the time consistency property of those measures is proved, according to the definition introduced there. Notice that the proof only requires the measure to have the properties of translation-invariance and monotonicity. See some variants in [7, 11, 14, 15, 31, 30, 39, 40, 42]. The particularization of the definition in our context is as follows:

Let \((\hat{\xi}^q, \hat{\eta}^q \forall q \in \mathcal{N}, \hat{\nu}^f \forall \omega \in \Omega, \hat{\alpha}^q_n \forall n \in \mathcal{N}^n, t \in \mathcal{T}_f, f \in \mathcal{F})\) denote any of the optimal solutions of the original ECVaR model (5), and \(z^n_{\text{ECVaR}}\) is the solution value in that model where only the terms related to the subtree \(\hat{\mathcal{A}}^n \cup \hat{\mathcal{S}}^n\) are considered. It can be expressed as

\[
z^n_{\text{ECVaR}} = \gamma \sum_{q \in \hat{\mathcal{A}}^n \cup \hat{\mathcal{S}}^n} w^q (a^q_1 \hat{\xi}^q + b^q_1 \hat{\eta}^q) + \sum_{f \in \mathcal{F}} \sum_{q \in \hat{\mathcal{S}}^n, t(q) \in \mathcal{T}_f} \rho_{t(q)}^f \left( w^q \hat{\alpha}^q_f - \frac{1}{\beta_f^q} \sum_{\omega \in \Omega} w^\omega \hat{\nu}^f_\omega \right)
\]

For any node \(n \in \mathcal{N}\), let us define the ECVaR\(^n\) submodel from (5) as follows:

- The subtree that supports submodel ECVaR\(^n\) includes the nodes in set \(\hat{\mathcal{A}}^n\) (from the original scenario tree) plus the subtree rooted in node \(n\) whose nodes are in set \(\hat{\mathcal{S}}^n\).
- The input data of the submodel is the same as in model (5) for any scenario tree of any scenario set \(\Omega\).
- Finally, in the submodel the variables in vectors \(x^q\) and \(y^q \forall q \in \hat{\mathcal{A}}^n \setminus \{n\}\) are fixed to the values in \(\hat{\xi}^q\) and \(\hat{\eta}^q\), res.

Let \(z^n_{\text{ECVaR}^n}\) denote the value of an optimal solution of the submodel ECVaR\(^n\). Therefore, the ECVaR measure is a time consistent one, since the following assertion is true:

\[
z^n_{\text{ECVaR}^n} = z^n_{\text{ECVaR}}
\]

Another consistent CVaR-based measure that may be considered as an alternative to model ECVaR (5) is the nested risk measure introduced in [33]; see also [25, 42]. The nested mechanism of the CVaR submodels for each period in set \(\mathcal{T}_f, f \in \mathcal{F}\), however, makes its decomposition more difficult.
Note: ECVaR, as well as any other ECRM, is very suitable for using decomposition algorithms (such as [18, 45], among others) for solving large-scale instances.

An interesting question (still an open one for us, at least) is: What version, either the time consistent one or the time inconsistent version of a risk averse measure, performs better for risk management? The computational experiments whose main results are reported in the next section may help to answer that question.

5. Computational experiments

5.1. Instances description

The instances used for testing the risk aversion approach are based on [4]. In that work a deterministic version of the problem is solved using real data from the forest company Forestal Millalem. It comprises 17 areas, geographically separated, each connected through public roads to demand nodes. It produces three wood qualities (for export, sawmills and pulp plants) that are sent to 7 different markets. In our work, different instances have been created by selecting subsets of the areas in order to obtain small realistic examples where the stochastic version could be solved in a reasonable computing time. The planning horizon considered is three years and each year is divided into two seasons (summer and winter).\( \mathcal{T} \), therefore, includes six time periods (also so-called stages). The first time period is considered to be summer.

Table 1 shows the main characteristics of the instances considered in this work. The headings are as follows: \( S \) number of stands; \( na \), number of areas; \( C \), number of stocking yards; \( I \), number of nodes; \( L^P \), number of potential roads; \( L^{Ed} \) and \( L^{Eg} \), number of existing roads in dirt and gravel, res.; and \( Ha \), total forest surface.

By contrast to [4], we consider that wood prices and demand are uncertain, and represent them using a multistage scenario tree. The scenario tree structure corresponds to a \( 1 \times 12 \times 12 \times 1^3 \) model, where the second stage has 12 nodes, each of the second-stage nodes has 12 sons, and the nodes in the rest of the stages (except stage 1) have only one son, resulting in \( 12 \times 12 = 144 \) scenarios (see Fig. 8). To build the data for the sons of a node, we have combined 4 different price scenarios and 3 different demand scenarios, obtaining 12 combinations.

Table 2 shows the size of the mathematical formulation of the deterministic model (only one scenario) and the compact formulation of the risk neutral approach of the stochastic model. The headings are as follows: \( m \), number of constraints, \( nc \), number of continuous variables, \( n01 \), number of binary variables,
$|\Omega|$, number of scenarios in the scenario tree, and $|\mathcal{N}|$, number of nodes in the scenario tree. Note: Only the objective function is considered, i.e., set $\mathcal{F}$ is a singleton.

The dimensions of the mathematical formulations for the risk management models TCVaR (4) and ECVaR (5) are very similar to the dimensions of the RN model (2). Model (4) adds one $\alpha$-variable for each stage for which the risk management is performed (i.e., $|\tilde{T}|$ variables), one $v$-variable for each node in the stages in $\tilde{T}$ (i.e. $k = \sum_{i \in \tilde{T}} |\mathcal{N}^{|i}|$), and the related $k$ constraints. Model (5) adds $k$ $\alpha$-variables, $|\Omega|$ $v$-variables, and the related $|\Omega|$ constraints. In our instances, there are fewer than 300 new constraints and continuous variables. Taking into account the dimensions of the RN model (2) (see table 2), there is less than 0.2% and 0.1% increments in the number of constraints and variables, res.

The computational experiments were conducted in the HW/SW platform given by a workstation under the Linux operating system (version Ubuntu GNU/linus 14.04.1) with 64 bits, 2 processors Intel(R) Xeon(R) CPU E5-2630 @ 2.3 GHz, 64 Gb of RAM DDR3 1600MHz ECC and 24 virtual cores. The model has been implemented with GAMS 24.3.2. The optimization uses one of the state-of-the-art commercial optimization engines, CPLEX 12.6.1.

5.2. Risk neutral approach: Deterministic versus stochastic models

Let us start with a comparison between the stochastic model with risk neutral policy (2) and the traditional deterministic approach, say EV (Expected Value), where the uncertain parameters have been replaced with their expected values. EEV is the Expected profit of the Expected Value, obtained by applying the EV solution to the scenarios. And WS (Wait-and-See) is the average of the profit obtained by the independent
Table 3: Risk neutral model (2) and deterministic model (1) solutions for instance i2

<table>
<thead>
<tr>
<th></th>
<th>EEV</th>
<th>RN</th>
<th>WS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution value</td>
<td>2415</td>
<td>2475</td>
<td>2553</td>
</tr>
<tr>
<td>Greatest scen. solution value</td>
<td>3525</td>
<td>3879</td>
<td>4228</td>
</tr>
<tr>
<td>Median</td>
<td>2399</td>
<td>2451</td>
<td>2519</td>
</tr>
<tr>
<td>c-VaR</td>
<td>1959</td>
<td>1959</td>
<td>2087</td>
</tr>
<tr>
<td>c-CVaR</td>
<td>1589</td>
<td>1851</td>
<td>2006</td>
</tr>
<tr>
<td>Smallest scen. solution value</td>
<td>587</td>
<td>1668</td>
<td>1894</td>
</tr>
<tr>
<td>CPU time $t_0$ (secs.)</td>
<td>11</td>
<td>34156</td>
<td>123</td>
</tr>
</tbody>
</table>

models related to each scenario, which is an upper bound on the expected profit of the original stochastic model. Notice that the WS solution does not usually satisfy the relaxed non-anticipativity constraints. The methodology for obtaining the EEV is very well established for the two-stage setting, see [8], but it is not for the multistage one, see [17]. Alternatively, we propose the following methodology for obtaining the EEV in a rolling horizon type of calculation (see [1] for more details): (1) The solution for the first stage is obtained from the EV solution, (2) Once the solution up to stage $t - 1$ is fixed, $|\mathcal{N}^t|$ independent scenario subtrees remain, (3) The EV solution is independently obtained for the scenario subtrees, whose root nodes are the nodes in $\mathcal{N}^t$, so that the solution for each root node is fixed to its EV solution, (4) The procedure continues until stage $T-1$, where the mixed 0-1 two-stage problem for each related node are solved. At the end of the process there is a solution for each scenario and EEV is obtained by weighting the solution values for the scenarios as calculated by the procedure.

Table 3 shows the main results related to the RN, WS and EEV solutions for instance i2. No data are reported for instances i1 and i3, since for the deterministic approach (i.e., the EV solution) the results are infeasible. For the EEV, RN and WS solutions, c-VaR is the 0.10-quantile of the profit vector for the set of scenarios (that is, 10% of the scenarios have a profit lower than c-VaR), and c-CVaR gives the expected profit for those scenarios whose profit is lower than c-VaR). We can observe that the EEV solution value (in our case, the expected profit) is 5.5% smaller than the RN profit and very similar to the computed Var profit (only 2.2% smaller). However, the computed CVaR and the smallest scenario-related profit obtained in the EEV approach are very poor, comparing with the ones obtained by the RN model (2). Therefore, even the RN maximization of the expected profit over the scenarios along the time horizon provides better results in terms of risk than the result EEV for the traditional EV approach.

Together with Table 3, we show the boxplot associated to the distribution of profits along the different scenarios. A boxplot is a standardized way of displaying the distribution of data based on the summary of the five statistical measures: minimum, first quartile, median, third quartile, and maximum. The central box spans the first quartile to the third quartile (the interquartile range or IQR). The segment inside the rectangle shows the median and whiskers above and below the box show the locations of the minimum and maximum.
Additionally, the mean (expected profit), the computed VaR (c-VaR) and CVaR (c-CVaR) are shown in each boxplot. From these boxplots, one can conclude that the solution provided by the EV approach is worse than the one provided by the RN model (2), since the different statistical measures are worse. It highlights the CVaR and the lower tail of the distribution, with very poor values for worst-case scenarios.

5.3. Risk-averse approaches

In this section the results for the three instances are reported by solving the different risk-averse models presented in Section 4. A computational comparison with the RN model (2) is presented in order to analyze the impact of those measures on the solution. Note: The optimality GAP bound has been set to 2%.

5.3.1. Time inconsistent TCVaR model

We have tested the impact of managing the risk in two points: the middle and the end of the planning horizon, that is, $\bar{T}_1 = \{3, 6\}$. The parameters $\beta_t$ for $t = 3, 6$ have been set to 0.10, and, to ascertain better the impact of the risk-averse term on the objective function, different combinations of the weight parameter $\rho_t$ have been tested, for $t \in \bar{T}_1$.

Table 4 shows the solution obtained by TCVaR model (4). The headings are as follows: $\rho_{3}^t$ and $\rho_{6}^t$, weight factors in the objective function for the risk management at periods $t = 3$ and 6, res.; $Z_{MIP}$, best upper bound for the optimal solution provided by the solver at the time at which the solver’s execution is stopped; $Z_{MIP}$, solution value of the incumbent solution; GAP, related optimality gap defined as $100 \frac{Z_{MIP} - Z_{MIP}}{Z_{MIP}}$; nn, number of nodes in the B&C tree that have been explored when stopping the solver’s execution; $t_0$, elapsed time instant (in secs.) at which the execution was stopped.

<table>
<thead>
<tr>
<th>Inst.</th>
<th>$\rho_{3}^1$</th>
<th>$\rho_{3}^6$</th>
<th>$Z_{MIP}$</th>
<th>$Z_{MIP}$</th>
<th>GAP</th>
<th>$t_0$ (secs.)</th>
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</thead>
<tbody>
<tr>
<td>i1</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{3}{4}$</td>
<td>2716.35</td>
<td>2663.12</td>
<td>1.96</td>
<td>18929.93</td>
</tr>
<tr>
<td>i1</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>9578.78</td>
<td>9411.52</td>
<td>1.75</td>
<td>30260.59</td>
</tr>
<tr>
<td>i1</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{3}{4}$</td>
<td>8966.35</td>
<td>8798.92</td>
<td>1.87</td>
<td>3039.89</td>
</tr>
<tr>
<td>i1</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{3}{4}$</td>
<td>14994.94</td>
<td>14723.37</td>
<td>1.81</td>
<td>16138.50</td>
</tr>
<tr>
<td>i2</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{3}{4}$</td>
<td>3428.40</td>
<td>3362.39</td>
<td>1.92</td>
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</tr>
<tr>
<td>i2</td>
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<td>$\frac{3}{4}$</td>
<td>12627.90</td>
<td>12380.34</td>
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<td>24414.99</td>
</tr>
<tr>
<td>i2</td>
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<td>11347.15</td>
<td>11129.96</td>
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<td>7190.29</td>
</tr>
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<td>$\frac{3}{4}$</td>
<td>20308.53</td>
<td>19999.71</td>
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<tr>
<td>i3</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{3}{4}$</td>
<td>2813.85</td>
<td>2743.07</td>
<td>2.52</td>
<td>&gt; 24h.*</td>
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<tr>
<td>i3</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{3}{4}$</td>
<td>9905.85</td>
<td>9711.63</td>
<td>1.96</td>
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<td>i3</td>
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<td>8634.14</td>
<td>8464.95</td>
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<tr>
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<td>1552.34</td>
<td>1521.90</td>
<td>1.96</td>
<td>72990.54</td>
</tr>
</tbody>
</table>

* limit of the elapsed time reached (24h)

Table 4: TCVaR: Results for model (4)
to the expected value for the WS approach up to the last period, since its value is ideally considered to be best (but infeasible), and set to 100 for comparison purposes. For the TC VaR measure different results are presented, depending on the pairs \((\rho_3^1, \rho_6^1)\) of the weight parameters that have been used for periods 3 and 6.

Those tables also show the expectation, the computed VaR, and computed CVaR of the profit up to the end of periods 1, 2, and 6. As can be observed, at the end of the planning horizon (period 6), all stochastic models practically provide the same expected profit, while the c-CVaR and c-VaR are slightly higher for the risk-averse models. However, the risk-averse TC VaR measure provides an expected profit for time periods 1 and 2 (especially for \(t = 1\)) much higher than the RN measure and a significant improvement in the c-VaR and c-CVaR. In short, we have observed that the risk-averse measure moves the profits to the early periods, and also that at the end of the planning horizon, the profit is only slightly deteriorated compared with that obtained by the RN model.

Besides those three statistics, it is also important to consider the distribution of profits over the set of scenarios, instead of just the expected profit. For that purpose, next to each table, we show the boxplots for the profit distribution at the end of the three periods that are considered for risk reduction in the instances. As in the previous analysis, the WS results are presented as a reference. The set of boxplots on the right figure of each table shows the profit distribution at the end of the planning horizon (period 6), the lower figure in the left part shows the profit at the end of the second period (corresponding to the end of the first year of harvesting planning horizon) and the upper left figure corresponds to the profit in the first period (corresponding to the first semester in the planning horizon). Note that in the latter case, the solution proposed by all the stochastic optimization models is the same for all scenarios (the boxplot is just a dot), since the non-anticipativity principle is satisfied, while the WS approach provides (by construction) a different profit for each scenario. It can also be observed that at the end of the second period, the TC VaR measure improves the results of the RN measure: not only are profits better on average (as shown in the tables), but also the improvement is due to the profit distribution over the set of scenarios.

It is worth analyzing the results shown in Tables 5, 6 and 7 related to the combination \((\rho_3^1 = 5, \rho_6^1 = 0.25)\). (Notice that period 6 is the last one in the time horizon considered in the experiment). It is a simulation where the decision-maker is assumed to give to the CVaR of period 3 a higher weight than to the CVaR of period 6. As a result we can observe that the expected profit and the profit distribution up to period 6 (i.e., the profit that considers the whole time horizon over the whole set of scenarios) are very poor with respect to the same results up to period 3, which are very good for periods 1 and 2 (as shown in the tables). It can be observed that the profit results are very different for a simulation with opposite priorities. This point is very interesting since the profit results could be very poor up to period 6 in the irreversible situation where the decision-maker adopts a shortsighted strategy. Note that this strategy is based on a shorter time horizon (say, up to period 3, whose results in the simulation are very good).
Table 5: TCVaR: Results and profit distribution for instance i1

<table>
<thead>
<tr>
<th>t Stat.</th>
<th>RN</th>
<th>$\rho_{1/4}^1\rho_{5/4}^6$</th>
<th>$\rho_{1/4}^2\rho_{5/4}^6$</th>
<th>$\rho_{1/4}^3\rho_{5/4}^6$</th>
<th>$\rho_{1/4}^4\rho_{5/4}^6$</th>
<th>$\rho_{5/4}^1\rho_{5/4}^6$</th>
<th>WS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Expect.</td>
<td>17.3</td>
<td>17.4</td>
<td>16.9</td>
<td>30.4</td>
<td>19.0</td>
<td>28.8</td>
<td></td>
</tr>
<tr>
<td>2 Expect.</td>
<td>37.9</td>
<td>39.4</td>
<td>39.6</td>
<td>51.8</td>
<td>41.4</td>
<td>49.9</td>
<td></td>
</tr>
<tr>
<td>c-VaR</td>
<td>33.9</td>
<td>36.0</td>
<td>35.4</td>
<td>49.9</td>
<td>39.1</td>
<td>31.8</td>
<td></td>
</tr>
<tr>
<td>c-CVaR</td>
<td>32.4</td>
<td>35.6</td>
<td>34.7</td>
<td>49.1</td>
<td>38.5</td>
<td>27.8</td>
<td></td>
</tr>
<tr>
<td>6 Expect.</td>
<td>92.9</td>
<td>92.5</td>
<td>92.6</td>
<td>74.2</td>
<td>91.2</td>
<td>100.0</td>
<td></td>
</tr>
<tr>
<td>c-VaR</td>
<td>69.3</td>
<td>69.4</td>
<td>69.6</td>
<td>-2.4</td>
<td>66.8</td>
<td>79.8</td>
<td></td>
</tr>
<tr>
<td>c-CVaR</td>
<td>64.0</td>
<td>64.2</td>
<td>64.9</td>
<td>-13.6</td>
<td>62.0</td>
<td>74.6</td>
<td></td>
</tr>
</tbody>
</table>

Table 6: TCVaR: Results and profit distribution for instance i2

<table>
<thead>
<tr>
<th>t Stat.</th>
<th>RN</th>
<th>$\rho_{1/4}^1\rho_{5/4}^6$</th>
<th>$\rho_{1/4}^2\rho_{5/4}^6$</th>
<th>$\rho_{1/4}^3\rho_{5/4}^6$</th>
<th>$\rho_{1/4}^4\rho_{5/4}^6$</th>
<th>$\rho_{5/4}^1\rho_{5/4}^6$</th>
<th>WS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Expect.</td>
<td>33.9</td>
<td>36.1</td>
<td>42.2</td>
<td>42.7</td>
<td>42.3</td>
<td>38.5</td>
<td></td>
</tr>
<tr>
<td>2 Expect.</td>
<td>51.3</td>
<td>53.4</td>
<td>57.2</td>
<td>58.3</td>
<td>57.1</td>
<td>56.3</td>
<td></td>
</tr>
<tr>
<td>c-VaR</td>
<td>47.3</td>
<td>50.3</td>
<td>54.0</td>
<td>56.1</td>
<td>54.1</td>
<td>43.7</td>
<td></td>
</tr>
<tr>
<td>c-CVaR</td>
<td>47.2</td>
<td>49.8</td>
<td>53.4</td>
<td>55.7</td>
<td>53.5</td>
<td>32.7</td>
<td></td>
</tr>
<tr>
<td>6 Expect.</td>
<td>96.5</td>
<td>96.5</td>
<td>95.5</td>
<td>92.2</td>
<td>95.4</td>
<td>100.0</td>
<td></td>
</tr>
<tr>
<td>c-VaR</td>
<td>75.4</td>
<td>75.7</td>
<td>77.0</td>
<td>71.8</td>
<td>76.6</td>
<td>80.3</td>
<td></td>
</tr>
<tr>
<td>c-CVaR</td>
<td>71.3</td>
<td>71.9</td>
<td>73.3</td>
<td>64.5</td>
<td>73.1</td>
<td>77.3</td>
<td></td>
</tr>
</tbody>
</table>

Table 7: TCVaR: Results and profit distribution for instance i3

<table>
<thead>
<tr>
<th>t Stat.</th>
<th>RN</th>
<th>$\rho_{1/4}^1\rho_{5/4}^6$</th>
<th>$\rho_{1/4}^2\rho_{5/4}^6$</th>
<th>$\rho_{1/4}^3\rho_{5/4}^6$</th>
<th>$\rho_{1/4}^4\rho_{5/4}^6$</th>
<th>$\rho_{5/4}^1\rho_{5/4}^6$</th>
<th>WS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Expect.</td>
<td>27.2</td>
<td>31.0</td>
<td>34.4</td>
<td>36.1</td>
<td>34.0</td>
<td>35.2</td>
<td></td>
</tr>
<tr>
<td>2 Expect.</td>
<td>42.9</td>
<td>45.8</td>
<td>47.5</td>
<td>48.8</td>
<td>46.8</td>
<td>51.2</td>
<td></td>
</tr>
<tr>
<td>c-VaR</td>
<td>36.6</td>
<td>42.2</td>
<td>45.6</td>
<td>47.2</td>
<td>45.1</td>
<td>40.2</td>
<td></td>
</tr>
<tr>
<td>c-CVaR</td>
<td>36.5</td>
<td>42.0</td>
<td>45.1</td>
<td>47.1</td>
<td>45.0</td>
<td>35.4</td>
<td></td>
</tr>
<tr>
<td>6 Expect.</td>
<td>94.5</td>
<td>96.0</td>
<td>93.4</td>
<td>89.9</td>
<td>92.6</td>
<td>100.0</td>
<td></td>
</tr>
<tr>
<td>c-VaR</td>
<td>69.5</td>
<td>71.7</td>
<td>72.5</td>
<td>65.1</td>
<td>70.9</td>
<td>76.8</td>
<td></td>
</tr>
<tr>
<td>c-CVaR</td>
<td>62.1</td>
<td>67.0</td>
<td>68.0</td>
<td>57.3</td>
<td>66.9</td>
<td>73.4</td>
<td></td>
</tr>
</tbody>
</table>
The ECVaR measure performs risk management for the instances we have experimented with at each of the given subtrees in the scenarios tree. Considering the tree structure in Fig. 8, period 2 is the only intermediate period where the risk can be controlled by ECVaR, then, $\tilde{T}_1 = \{1, 2\}$. The parameters $\beta_1$ and $\beta_2$ have been set up to $0.10$ and $0.18$. (Note that the nodes of period $t = 2$ have only 12 sons each, thus each weight is 0.0833; we then decided to use a greater value for the $\beta$-parameter, in order to penalize not just one scenario per subtree, but two at least). For a better assessment of the impact of the risk-averse term in the objective function, several combinations of the weight parameter $\rho_t$ for $t = 1, 2$ have been considered; in particular, the following ones have been tested: $\rho_1 = \{0.25, 5\}$ and $\rho_2 = \{0, 0.25, 5\}$. Notice that $\rho_2 = 0$ represents the traditional CVaR measure, where the risk management considers the profit along the whole planning horizon and no control is performed at intermediate periods.

Table 8 shows the solution obtained by ECVaR model (5). We can observe the high elapsed time required for solving the problem compared with that required by the TCVaR model (4) for solving instance i1 (see table 4). The conclusion is that this is unacceptable; see in Section 6 an outline of our future research plans on the subject.

Table 9 shows the expected profit, computed VaR and computed CVaR up to the end of periods 1, 2, and 6. As in the previous tables, the last column shows the results for the WS approach, the values being normalized. For the ECVaR measure different results are presented, depending on the values of the

<table>
<thead>
<tr>
<th>Inst.</th>
<th>$\rho_1$</th>
<th>$\rho_2$</th>
<th>$Z_{MIP}$</th>
<th>$Z_{MIP}$</th>
<th>GAP</th>
<th>$t_0$ (secs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>i1</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>2444.42</td>
<td>2396.36</td>
<td>1.96</td>
<td>35191.80</td>
</tr>
<tr>
<td>i1</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{4}$</td>
<td>2875.61</td>
<td>2819.26</td>
<td>1.96</td>
<td>1127.48</td>
</tr>
<tr>
<td>i1</td>
<td>$\frac{1}{2}$</td>
<td>5</td>
<td>1114.98</td>
<td>1093.40</td>
<td>1.93</td>
<td>14000.09</td>
</tr>
<tr>
<td>i1</td>
<td>5</td>
<td>0</td>
<td>9321.70</td>
<td>9139.42</td>
<td>1.95</td>
<td>80663.08</td>
</tr>
<tr>
<td>i1</td>
<td>5</td>
<td>$\frac{1}{2}$</td>
<td>9754.30</td>
<td>9563.34</td>
<td>1.96</td>
<td>1223.46</td>
</tr>
<tr>
<td>i1</td>
<td>5</td>
<td>5</td>
<td>18000.40</td>
<td>17651.02</td>
<td>1.99</td>
<td>1186.97</td>
</tr>
<tr>
<td>i2</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>1419.94</td>
<td>1383.30</td>
<td>1.96</td>
<td>11198.13</td>
</tr>
<tr>
<td>i2</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{4}$</td>
<td>3025.71</td>
<td>2966.38</td>
<td>1.96</td>
<td>18626.89</td>
</tr>
<tr>
<td>i2</td>
<td>$\frac{1}{2}$</td>
<td>5</td>
<td>3576.17</td>
<td>3506.06</td>
<td>1.96</td>
<td>17930.90</td>
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<tr>
<td>i2</td>
<td>5</td>
<td>0</td>
<td>12259.96</td>
<td>12025.93</td>
<td>1.91</td>
<td>16702.38</td>
</tr>
<tr>
<td>i2</td>
<td>5</td>
<td>$\frac{1}{2}$</td>
<td>12814.18</td>
<td>12565.30</td>
<td>1.94</td>
<td>960.37</td>
</tr>
<tr>
<td>i2</td>
<td>5</td>
<td>5</td>
<td>23263.51</td>
<td>22859.61</td>
<td>1.73</td>
<td>2732.10</td>
</tr>
<tr>
<td>i3</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>2501.44</td>
<td>2452.40</td>
<td>1.96</td>
<td>66026.98</td>
</tr>
<tr>
<td>i3</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{4}$</td>
<td>2967.05</td>
<td>2882.59</td>
<td>2.85</td>
<td>&gt; 24h*</td>
</tr>
<tr>
<td>i3</td>
<td>$\frac{1}{2}$</td>
<td>5</td>
<td>11367.54</td>
<td>11185.94</td>
<td>1.60</td>
<td>38454.60</td>
</tr>
<tr>
<td>i3</td>
<td>5</td>
<td>0</td>
<td>9655.31</td>
<td>9471.58</td>
<td>1.90</td>
<td>19253.64</td>
</tr>
<tr>
<td>i3</td>
<td>5</td>
<td>$\frac{1}{2}$</td>
<td>10084.57</td>
<td>9892.14</td>
<td>1.90</td>
<td>24718.45</td>
</tr>
<tr>
<td>i3</td>
<td>5</td>
<td>5</td>
<td>18462.22</td>
<td>18113.08</td>
<td>1.89</td>
<td>43353.94</td>
</tr>
</tbody>
</table>

* limit of the elapsed time reached (24 hours)
pair \((\rho_1, \rho_2)\) of the weight parameters that have been used. Again, as may be observed, at the end of the planning horizon (period 6), all of the stochastic models practically provide the same expected profit, while VaR and CVaR are slightly higher in the ECVaR model than in RN. However, at the end of periods 1 and 2, the ECVaR measure provides an expected profit much higher than that provided by the RN measure; observe that the expected profit for ECVaR in period 1 is 40-45% greater than for RN. In period 2, the results are also better, especially for the instance i1-5, where the weight for risk management has been set up to 5. As in TCVaR (see table 5), it can be concluded that the risk-averse ECVaR measure moves the profits to the early stages, but at the end of the planning period the profits are similar to those obtained by the RN measure. Observe in boxplot that the conclusion is even stronger, the profit distributions in ECVaR for periods 1 and 2 are much better than those provided by RN, being almost equal at the end of the planning horizon.

Tables 10 and 11 show the results for instances i2 and i3, res. The conclusions are very similar to the ones obtained for instance i1.
Table 10: EC VaR: Results and profit distribution for instance i2

<table>
<thead>
<tr>
<th>t</th>
<th>Stat.</th>
<th>RN</th>
<th>$\rho_1^t$</th>
<th>$\rho_2^t$</th>
<th>$\rho_1^t$</th>
<th>$\rho_2^t$</th>
<th>$\rho_1^t$</th>
<th>$\rho_2^t$</th>
<th>$\rho_1^t$</th>
<th>$\rho_2^t$</th>
<th>$\rho_1^t$</th>
<th>$\rho_2^t$</th>
<th>WS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Expect.</td>
<td>33.9</td>
<td>41.7</td>
<td>41.7</td>
<td>34.8</td>
<td>41.8</td>
<td>41.8</td>
<td>41.4</td>
<td>38.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Expect.</td>
<td>51.3</td>
<td>56.6</td>
<td>56.7</td>
<td>56.9</td>
<td>58.5</td>
<td>58.6</td>
<td>59.2</td>
<td>56.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>c-VaR</td>
<td>47.3</td>
<td>53.5</td>
<td>53.5</td>
<td>51.6</td>
<td>55.7</td>
<td>56.1</td>
<td>55.5</td>
<td>43.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Expect.</td>
<td>96.5</td>
<td>96.0</td>
<td>96.0</td>
<td>95.7</td>
<td>95.6</td>
<td>95.7</td>
<td>95.9</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>c-VaR</td>
<td>75.4</td>
<td>76.8</td>
<td>76.8</td>
<td>75.9</td>
<td>77.4</td>
<td>77.4</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>c-CVaR</td>
<td>71.3</td>
<td>73.1</td>
<td>73.2</td>
<td>72.5</td>
<td>73.9</td>
<td>73.6</td>
<td>73.6</td>
<td>77.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 11: EC VaR: Results and profit distribution for instance i3
5.3.3. Comparing ECVaR and TCVaR models with a mixture of both

A computational comparison is performed between the measures TCVaR (4), ECVaR (5) and a mixture of both, so-called MCVaR. (Notice that CVaR (3) is the particular case of TCVaR for $\tilde{T} = \{6\}$). The MCVaR approach combines both models, such that the risk management with the related profiles is carried out by using the ECVaR measure in periods 2 and the TCVaR measure in periods 3 and 6; that is, MCVaR performs a risk management at period 6 but individualized for each group of scenarios in period 2 and, simultaneously, a risk management for the whole set of scenarios up to periods 3 and 6.

Table 12 shows the solution obtained by MCVaR as a mixture of models (4) and (5). It is not surprising that a high computing time is required, given the difficulty of each of those models.

Fig. 9, 10 and 11 show the boxplot of the profit distribution obtained for instances i1, i2 and i3, res., by the measures RN (2), CVaR (3), ECVaR (5), MCVaR and TCVaR (4). In all instances, the weight $\rho$-parameters of the risk term in the objective function, has been set to 0.25. The main conclusions are as follows:

- All the stochastic models provide similar expected profit at the end of the planning horizon ($t = 6$). The profit distribution is also very similar for the four models, although the risk-averse measures have better (i.e. smaller) lower tails for instance i3.

- The risk-averse measures provide better profit than the risk neutral strategy for periods 1 and 2.

- For period 1, the measures MCVaR and TCVaR give a higher profit than the other two strategies, mainly in instances i2 and i3. Observe the risk-averse performance in the profit obtained up to period 3 (i.e., an intermediate one in the time horizon).

- For period 2, the measure MCVaR is the champion in instances i2 and i3.

6. Conclusions

In this work we have formulated and solved a multiperiod stochastic mixed 0-1 forestry model for planning forest harvesting and road building. We consider uncertainties in product demand and prices along the planning horizon, by analyzing of a finite set of discrete scenarios, contrary to the traditional approach that uses average values for uncertain parameters. The main contribution of the work has been to formulate risk management on the solution of the risk neutral (RN) strategy, by including two versions of
Figure 9: MCVaR: Profit distribution for instance i1

Figure 10: MCVaR: Profit distribution for instance i2

Figure 11: MCVaR: Profit distribution for instance i3
the very popular Condition Value-at-Risk (CVaR) measure in the stochastic model we had presented earlier (a tighter version is presented in Appendix A). One version of the risk averse measure is the time consistent Expected CVaR (ECVaR). It allows the risk reduction on the expected values of the given function (in our case, the harvesting profit) to be personalized for the whole time horizon in the groups of scenarios whose 1-to-1 corresponding nodes in the scenario tree belong to a modeler-driven period subset. The other version is the time inconsistent Time CVaR (TCVaR). It allows risk reduction to be performed on the expected values of the given function up to a modeler-driven period subset of the time horizon in the whole set of scenarios. The latter type of risk averse measure allows risk management to be performed at intermediate periods, a very useful additional tool for a decision-maker who needs to plan for a long time horizon. We have analyzed the performance of both risk averse measures compared with the traditional EV and RN strategies, by comparing the solutions obtained for a set of large real-life instances of the forest harvesting planning problem. We have observed that the main advantage of the risk averse measures is that, with a very small deterioration in the expected profit at the end of the planning horizon, they advance the profit to early periods. That is, both strategies provide higher profits at early periods than the EV and RN strategies, in addition to reducing the variability of the profits for unwanted scenarios (i.e., low-probability scenarios with a profit in unwanted quantiles).

An interesting advantage of TCVaR over ECVaR is that the former can be used in all periods (except for the first one, of course). On the contrary, by construction, the periods to be used for building the groups of scenarios for ECVaR can only be chosen among those that have a 1-to-1 correspondence with the nodes in the scenario tree that are roots of the related subtrees. Observe that in the instances we have experimented with, risk reduction can be performed for the periods $t=2,3,4,5,6$ by using TCVaR, while it can only be performed for periods 1 and 2 when using ECVaR. Notice that, by construction, TCVaR for $t = 6$ is the same as ECVaR for $t = 1$. At any rate, our provisional conclusion is that the mixture MCVaR takes advantage of both risk-averse measures. Its obvious drawback is the computing time. See below our plan to address this.

On the other hand, as expected given the tightness of the RN model (2), the computing time that is required by plain use of the state-of-the-art MIP solver CPLEX is very reasonable (up to 4 hours, approx) for the HW/SW platform that we have used for such large-sized instances (up to 250,000 constraints, 400,000 continuous variables and 36,000 0-1 variables). (Notice the WS bound and EV strategy required up to 123 and 11 seconds, res.). The measures TCVaR (4), ECVaR (5) and the mixture MCVaR, however, required a much higher computing effort (very frequently reaching the time limit of 24 hours, even though the optimality gap was not greater than 2%).

In our future research plan, however, we will develop a decomposition methodology for dealing with the risk averse mixture MCVaR. Notice that the model requires groups of cross scenario constraints (as many groups as the number of nodes in the total chosen period subset) to be considered, such that the nice structure of the scenario tree based constraints is destroyed. And, so, typical decomposition algorithms cannot be used with an affordable computational effort for solving large sized problems. Hence, given the large dimensions of the instances and the problems’ complexity, it is unrealistic to seek for an optimal solution.
Our research effort in developing suitable decomposition algorithms is twofold: On one hand, our effort will be concentrated on cluster Lagrangean decomposition and relaxation algorithms for providing strong upper bounds on the solution value of the problem and a type of Lagrangean heuristic for obtaining (hopefully, good) feasible solutions with guaranteed goodness gap for the TCVaR part of the mixture measure. On the other hand, given the structure of the scenario tree and that the groups belong to a modeler-driven period subset, the ECVaR part of the mixture measure has a suitable structure to be exploited. The exploitation could be performed by Stochastic Nested Decomposition algorithms, based on the reasons given in this work, to also obtain good feasible solutions.

Appendix A Deterministic model

Notation

The parameters and variables are denoted with capital and small letters, res.

Sets

- \( T = \{1, \ldots, T\} \), set of time periods (summer and winter seasons).
- \( T^S, T^W \), set of summer and winter time periods, res. \( T = T^S \cup T^W, T^S \cap T^W = \emptyset \).
- \( Q = \{q_1, q_2, \ldots, q_K\} \), harvest product. It can be considered an ordered set, such that product \( q \) has higher quality than product \( q' \) provided that \( q < q' \). A higher-level quality can be used for lower level purposes, at a loss in sale price.
- \( C \), set of stocking yard.
- \( I^O \), set of origin nodes.
- \( I^I \), set of intermediate nodes. Each intermediate node represents a junction in the roads network. Note that an origin node can also be located in a junction, but it has some stands associated.
- \( I^F \), set of final destination nodes. Each final destination node is directly connected to the markets.
- \( I \), set of nodes in the road network, \( I = I^O \cup I^I \cup I^F \cup C \).
- \( S \), set of stands.
- \( S_i \), set of stands associated with origin node \( i \), \( \forall i \in I^O \), \( S_i \subseteq S \).
- \( M \), set of markets.
- \( M_i \), set of markets served from node \( i \) (where \( i \) is a final destination or stocking yard), \( \forall i \in I^F \cup C \). Note: \( M_i \subseteq M \).
- \( L \), set of links (potential or existing) in the road network. A link is an edge linking two consecutive nodes in the road network.
• \( \mathcal{R} \), road standards, where \( \{d, g\} \in \mathcal{R} \), \( d \) being for dirt and \( g \) for gravel.

• \( \mathcal{L}^E, \mathcal{L}^P \), set of existing and potential links, res.

• \( \mathcal{L}^{Ed}, \mathcal{L}^{Eg} \), set of existing links in dirt and gravel, res. Note 1: All public roads exist from the beginning of the time horizon and they are in gravel. Note 2: Existing roads in dirt (i.e., private roads) can be upgraded to gravel. Note 3: Existing or potential dirt links cannot be used in winter, so, they should be upgraded to gravel in case they are to be used.

• \( \Gamma(i) \), adjacency set of node \( i, i \in \mathcal{I} \). Note: \( j \in \Gamma(i) \iff \{i, j\} \in \mathcal{I} \iff i \in \Gamma(j) \).

**Constraint-related parameters**

• \( B_{qs}^t \), amount of timber (\( \text{m}^3 \)) of quality \( q \) produced per hectare in stand \( s \) if harvested in period \( t \), (this parameter is determined through a growth simulator), \( \forall s \in \mathcal{S}, q \in Q, t \in \mathcal{T} \).

• \( \bar{A}_s \), upper bound in the area (hectares) of stand \( s \) that can be harvested, \( \forall s \in \mathcal{S} \).

• \( \underline{A}_s \), lower bound in the area (hectares) of stand \( s \) to be harvested in any time period, if any, \( \forall s \in \mathcal{S} \).

• \( N_s \), maximum number of periods that stand \( s \) can be harvested. Note 1: It depends on \( \bar{A}_s \), such that combined with \( \underline{A}_s \) it tries to concentrate the harvesting of a stand in a reasonable number of time periods with at least a minimum area to be harvested. Note 2: A possible value for \( N_s \) could be \( \left\lceil \frac{\bar{A}_s}{\underline{A}_s} \right\rceil \), \( \forall s \in \mathcal{S} \).

• \( U_{ijr}^t \), flow capacity (\( \text{m}^3 \)) on link \( \{i, j\} \) built in standard \( r \) available in period \( t \), \( \forall \{i, j\} \in \mathcal{L}^P, r \in \mathcal{R}, t \in \mathcal{T} \). Note: The flow in a link can be in both directions.

• \( \overline{U}_{ijr}^t \), flow capacity (\( \text{m}^3 \)) on existing link \( \{i, j\} \) at the beginning of the time horizon built in standard \( r \) available in period \( t \), \( \forall \{i, j\} \in \mathcal{L}^P, r \in \mathcal{R}, t \in \mathcal{T} \).

• \( Z_{km}^t, \overline{Z}_{km}^t \), lower and upper bound of demand (\( \text{m}^3 \)) of product \( k \) at destination \( m \) in period \( t \), \( \forall q \in Q, m \in \mathcal{M}, t \in \mathcal{T} \).

• \( C_c \), capacity (\( \text{m}^3 \)) of stocking yard \( c, \forall c \in \mathcal{C} \).

• \( \tau_d \), latency (i.e., number of periods) required for making available a potential link in dirt since the time period one decided to build it. Note: If a link is available in any of the first \( \tau_d - 1 \) periods, then it is assumed that the decision to build it is made before the beginning of the time horizon.

• \( \tau_g \), latency (i.e., number of periods) required for making available a potential link in gravel (or for upgrading it from dirt to gravel) since the time period it is built. Note: If a link is available in any of the first \( \tau_g - 1 \) periods, then it is assumed that the decision to build it was made before the beginning of the time horizon.
• Note: It is assumed that the latency is the same for all potential links along the time horizon. Observe that one distinguishes between the period when the decision was made to build a link and the period when the link becomes available.

**Objective function parameters**

- \( R_{qt}^m \), unit selling price of quality timber \( q \) in market \( m \) in period \( t \), \( \forall q \in Q, m \in \mathcal{M}, t \in \mathcal{T} \).
- \( S_{qt}^m \), unit penalization cost for unmet demand of quality timber \( q \) in market \( m \) in period \( t \), \( \forall q \in Q, m \in \mathcal{M}, t \in \mathcal{T} \). Note: \( S_{qt}^m \gg R_{qt}^m \)
- \( P_t^s \), unit harvesting cost per Ha. in stand \( s \) in period \( t \), \( \forall s \in S, m \in \mathcal{M}, t \in \mathcal{T} \).
- \( P_{qt}^i \), unit production cost per m\(^3\) of quality timber \( q \) in node \( i \) in period \( t \), \( \forall i \in \mathcal{I}_O, q \in Q, t \in \mathcal{T} \).
- \( D_{qt}^{ir} \), unit transportation cost of quality timber \( q \) through link \( \{i, j\} \) in standard \( r \) in period \( t \), \( \forall \{i, j\} \in \mathcal{L}, r \in \mathcal{R}, q \in Q, t \in \mathcal{T} \).
- \( D_{qm}^{it} \), unit transportation cost of quality timber \( q \) from node \( i \) to market \( m \) in period \( t \), \( \forall i \in \mathcal{I}_F \cup \mathcal{C}, m \in \mathcal{M}_i, q \in Q, t \in \mathcal{T} \).
- \( H_{ijr}^t \), cost of building link \( \{i, j\} \) in standard \( r \), \( \forall \{i, j\} \in \mathcal{L}_P, r \in \mathcal{R} \).
- \( \tilde{H}_{ij}^t \), cost of upgrading link \( \{i, j\} \) from standard dirt to gravel in period \( t \), \( \forall \{i, j\} \in \mathcal{L}_P \cup \mathcal{L}_{Ed}, t \in \mathcal{T} \).
- \( \hat{H}_c^t \), unit stocking cost in yard \( c \) in period \( t \), \( \forall c \in \mathcal{C}, t \in \mathcal{T} \).

**Binary variables**

- \( w_{ijr}^t = 1 \) if link \( \{i, j\} \) is built in standard \( r \) by time period \( t \) (it can be used \( \tau_r \) time periods later), and otherwise, 0, \( \forall \{i, j\} \in \mathcal{L}_P, t \in \mathcal{T}, r \in \mathcal{R} \). Notice that \( w_{ijr}^t \) is a so-called step variable that makes the model stronger than when using the impulse variables, see e.g., [20] for forest harvesting as well as in other planning problems, see [1] and others.
- \( v_{ij}^t = 1 \) if link \( \{i, j\} \) is upgraded from dirt to gravel by time period \( t \) (the upgrade is valid \( \tau_u \) time periods later, and the link cannot be upgraded in the same time period it is built), and otherwise, 0, \( \forall \{i, j\} \in \mathcal{L}_P, t \in \mathcal{T} \).
- \( e_s^t = 1 \), if stand \( s \) is harvested at period \( t \), and otherwise, 0, \( \forall s \in \mathcal{S}, t \in \mathcal{T} \). Notice that, by construction, it is a so-called impulse variable.

**Continuous variables**

- \( x_s^t \), area (i.e., number of hectares) of stand \( s \) harvested in period \( t \), \( \forall s \in \mathcal{S}, t \in \mathcal{T} \).
- \( y_{qt}^{i}\), volume (m\(^3\)) of timber of quality \( q \) harvested in all stands associated with origin \( i \) during period \( t \), \( \forall i \in \mathcal{I}_O, q \in Q, t \in \mathcal{T} \).
• $f_{ijr}^{qt}$, flow (m$^3$) of timber of quality $q$ transported on link $\{i, j\}$ built in standard $r$ in period $t$, $\forall \{i, j\} \in \mathcal{L}, r \in \mathcal{R}, q \in \mathcal{Q}, t \in \mathcal{T}$. Note: $f_{ijr}^{qt} = 0$ for all existing links in dirt ($\{i, j\} \in \mathcal{L}^E$ and $r = d$) and for all potential links in dirt in winter ($\{i, j\} \in \mathcal{L}^P, r = d$ and $t \in \mathcal{T}^W$).

• $f_{icr}^{qt}$, flow (m$^3$) of timber of quality $q$ transported from node $i$ on link $\{i, c\}$ in standard $r$ to its adjacent $c$ in period $t$, $\forall i \in \Gamma(c), c \in \mathcal{C}, r \in \mathcal{R}, q \in \mathcal{Q}, t \in \mathcal{T}$.

• $f_{im}^{qt}$, flow (m$^3$) of timber of quality $q$ transported from node $i$ (i.e., a final destination or a stoking yard) to market $m$ at period $t$, $\forall i \in \mathcal{I}_F \cup \mathcal{C}, q \in \mathcal{Q}, m \in \mathcal{M}_i, t \in \mathcal{T}$.

• $z_{mt}^{qt}$, amount (m$^3$) of timber delivered as quality $q$ to destination $m$ in period $t$, $\forall q \in \mathcal{Q}, m \in \mathcal{M}, t \in \mathcal{T}$. Note: The timber delivered to the market at the price of quality $q$ can actually be (in part or totally) of a higher quality.

• $z_{mt}^{-qt}$, unmet timber demand (m$^3$) of quality $q$ requested at destination $m$ in period $t$, $\forall q \in \mathcal{Q}, m \in \mathcal{M}, t \in \mathcal{T}$.

Constraints

1. Road network design

   (a) Decisions about new links or upgrades to gravel can be taken only in summer (since is the only period for work-load)

   $$w_{ijr}^{t-1} \leq w_{ijr}^t, \quad \forall \{i, j\} \in \mathcal{L}^P, t \in \mathcal{T}^S$$

   (6a)

   $$w_{ijr}^t = w_{ijr}^{t-1}, \quad \forall \{i, j\} \in \mathcal{L}^P, t \in \mathcal{T}^W$$

   (6b)

   $$v_{ij}^t \leq v_{ij}^{t-1}, \quad \forall \{i, j\} \in \mathcal{L}^P \cup \mathcal{L}^d, t \in \mathcal{T}^S$$

   (6c)

   $$v_{ij}^{t-1} = v_{ij}^t, \quad \forall \{i, j\} \in \mathcal{L}^P, t \cup \mathcal{L}^d \in \mathcal{T}^W$$

   (6d)

   (b) A link cannot be upgraded from dirt to gravel if it has not been built two periods (i.e., one year) earlier, at least

   $$v_{ij}^t \leq w_{ijd}^{t-2}, \quad \forall \{i, j\} \in \mathcal{L}^P, t \in \mathcal{T}$$

   (6e)

   (c) Road incompatibility: A link cannot simultaneously be built in dirt and in gravel

   $$\sum_{r \in \mathcal{R}} w_{ijr}^t \leq 1, \quad \forall \{i, j\} \in \mathcal{L}^P, t \in \mathcal{T}^W$$

   (6f)

   Note: A link built in dirt can be upgraded to gravel, but cannot be built in gravel.

   (d) Origins-to-roads triggers: If a stand is not connected to a existing link at the beginning of the time horizon and it is harvested, then one potential link has to be built, at least. Furthermore, if a stand is not connected to a existing link and it is harvested in winter, then one potential link has to be built in gravel, at least, by this time period. So, the following constraint can be
added for those stands in order to tighten the model, see [4] for more details. For origins that are not connected to any existing link at the beginning of the time horizon, then add the following constraint system \( \{ i, j \} \in L^E : j \in \Gamma(i) \} = \emptyset \forall i \in \mathcal{I}^O, \)

\[ \sum_{\nu \in T^W : \nu \leq t} e^\nu_s \leq \min\{ t, N_s \} \sum_{\{ i, j \} \in L^P} \sum_{r \in R} w_{i,j,r}^t, \quad \forall s \in \mathcal{S}, t \in \mathcal{T} \quad (6g) \]

\[ \sum_{\nu \in T^W : \nu \leq t} e^\nu_s \leq \min\{ N^W_t, N_s \} \sum_{\{ i, j \} \in L^P} \left( w_{i,j,r}^t + v_{i,j}^t \right), \quad \forall s \in \mathcal{S}_i \cup \mathcal{S}_j, t \in \mathcal{T}^W, \quad (6h) \]

where \( N^W_t = |\{ t' \in \mathcal{T}^W : t' \leq t \}|. \)

(e) Road-to-road triggers [4, 20]: If a potential link \( \{ i, j \} \) that is not connected to an existing link is built, then, at least one of the link connecting \( \{ i, j \} \) must be built, in order to avoid disconnecting the link network:

\[ \sum_{r \in R} w_{i,j,r}^t \leq \sum_{\{ i', j' \} \in L^{(i,j)}} \sum_{r \in R} w_{i',j',r}^t, \quad \forall \{ i, j \} \in L^P : L^{(i,j)} \cap L^E = \emptyset, t \in \mathcal{T} \quad (6i) \]

where \( L^{(i,j)} \) is the set of links adjacent to \( \{ i, j \} \).

2. Harvesting decisions

(a) Bounds in the harvested area per stand and time period

\[ A_s e^t_s \leq x^t_s \leq \bar{A}_s e^t_s, \quad \forall s \in \mathcal{S}, t \in \mathcal{T} \quad (7a) \]

(b) Maximum area harvested per stand

\[ \sum_{t \in \mathcal{T}} x^t_s \leq \bar{A}_s, \quad \forall s \in \mathcal{S} \quad (7b) \]

(c) Maximum number of periods that a stand can be harvested

\[ \sum_{t \in \mathcal{T}} e^t_s \leq N_s \quad \forall s \in \mathcal{S} \quad (7c) \]

Notice that (7c) is redundant for the 0-1 model but it is tightening its LP relaxation.

3. Production by origin

\[ \sum_{s \in \mathcal{S}_i} B_s^q x^t_s = y^q_t, \quad \forall i \in \mathcal{I}^O, q \in \mathcal{Q}, t \in \mathcal{T} \quad (8) \]

4. Flow constraints at nodes
(a) For origin nodes
\[ y_i^q = \sum_{r \in R} \sum_{j \in \Gamma(i)} f^q_{rj} = \sum_{r \in R} \sum_{j \in \Gamma(i)} f_{rj}^q, \quad \forall i \in I^O, q \in Q, t \in T \] (9a)

(b) For intermediate nodes
\[ \sum_{r \in R} \sum_{j \in \Gamma(i)} f^q_{rj} = \sum_{r \in R} \sum_{j \in \Gamma(i)} f_{rj}^q, \quad \forall i \in I^I, q \in Q, t \in T \] (9b)

(c) For final destination nodes
\[ \sum_{r \in R} \sum_{j \in \Gamma(i)} f^q_{rj} = \sum_{r \in R} \sum_{j \in \Gamma(i)} f_{rj}^q + \sum_{m \in M} f^q_{im}, \quad \forall i \in I^F, q \in Q, t \in T \] (9c)

(d) For stocking yards
   i. Arrivals in summer must be equal to dispatches in winter
   \[ \sum_{r \in R, i \in \Gamma(c)} f^q_{ir} = \sum_{m \in M} f^{q+1}_{cm}, \quad \forall c \in C, q \in Q, t \in T^S \] (9d)
   ii. There are neither arrivals in winter nor dispatches in summer
   \[ f^q_{ir} = 0, \quad \forall r \in R, c \in C, i \in \Gamma(c), q \in Q, t \in T^W, \] (9e)
   \[ f^{q+1}_{cm} = 0, \quad \forall q \in Q, c \in C, m \in M_c, t \in T^S \] (9f)

5. Demand constraints

(a) The amount of timber delivered to a market as quality \( q \) (which, if needed, may actually be of higher quality, as stated above, at a loss) is bounded by the flow arriving to the nodes that can served that market.
\[ \sum_{q' \in Q: q' \leq q} z^{q'}_m \leq \sum_{q' \in Q: q' \leq q} \sum_{i \in I^F \cup C} f^{q'}_{im}, \quad \forall q \in Q, m \in M, t \in T \] (10a)

(b) Demand bounds
\[ Z^q_m \leq z^{q-1}_m \leq \overline{z}^q_m, \quad \forall q \in Q, m \in M, t \in T \] (10b)

6. Capacity constraints

(a) Capacity at stocking yards (in summer)
\[ \sum_{q \in Q} \sum_{r \in R} \sum_{i \in \Gamma(c)} f^q_{ir} \leq C_c, \quad \forall c \in C, t \in T^S \] (11a)
(b) Flow capacity on roads built in dirt

Flow through a potential link in dirt (i.e., \( r = d \)) is only possible in any period if the link has been previously built in dirt and not upgraded to gravel.

\[
\sum_{q \in Q} (f_{ijd}^{qt} + f_{jid}^{qt}) \leq U_{ijd}^t (w_{ijd}^{t-\tau_d} - v_{ij}) \quad \forall \{i, j\} \in \mathcal{L}^P, t \in \mathcal{T}^S
\]  

(11b)

If there is any flow (on dirt) between the beginning of the time horizon and period \( t \), the link must have been built by time period \( t \)

\[
\sum_{t' \in T : t' \leq t} \sum_{q \in Q} (f_{ijd}^{qt} + f_{jid}^{qt}) \leq U_{ijd}^t \quad \forall \{i, j\} \in \mathcal{L}^P, t \in \mathcal{T}^S, r = d (\text{dirt})
\]

(11c)

Even more, if there is some flow (on dirt) during or after time period \( t \), the link cannot have been upgraded by time period \( t \)

\[
\sum_{t' \in T : t' \leq t} \sum_{q \in Q} (f_{ijd}^{qt} + f_{jid}^{qt}) \leq U_{ijd}^t (1 - v_{ij}) \quad \forall \{i, j\} \in \mathcal{L}^P, t \in \mathcal{T}^S
\]

(11d)

The dirt roads are only available in summer, so, no flow is possible on them in winter

\[
f_{ijd}^{qt} + f_{jid}^{qt} = 0, \quad \forall q \in Q, \{i, j\} \in \mathcal{L}^P, t \in \mathcal{T}^W
\]

(11e)

(c) Flow capacity on roads built in gravel or upgraded to gravel

Flow through a potential link in gravel is only possible at any time period if the link has been built in gravel or upgraded from dirt to gravel by that period

\[
\sum_{t' \in T : t' \leq t} \sum_{q \in Q} (f_{ijg}^{qt} + f_{jig}^{qt}) \leq U_{ijg}^t (w_{ijg}^{t-\tau_g} + v_{ij}) \quad \forall \{i, j\} \in \mathcal{L}^P, t \in \mathcal{T}^S
\]

(11f)

(d) Flow capacity of roads (links) that exist from the beginning of the time horizon

For such links in dirt, flow is only allowed provided they have not been upgraded to gravel.

\[
\sum_{t' \in T : t' \leq t} \sum_{q \in Q} (f_{ijd}^{qt} + f_{jid}^{qt}) \leq U_{ijd}^t (1 - v_{ij}) \quad \forall \{i, j\} \in \mathcal{L}^{Ed}, t \in \mathcal{T}^S
\]

(11g)

For an existing link in dirt, flow on gravel is allowed only if it has been upgraded

\[
\sum_{t' \in T : t' \leq t} \sum_{q \in Q} (f_{ijg}^{qt} + f_{jig}^{qt}) \leq U_{ijg}^t v_{ij} \quad \forall \{i, j\} \in \mathcal{L}^{Ed}, t \in \mathcal{T}^S
\]

(11h)

For existing links in gravel, flow is upper bounded

\[
\sum_{q \in Q} (f_{ijg}^{qt} + f_{jig}^{qt}) \leq U_{ijg}^t \quad \forall \{i, j\} \in \mathcal{L}^{Ed}, t \in \mathcal{T}
\]

(11i)
7. Nature of variables

\[ w_{ijr}^t \in \{0, 1\} \quad \forall \{i, j\} \in \mathcal{L}^P, t \in \mathcal{T}, r \in \mathcal{R} \]
\[ v_{ij}^t \in \{0, 1\} \quad \forall \{i, j\} \in \mathcal{L}^P, t \in \mathcal{T} \]
\[ e_s^t \in \{0, 1\} \quad \forall s \in \mathcal{S}, t \in \mathcal{T} \]
\[ x_s^t \geq 0 \quad \forall s \in \mathcal{S}, t \in \mathcal{T} \]
\[ y_{i}^{qt} \geq 0 \quad \forall i \in \mathcal{I}^O, q \in \mathcal{Q}, t \in \mathcal{T} \]
\[ f_{ijr}^{qt} \geq 0 \quad \forall \{i, j\} \in \mathcal{L}, r \in \mathcal{R}, q \in \mathcal{Q}, t \in \mathcal{T} \]
\[ f_{im}^{qt} \geq 0 \quad \forall q \in \mathcal{Q}, i \in \mathcal{I}^F \cup \mathcal{C}, m \in \mathcal{M}_i, t \in \mathcal{T} \]
\[ z_{m}^{qt} \geq 0 \quad \forall q \in \mathcal{Q}, m \in \mathcal{M}, t \in \mathcal{T} \]
\[ z_{m}^{-qt} \geq 0 \quad \forall q \in \mathcal{Q}, m \in \mathcal{M}, t \in \mathcal{T} \]

**Objective Function**

A typical objective function consists of maximizing the net present value of the profit, i.e. income minus cost. It can be expressed as follows,

\[
\begin{align*}
\text{max} & \quad \sum_{q \in \mathcal{Q}} \sum_{m \in \mathcal{M}} \sum_{t \in \mathcal{T}} P_{ijr}^{qt} z_{ijr}^{qt} - \\
& \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}} P_s x_s^t - \\
& \sum_{i \in \mathcal{I}^O} \sum_{q \in \mathcal{Q}} \sum_{t \in \mathcal{T}} T_i^t y_{i}^{qt} - \\
& \sum_{\{i, j\} \in \mathcal{L}} \sum_{r \in \mathcal{R}} \sum_{q \in \mathcal{Q}} \sum_{t \in \mathcal{T}} D_{ijr}^{qt} f_{ijr}^{qt} - \\
& \sum_{i \in \mathcal{I}^O \cup \mathcal{C}} \sum_{m \in \mathcal{M}_i} \sum_{q \in \mathcal{Q}} \sum_{t \in \mathcal{T}} D_{im}^{qt} f_{im}^{qt} - \\
& \sum_{\{i, j\} \in \mathcal{L}^P} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} H_{ijr}^t w_{ijr}^t - \\
& \sum_{\{i, j\} \in \mathcal{L}^P \cup \mathcal{L}^D} \sum_{t \in \mathcal{T}} H_{ijr}^t \bar{w}_{ij}^t - \\
& \sum_{q \in \mathcal{Q}} \sum_{m \in \mathcal{M}} \sum_{t \in \mathcal{T}} S_{m}^{qt} z_{m}^{-qt} - \\
& \sum_{e \in \mathcal{E}} \sum_{q \in \mathcal{Q}} \sum_{t \in \mathcal{T}} H_{e}^t \sum_{r \in \mathcal{R}} \sum_{i \in \mathcal{I}(e)} f_{icr}^{qt}
\end{align*}
\]

income

harvesting cost (13a)

production cost (13b)

transportation cost (13c)

sending to markets cost (13d)

link-building cost (13e)

link-upgrading cost (13f)

shortfall cost (13g)

stocking cost (13h)
References


