Co-optimization of Demand Response and Reserve Offers for a Major Consumer

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***Abstract*—In this paper we present a stochastic optimization problem for a strategic major consumer who has flexibility over its consumption and can offer reserve. Our model is a bi-level optimization model (reformulated as a mixed-integer program) that embeds the optimal power flow problem, in which electricity and reserve are co-optimized. We implement this model for a large consumer of electricity in the New Zealand Electricity Market (NZEM). To reduce the solution time of the large mixed integer program, we explore the specific properties of the optimal power flow to reformulate the model. We show that by adding further constraints to tighten the LP relaxation we manage to improve the performance of the model.**

***Index Terms*—Demand-side management, mathematical programming, integer programming.**

I. Introduction

Electricity markets have improved short and long run efficiency in the electricity sector. While there has been a large amount of research undertaken on optimization of the generation side [1]–[6] as well as improving market regulations [7]–[10], there has been less research on demand response. This is partly because short term demand for electricity was perceived as inelastic. With the introduction of disruptive technologies (e.g. solar PVs and battery storage), there has recently been more work on household demand response [11]–[14].

Since the early 1980s, a number of authors have discussed demand response in restricted formats, e.g. [15]–[17] and have examined their potential and limitations [18], [19]. Chao, in [20] and [21], discusses efficient pricing of demand response based on an estimate of a customer's "baseline", where the baseline is the counterfactual consumption in absence of demand response. Recently Ferris and Liu [22] have extended this notion and developed a model for the Independent System Operator (ISO) to control demand response efficiently.

The question we tackle in this paper is different from the perspectives of the cited papers. We construct a comprehensive demand response model for a large consumer, over a single trading period, with the following attributes:

- We consider a large consumer who is not only capable of reducing consumption, but also of offering interruptible load reserve (ILR) in a co-optimized energy and reserve market.
- The consumer submits a consumption bid curve, as well as a reserve supply function that are incorporated into the ISO's social welfare maximizing dispatch. In the NZEM, the wholesale market clears to optimize the sum of demand and supplier benefits in both energy and reserve markets.
- We start with a deterministic model and extend it to a full-scale stochastic version, and we report the computational results.

Our problem is important because world wide it is increasingly difficult for manufacturers (e.g. steel manufacturers) to compete on price. Many such large manufacturers are major consumers of electricity and they need to make smart decisions on their consumption. Furthermore, there has been a flurry of recent efforts to strive for more efficient electricity markets by utilizing demand response (see e.g. FERC order 745 [23] and the NZEM's dispatchable demand framework [24]). Cleland et al. [25], [26] approach this problem through simulation and optimization over a discrete set of possible offer stacks. Our contribution is to model the problem comprehensively and consider all eligible demand and reserve supply levels.

The paper is laid out as follows. In section II we introduce a demand-side co-optimization bi-level model for a price making, major consumer in a deterministic setting. We reformulate the bi-level problem to a mixed integer program that maximizes the consumer's profit while taking into account the consumer's impact on prices obtained from the dispatch problem. Using this model we conduct an experiment to illustrate the impacts of co-optimization. We then proceed to the more realistic, stochastic version of this model. The major consumer submits an interruptible load reserve (ILR) offer curve and a demand bid curve. Market regulations require that these curves be monotone stepwise functions, which we enforce using monotonicity constraints presented in section IV. In section V we explore alternative MIP reformulations that take advantage of the nature of our problem. In section VI we compare the performance of the reformulations and present our computational results for a large electricity consumer in the NZEM. Section VII concludes the paper.

II. Price-Making Major Consumer

We view our consumer as a strategic, price-maker agent, which is a reasonable model for imperfectly competitive markets (e.g. the NZEM, Singapore, and some European jurisdictions). We start by assuming that this consumer maximizes its utility by consuming a Cournot quantity (y^d), and offering a Cournot quantity (y^r) for ILR. As the consumer is a price maker, the electricity market clearing problem is embedded within the consumer's profit optimization problem, rendering a leader-follower type model captured as a bi-level program. We use the following notation to formulate this model:

- $\mathcal{A}$ is the set of all arcs in the network.
- $\mathcal{N}_e$ is the set of all nodes in island e.
- f_{ij} is the flow between node i and j.
- L is the loop constraint matrix, where $L_{l,ij}$ corresponds to row l (associated with each loop) and the column ij corresponds to arc ij.
- ${\mathcal{T}'}_c^n$ is the set of interruptible consumption tranches.
- K_{ij} is the line capacity in arc ij.
- r_e is the reserve level required in island e.
- $\{N, S\}$ is the set of islands, north and south.
- $\mathcal{N}$ is the set of all nodes in the network. Note that in our model we have one agent per node.
- $\mathcal{Z} = \{c, g, rc, rg\}$ is the index set for tranche types, i.e. consumption, generation, ILR and reserve.
- $\mathcal{T}_z$ is the set of all offered tranches of type z.
- $\mathcal{T}_z^n$ is the set of all tranches of type z at node n.
- x_t^z is the dispatch quantity of a tranche of type z.
- p_t^z is the price associated with the tranche type z.
- q_t^z is the quantity of a tranche of type z.
- r_e is the reserve level required in island e.
- e_n determines the island that node n is located in.
- V_n is the minimum amount of difference between reserve and consumption level, at node n.
- B_n is the proportion of generation allowed to be offered at reserve, at node n.
- W_n is the maximum total amount of generation and reserve of a generator at node n.
- U and F are big-M parameters.
- Variables in [ ] indicate the duals of their associate constraints.

To present a general model, we do not restrict the major consumer to be located at a single node. We define the strategic nodes by allowing the strategic consumption bid y_n^d and y_n^r to be positive. Hence, we set all other non-strategic nodes' C_n^d (maximum consumption) and C_n^r (maximum ILR) to be zero. In addition we define the utility function over consumption of electricity $\mathcal{U}(x) = ux$ where u is a constant. However the results are applicable for any concave utility/costs function $\mathcal{U}(x)$. The objective is to maximize the profit, taking into account revenues obtained through ILR. We lay out the bi-level optimization problem, that we call [PMP], below:

$$\max_{y^d, y^r} \quad \sum_{n \in \mathcal{N}} \left(u y_n^d - \pi_n^d y_n^d + \pi_{e_n}^r y_n^r \right)$$

$$\text{s.t.} \quad 0 \le y_n^d \le C_n^d \tag{II.1}$$

$$0 \le y_n^r \le C_n^r \tag{II.2}$$

$$y_n^d - y_n^r \ge V_n \tag{II.3}$$

$$\max_{x^{z|z\in\mathcal{Z}}} \sum_{t_c \in \mathcal{T}_c} p_{t_c}^c x_{t_c}^c - \sum_{t_g \in \mathcal{T}_g} p_{t_g}^g x_{t_g}^g - \sum_{t_{rg} \in \mathcal{T}_{rg}} p_{t_{rg}}^{rg} x_{t_{rg}}^{rg} - \sum_{t_{rc} \in \mathcal{T}_{rc}} p_{t_{rc}}^{rc} x_{t_{rc}}^{rc}$$

$$\text{s.t.} \quad \sum_{t_c \in \mathcal{T}_c^n} x_{t_c}^c + \sum_{i | ni \in \mathcal{A}} f_{ni} - \sum_{i | in \in \mathcal{A}} f_{in} = \sum_{t_g \in \mathcal{T}_g^n} x_{t_g}^g - y_n^d \quad [\pi_n^d] \tag{II.4}$$

$$- \sum_{n \in \mathcal{N}_e} \sum_{z \in \{rc, rg\}} \sum_{t_z \in \mathcal{T}_z^n} x_{t_z}^z = \sum_{n \in \mathcal{N}_e} y_n^r - r_e \quad [\pi_e^r] \tag{II.5}$$

$$\sum_{ij \in \mathcal{A}} L_{l,ij} f_{ij} = 0 \quad [\lambda_l] \tag{II.6}$$

$$- K_{ij} \le f_{ij} \le K_{ij} \quad [\eta_{ij}^+, \eta_{ij}^-] \tag{II.7}$$

$$0 \le x_{t_z}^z \le q_{t_z}^z \quad [\nu_{t_z}^{z+}, \nu_{t_z}^{z-}] \tag{II.8}$$

$$\sum_{t_{rc} \in \mathcal{T}_{rc}^n} x_{t_{rc}}^{rc} - \sum_{t_c \in {\mathcal{T}'}_c^n} x_{t_c}^c \le 0 \quad [\theta_n] \tag{II.9}$$

$$\sum_{t_{rg} \in \mathcal{T}_{rg}^n} x_{t_{rg}}^{rg} \le B_n \sum_{t_g \in \mathcal{T}_g^n} x_{t_g}^g \quad [\phi_n] \tag{II.10}$$

$$\sum_{t_{rg} \in \mathcal{T}_{rg}^n} x_{t_{rg}}^{rg} + \sum_{t_g \in \mathcal{T}_g^n} x_{t_g}^g \le W_n \quad [\phi'_n]. \tag{II.11}$$

Here (II.1) to (II.4) hold $\forall n \in \mathcal{N}$. (II.5) holds $\forall e \in \{N, S\}$. (II.6) holds for each loop, indicating that sum of impedance adjusted flows across the loop must be zero. (II.7) holds $\forall ij \in \mathcal{A}$. (II.8) holds $\forall t_z \in \mathcal{T}_z, \forall z \in \mathcal{Z}$.

Note that the consumer also has a limit on their ILR offer constituted by their level of consumption and a load margin V_n that can not be interrupted.

A common approach to solve a bi-level problem is through reformulating it as an MPEC, by replacing the lower level problem with its equivalent optimality conditions. However, most nonlinear optimization solvers will not guarantee global optimality due to non-convexity coming from the optimality conditions of the lower level problem. Therefore we consider a mixed-integer reformulation to find globally optimal solutions.

III. Mixed Integer Program Reformulation

Here we use reformulations to convert [PMP] into a mixed integer program [MIP]. Given that the dispatch model is convex, we are able to replace it by its set of KKT conditions and we present the new formulation as an MPEC. The consumer's profit maximization problem includes the primal and dual feasibility constraints of the market clearing problem as well as the complementarity conditions. We assume natural bounds on the primal and dual variables. The prices are bounded by the value of lost load. Following [27] we have used big-M to linearize the complementary constraints. The big-M parameters are defined by U and F, which are sufficiently large for their corresponding constraints. This formulation has a bi-linear objective function and linear constraints with binary variables, we call this model [MIP]:

$$\max_{y^d, y^r} \quad \sum_{n\in\mathcal{N}} \left(u y_n^d - \pi_n^d y_n^d + \pi_{e_n}^r y_n^r \right)$$

s.t. (II.1) – (II.11)

$$p_{t_c}^c = \pi_n^d + \nu_{t_c}^{c+} - \nu_{t_c}^{c-} - \theta_n \tag{III.1}$$

$$p_{t_g}^g = \pi_n^d - \nu_{t_g}^{g+} + \nu_{t_g}^{g-} + B_n\phi_n - \phi_n' \tag{III.2}$$

$$p_{t_{rc}}^{rc} = \pi_{e_n}^r - \nu_{t_{rc}}^{rc+} + \nu_{t_{rc}}^{rc-} - \theta_n \tag{III.3}$$

$$p_{t_{rg}}^{rg} = \pi_{e_n}^r - \nu_{t_{rg}}^{rg+} + \nu_{t_{rg}}^{rg-} - (\phi_n + \phi_n') \tag{III.4}$$

$$\pi_i^d - \pi_j^d + \eta_{ij}^+ - \eta_{ij}^- + \lambda_l L_{l,ij} = 0 \tag{III.5}$$

$$\nu_{t_z}^{z+} \le F_{t_z}^{(\nu^{z+})} s_{t_z}^{(\nu^{z+})} \tag{III.6}$$

$$x_{t_z}^z - q_{t_z}^z \ge -U_{t_z}^{(\nu^{z+})}(1 - s_{t_z}^{(\nu^{z+})}) \tag{III.7}$$

$$\nu_{t_z}^{z-} \le F_{t_z}^{(\nu^{z-})} s_{t_z}^{(\nu^{z-})} \tag{III.8}$$

$$x_{t_z}^z \le U_{t_z}^{(\nu^{z-})}(1 - s_{t_z}^{(\nu^{z-})}) \tag{III.9}$$

$$\phi_n \le F^{(\phi_n)} s^{(\phi_n)} \tag{III.10}$$

$$\sum_{t_{rg}\in\mathcal{T}_{rg}^n} x_{t_{rg}}^{rg} - B_n \sum_{t_g\in\mathcal{T}_g^n} x_{t_g}^g \ge -U^{(\phi_n)}(1 - s^{(\phi_n)}) \tag{III.11}$$

$$\phi_n' \le F^{(\phi_n')} s^{(\phi_n')} \tag{III.12}$$

$$\sum_{t_{rg}\in\mathcal{T}_{rg}^n} x_{t_{rg}}^{rg} + \sum_{t_g\in\mathcal{T}_g^n} x_{t_g}^g - W_n \ge -U^{(\phi_n')}(1 - s^{(\phi_n')}) \tag{III.13}$$

$$\theta_n \le F^{(\theta_n)} s^{(\theta_n)} \tag{III.14}$$

$$\sum_{t_{rc}\in\mathcal{T}_{rc}^n} x_{t_{rc}}^{rc} - \sum_{t_c\in\mathcal{T}_c^n} x_{tc}^c + V_n \ge -U^{(\theta_n)}(1 - s^{(\theta_n)}) \tag{III.15}$$

$$\eta_{ij}^+ \le F_{ij}^{(\eta^+)} s_{ij}^{(\eta^+)} \tag{III.16}$$

$$f_{ij} - K_{ij} \ge -U_{ij}^{(\eta^+)}(1 - s_{ij}^{(\eta^+)}) \tag{III.17}$$

$$\eta_{ij}^- \le F_{ij}^{(\eta^-)} s_{ij}^{(\eta^-)} \tag{III.18}$$

$$f_{ij} + K_{ij} \le U_{ij}^{(\eta^-)}(1 - s_{ij}^{(\eta^-)}) \tag{III.19}$$

$$\eta_{ij}^+, \eta_{ij}^- \ge 0 \tag{III.20}$$

$$s_{ij}^{(\eta^+)}, s_{ij}^{(\eta^-)} \in \{0,1\} \tag{III.21}$$

$$\nu_{t_z}^{z+}, \nu_{t_z}^{z-} \ge 0, \tag{III.22}$$

$$s_{t_z}^{(\nu^{z+})}, s_{t_z}^{(\nu^{z-})} \in \{0,1\} \tag{III.23}$$

$$s^{(\theta_n)}, s^{(\phi_n)}, s^{(\phi_n')} \in \{0,1\}. \tag{III.24}$$

Here (III.1) to (III.4), (III.10) to (III.15) and (III.24) hold $\forall n \in \mathcal{N}$. (III.5), (III.16) to (III.21) hold $\forall ij \in \mathcal{A}$. (III.1) holds $\forall t_c \in \mathcal{T}_c^n$. (III.2) holds $\forall t_g \in \mathcal{T}_g^n$. (III.3) holds $\forall t_{rc} \in \mathcal{T}_{rc}^n$. (III.4) holds $\forall t_{rg} \in \mathcal{T}_{rg}^n$. (III.6) to (III.9), (III.22) and (III.23) hold $\forall t_z \in \mathcal{T}_z, z \in \mathcal{Z}$. To linearize [MIP]'s objective function, we substitute y_n^d, y_n^r, π_n^d and $\pi_{e_n}^r$ respectively with their equivalents in (II.4), (II.5) and (III.1) – (III.4). We also utilize the fact that $\nu_{t_z}^{z+}$, $\nu_{t_z}^{z-}$, θ_n, ϕ_n and ϕ_n' are the dual variables for (II.7), (II.9), (II.10) and (II.11) respectively. Hence we obtain a linear objective function:

$$\begin{aligned}
&\sum_{n\in\mathcal{N}} (u - \pi_n^d) y_n^d + y_n^r \pi_{e_n}^r \\
&= u \sum_{n\in\mathcal{N}} y_n^d + \sum_{e\in\{S,N\}} r_e \pi_{e_n}^r \\
&+ \sum_{t_c\in\mathcal{T}_c} p_{t_c}^c x_{t_c}^c - \sum_{t_g\in\mathcal{T}_g} p_{t_g}^g x_{t_g}^g \\
&- \sum_{t_{rg}\in\mathcal{T}_{rg}} p_{t_{rg}}^{rg} x_{t_{rg}}^{rg} - \sum_{t_{rc}\in\mathcal{T}_{rc}} p_{t_{rc}}^{rc} x_{t_{rc}}^{rc} \\
&- \phi_n' W_n + \theta_n V_n - \sum_{z\in\mathcal{Z}} \sum_{t_z\in\mathcal{T}_z} q_{t_z}^z \nu_{t_z}^{z+} \\
&- \sum_{ij\in\mathcal{A}} [\eta_{ij}^+ + \eta_{ij}^-] K_{ij}.
\end{aligned}$$

A. Single-node Example

In this example we examine the effects of demand response in the electricity and reserve market in a toy model. We use the MIP model where we only have a single node (node A). Although a single-node market does not model the network effects, namely price differences between nodes, it still captures the effects that a major consumer has on the prices of energy and reserve. At this node we have 3 agents. A generator who is able to submit an energy offer stack and a reserve offer stack, inelastic demand (D_A), and our major consumer. For the sake of simplicity we only apply one of the inverse bathtub constraints (II.11) for the generator. The inelastic demand can be interpreted as a consumer's bid tranche in MIP with the length of D_A and a very large price. The major consumer is the strategic agent in MIP.

To show the impacts of co-optimization of electricity and reserve we construct two versions of this example. First we assume that the major consumer is only allowed to submit its quantity of consumption (y^d_A), and not allowed to offer in ILR; we call this version VS.1. In the second version the major consumer is able to offer a variable quantity of ILR (y^r_A), as well as deciding on the consumption quantity; we call this version VS.2.

To present a comprehensive illustration, we apply changes in inelastic demand in node A. We solve VS.1 and VS.2 for several levels of inelastic demand (from 65 to 130 MWh with steps of 1 MWh), and we plot the optimal values.

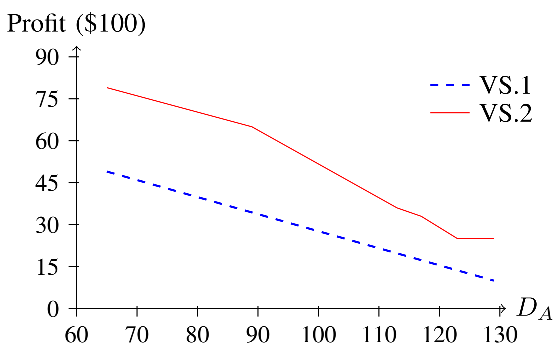

Fig. 1: Profit Maximization Objective Function

Fig. 1 demonstrates the optimal utility of the large consumer from VS.1 and VS.2 experiments. It shows that the bi-level problem's optimal objective value decrease as the demand in node A increases. Also the figure depicts the difference in optimal objective values obtained from experiments VS.1 and VS.2 that, as anticipated, shows more profit in co-optimized version, VS.2.

Next we report on the optimal values of y^d_A and y^r_A for the two experiments.

Note that, as shown in Fig. 2, in VS.2, the ILR offer curve is always below the electricity consumption (dotted curve), indicating the consumer can not offer any more ILR (y^r_A) than the quantity consumed (y^d_A). The flat areas of ILR offer curve shows that, initially the consumer withholds from offering reserve at full capacity to utilize

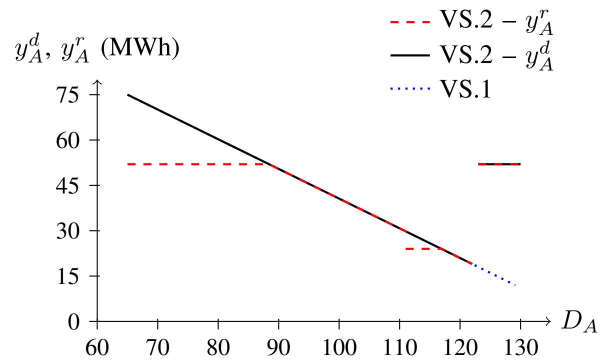

Fig. 2: Strategic Consumer Decision Variables

the higher prices of reserve. We see this pattern until $D_A = 122$, where we observe a sudden increase in both electricity consumption and ILR offer. Although more consumption results in higher electricity prices, this cost is offset by being able to offer more ILR. On the other

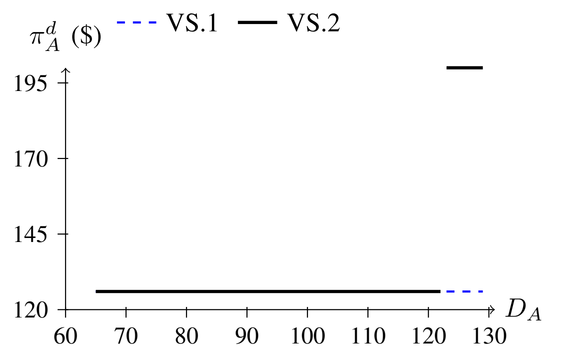

Fig. 3: Electricity Prices

hand, in the absence of the option to supply ILR, in VS.1, the curve indicating (y^d_A) decreases constantly to keep the total demand (major consumer plus the inelastic demand) constant.

The last set of results pertains to the clearing prices for energy π^d_A for the experiments (illustrated in Fig. 3) and reserve prices π^r_A, (illustrated in Fig. 4). We observe a jump in electricity price in VS.2, which is a result of sudden increase in electricity consumption. For the major consumer, this rise in electricity cost is compensated by being paid more for ILR. In VS.1, the price is constant. The major consumer is able to keep the price constant, by decreasing its own demand to cancel out the increase in the inelastic demand.

The reserve price is also affected by ILR offers of the strategic agent. In VS.2, the reserve prices are at most equal to the VS.1 reserve prices. Note that in VS.2, reserve prices are more competitive with the participation of the major consumer in the reserve market. These

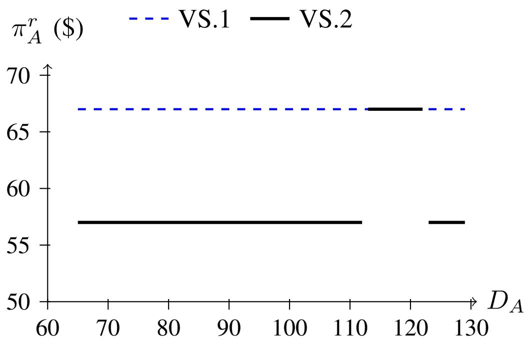

Fig. 4: Reserve Prices

results show that a major consumer who co-optimizes its energy bids and reserve offers strategically is able to improve its profit.

IV. Stochastic Version

In section II, we assumed full information about the electricity market, including information on bids and offers of other market participants. While a deterministic model is a useful starting point, in reality we are exposed to uncertainty and a lack of information regarding the market data. To address this issue, we develop a stochastic version of our model, where the consumer faces a set of scenarios $\omega \in \Omega$ with probability ρ_ω for scenario ω. These scenarios can capture different levels of system demand for instance, or different generation offers. The optimal solution of this problem consists of two stacks (the consumer's demand-side bid and ILR offer). The admissible stacks are monotone step functions. In particular, the bid stack must be decreasing and the ILR stack increasing. Initially we dispense with the monotonicity requirement, but we address this subsequently.

Consider the single scenario case. Here, in the absence of reserve and over a single-node market, the optimal consumption level is effectively singled out by the quantity that determines the dispatch on the (aggregate market) residual supply function. The seminal paper of Klemperer and Meyer [28] lays out the premise for using supply functions as these kinds of offers adapt better to uncertain environments, faced with multiple scenarios.

Cleland et al. introduce a stochastic model in which they lay out a co-optimized electricity bid and reserve offer using simulation-optimization [25], [26]. In this section we extend our bi-level model from section III to a stochastic optimization problem where the upper level problem maximizes the expected profit for a strategic consumer, subject to a lower level problem which maximizes social welfare for each scenario. To present a general model we do not restrict the major consumer to be located at a single node. We define the strategic nodes by allowing the strategic consumption bid $y_n^{d^\omega}$ and $y_n^{r^\omega}$ to be positive. Hence, we set the parameters $C_n^{d^\omega}$ (maximum consumption) and $C_n^{r^\omega}$ (maximum ILR) of all other nodes to be zero.

As in section III, we reformulate the bi-level problem for each scenario as a mixed-integer program (MIP) using the big-M method, and solve these simultaneously. We call this model [SMIP]. The objective function of [SMIP] is:

$$\begin{aligned}\text{Max.} \sum_{\omega\in\Omega}\rho^\omega \sum_{n\in\mathcal{N}} \Big(& r_e^\omega \pi_{e_n}^{r^\omega} + u^\omega y_n^{d^\omega} \\ & + \sum_{t\in\mathcal{T}_c^{n^\omega}} p_t^c x_t^c - \sum_{t\in\mathcal{T}_g^{n^\omega}} p_t^g x_t^g - \sum_{t\in\mathcal{T}_{rg}^{n^\omega}} p_t^{rg} x_t^{rg} \\ & - \sum_{t\in\mathcal{T}_{rc}^{n^\omega}} p_t^{rc} x_t^{rc} - \phi_n'^\omega W_n^\omega + \theta_n^\omega V_n^\omega \\ & - \sum_{z\in\mathcal{Z}} \sum_{t\in\mathcal{T}_z^{n^\omega}} q_t^z \nu_t^{z+} - \sum_{ij\in\mathcal{A}} [\eta_{ij}^{+^\omega} + \eta_{ij}^{-^\omega}] K_{ij}^\omega \Big),\end{aligned}$$

where the superscript $\omega \in \Omega$ denotes the scenarios. To avoid repetition we omit the constraints of [SMIP] since they are identical in form to those in [MIP], the only difference being that they and the corresponding variables are indexed over the set of scenarios Ω.

It is tempting to determine the optimal quantity of consumption for each scenario, in isolation. The problem with this approach is that the sequence of these consumption decisions may not support a monotone curve. Fig. 5 demonstrates the optimal points obtained by solving [SMIP]. Note that there is no single monotone decreasing demand curve that can pass through these 4 points.

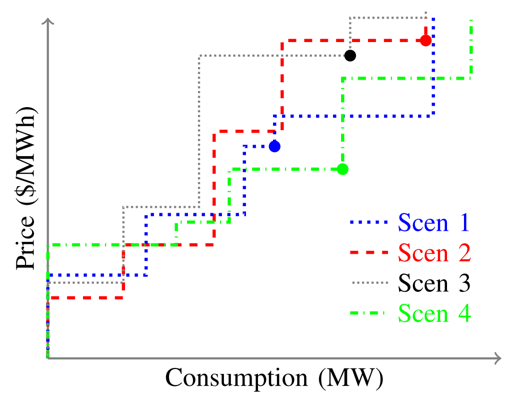

Fig. 5: Optimal wait-and-see bids.

To construct admissible bids, we add monotonicity constraints on the resulting bid and offer curves. To enforce monotonicity, we define binary variables $\zeta_{n,ij}$ to ensure that the energy price in scenario i is higher than that of j provided the quantity consumed in scenario i is less than in scenario j.

$$y_n^{d^{\omega_i}} \leq y_n^{d^{\omega_j}} + M^d \zeta_{n,ij}^d \quad \text{(IV.1)}$$
$$y_n^{r^{\omega_j}} \leq y_n^{r^{\omega_i}} + M^r \zeta_{n,ij}^r \quad \text{(IV.2)}$$
$$\pi_n^{d^{\omega_j}} \leq \pi_n^{d^{\omega_i}} + M^c \zeta_{n,ij}^d \quad \text{(IV.3)}$$
$$\pi_{e_n}^{r^{\omega_j}} \leq \pi_{e_n}^{r^{\omega_i}} + M^{rc} \zeta_{n,ij}^r \quad \text{(IV.4)}$$
$$\zeta_{n,\omega_i\omega_j}^d + \zeta_{n,\omega_j\omega_i}^d = 1 \quad \text{(IV.5)}$$
$$\zeta_{n,\omega_i\omega_j}^r + \zeta_{n,\omega_j\omega_i}^r = 1 \quad \text{(IV.6)}$$
$$\zeta_{n,\omega_i\omega_j}^d, \zeta_{n,\omega_i\omega_j}^r \in \{0,1\}. \quad \text{(IV.7)}$$

Here (IV.1) to (IV.7) hold $\forall n \in \mathcal{N}, \forall \omega_i \in \Omega, \omega_j \in \Omega, \omega_i \neq \omega_j$.

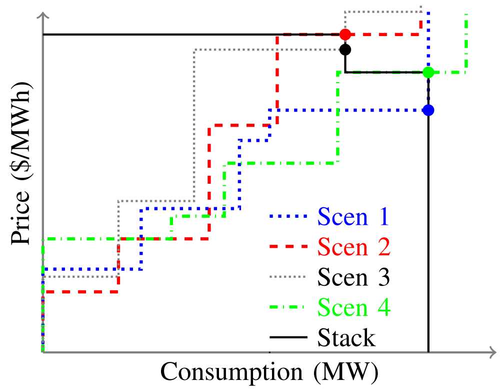

Fig. 6: Optimal monotone consumer bid.

Once the problem is solved with the monotonicity constraint included, we obtain a monotone solution as depicted in Fig. 6.

When solving the problem with multiple scenarios, the number of constraints and variables increase proportionally with the number of scenarios. In addition, the monotonicity constraints link the scenarios together, meaning that decisions in one scenario may affect the other scenarios' optimal solutions. This yields a large MIP, which can become computationally intractable with a large number of scenarios. Thus we consider reformulating to improve the computational times.

V. Alternative Reformulations

In order to reduce the solution time, we simplify and strengthen our [SMIP] model with different approaches, using the features of optimal power flow models.

A. Piecewise Linear Reformulation

Note that each supply function, or offer stack, consists of a number of price-quantity pairs that define the tranches and is therefore a piecewise step function. We define the tranches by two parametric functions called $q(t)$ and $p(t)$, using SOS type 2 variables [29]. Here the parameter t is the distance travelled from the origin along the offer stack in question. Due to the stepwise nature of the offer stacks, $p(t)$ is constant when $q(t)$ is changing and vice versa. Therefore, the function $\Pi(t) = p(t) \times q(t)$, when $q(t)$ and $p(t)$ are obtained from an offer stack, is a piecewise linear function. By using $q(t), p(t)$, instead of using separate variables for determining the dispatched tranches, we can reduce the number of binary variables used in the KKTs of the dispatch model. This is a consequence of the piecewise linearity of our objective function and the fact that by using SOS-2 variables a significant number of binary variables are eliminated; this approach was previously used in [30] for a similar type of bi-level program. We have chosen to omit the formulation of this model, since it mirrors the modifications presented in [30], but will present results from this model, in terms of both model size and solve time, in section VI.

B. Big-M Strengthening

Given the stepwise nature of offer stacks, by applying the same concept as for the piecewise linear formulation, we impose constraints that strengthen the linear program (LP) relaxation, thereby reducing the branch and bound time. The idea is to ensure that the duals on the tranche constraints comply with the increasing order of the tranches for each node. For instance, we could never see the highest priced tranche at its upper bound when lower priced tranches are not fully dispatched. To incorporate this intuition in the model, we impose additional constraints on the binary variables associated with complementary slackness of tranche capacities $s_{t_z}^{(\nu^{z+})}$ and $s_{t_z}^{(\nu^{z-})}$. The new strengthened [SMIP] will have the additional constraints below. An analogous pair of constraints are applied to the reserve stack.

$$s_t^{(\nu^{z+})} \geq s_{t+1}^{(\nu^{z+})} \quad \forall t \in \mathcal{T}_z^{n^\omega}, \forall n \in \mathcal{N}, \forall z \in \mathcal{Z}, \forall \omega \in \Omega,$$
$$s_t^{(\nu^{z-})} \leq s_{t+1}^{(\nu^{z-})} \quad \forall t \in \mathcal{T}_z^{n^\omega}, \forall n \in \mathcal{N}, \forall z \in \mathcal{Z}, \forall \omega \in \Omega.$$

C. Other Methodologies

We tested several other approaches that were not computationally effective. We applied disjunctive reformulations [31], in which we linearized the objective function by using disjunctives and strong duality theorem. As another reformulation for our model, we defined KKTs by using SOS2 variables [22], instead of Big-M parameters. In order to strengthen the LP relaxation, we used triangle inequality constraints, where redundant constraints are added on binary variables associated with monotonicity constraints. Furthermore, we tried indicator variables, instead of using Big-M parameters. Lastly we attempted to directly solve the bi-linear program, given there are non-linear solvers than can handle bi-linear terms well (e.g. BARON [32]). However, because the computational results demonstrated that these methodologies are not effective, we only report on the first three reformulations.

VI. Computational results

This sections presents some computational results. We start by implementing a four node model to track the impacts of the parameters, reformulations and number of scenarios more easily and comprehensively (section VI-A). We present the results for the full NZEM model in section VI-B to assess the effectiveness of our approach.

A. 4-Node Network

This network has one loop (impedance equals 1.0 for all arcs), two generators (buses B and C), one consumer with dispatchable demand (bus D) and one strategic consumer (bus A), as depicted in Figure 7. The required reserve level is 96 MWh. The maximum amount of aggregated reserve and energy that can be dispatched simultaneously (application of the inverse bathtub constraint) for generators B and C are 38 and 60 MWh, respectively. The line capacities are shown on Figure 7. Tables I and II, present the offer and bid data.

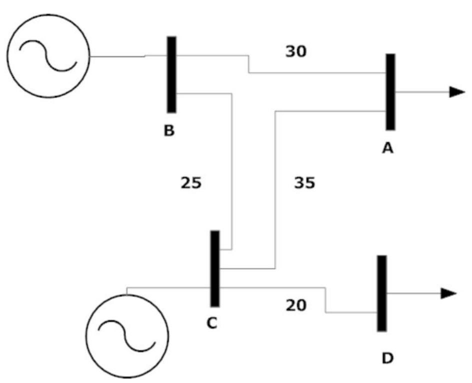

Fig. 7: The 4 node network

TABLE I: Energy offer and bid stack data.

Bus B		Bus C		Bus D	
quantity	price	quantity	price	quantity	price
20	2	15	1.5	19	16
20	5	20	4	10	5
20	12	15	7.5	15	3

TABLE II: Reserve offer stack data.

Bus B		Bus C		Bus D	
quantity	price	quantity	price	quantity	price
21	3	13	2.5	14	1.3
12	6.5	24	4	12	5.7
10	4	14	9	11	6.7

We construct our stochastic model by introducing a set of scenarios. To generate it, we add uniformly distributed inelastic demand to nodes B and C (both distributed $U[0, 25]$ MWh), and define scenarios by sampling the two inelastic demands independently.

Adding the monotonicity constraints has a significant effect on the solution time. To demonstrate this impact, we first solve the model without enforcing monotonicity constraints (Table III). Second, we solve the model with monotonicity constraints enforced (Table IV). These results show that the strengthened big-M method performs better than the other formulations. Although we implement the same intuition as the piecewise linear formulation in this model, the significant performance level could be due to the way the solvers handle branch and bound of SOS2 variables.

TABLE III: Non monotone solution time.

(50 Scen)	vars	binary vars	time(s)	rltv gap
Big-M	41,901	30,600	1.9	0
P-w Linear	32,451	21,800	1.3	0
Stgth Big-M	41,901	30,600	1.6	0

TABLE IV: Monotone solution time.

(20 Scen)	vars	binary vars	time(s)	rltv gap
Big-M	11,961	7,440	7.3	0
P-w Linear	7,541	1,520	1000	0.01
Strgh Big-M	11,961	7,440	1.25	0
(30 Scen)				
Big-M	20,341	13,560	1000	0
Strgh Big-M	20,341	13,560	171.3	0

B. Full Network Simulation

We report on experiments conducted to find the optimal consumption bid curve and reserve offers for a large industrial consumer of electricity in New Zealand. Our problem embeds the NZEM's market clearing problem. This problem is solved every half hour and an exact replica of it, called vectorized scheduling, pricing and dispatch (vSPD), is publicly available [33]. For the sake of simplicity we eliminate network losses and assume exogenous island reserve levels although in reality these levels are somewhat endogenous. All data for populating the model are historical. We used Gurobipy 6.5 and IBM ILOG Cplex 12.6.3, on an Intel Core i7-4770 CPU @ 3.40 GHz desktop computer to solve our MIPs.

Our aim is to assess the value of enlarging the number of scenarios that the stochastic programs are built on. The key uncertainty factor in market clearing over short time horizons such as considered in this paper, is that of "rest of New Zealand" demand. We constructed 20 scenarios based on historical data. To define our sample space, we used the data of morning peaks (7–9 AM) of Monday to Friday from a mid January week in 2016. These scenarios vary in energy offers, demand, etc.

To reduce the problem size and complexity, we relaxed the flow capacities within the North Island lines, as the node of interest to our consumer is located at the bottom of the South Island, and the North Island effects are mostly captured through the HVDC, rather than intra-island congestions. For sample sizes 1,2 and 4 that could be solved to optimality, we verified that this alteration did not change the optimal value of the variables related to the strategic consumer. This reduced the solution time, when solving for a larger number of scenarios. Nevertheless, solving the model for several scenarios to zero integrality gap would not be possible in 3600 seconds. Therefore, practically, we were faced with a choice, namely, to limit the number of scenarios and solve the models more accurately, or include more scenarios and terminate with a larger gap.

We randomly chose subsets of 1, 2, 4, 6 and 8 scenarios (described above), and solved the resulting stochastic model. We repeat this experiment 6 times (a–f) for each group of 1, 2, 4, 6 or 8 scenarios; the random sample of scenarios is different in the different experiments. Table V shows how the problem size scales with the sample size. Subsequently, we simulated the performance of the resulting optimal ILR offer and bid stack for each problem, under each of the 20 scenarios in our scenario set. We computed the utility function value for the strategic consumer with realized prices and quantities in the dispatch model. Table VI shows the expected utility for optimal solution values as computed by simulation for each of sets of scenarios. The columns are the number of scenarios selected for each set and the rows are the 6 different samples and the mean of these values. Line seven in Table VI demonstrates that on average as the sample size increases the revenue increases. In general the utility increases as more scenarios are added (although there may be some exceptions). Furthermore, the average utility over each of the 6 experiments (a–f) increases as we increase the number of scenarios in each problem. Sample sizes 4 and 6 provide better expected utilities and standard deviations compared to the others.

TABLE V: Problem size

sample size	constraints	vars	binaries
2	30833	26677	4314
4	61631	53299	8607
6	92534	79991	13187
8	123225	106485	17306

TABLE VI: Expected utility versus sample size.

	Sample size				
Experiment	1	2	4	6	8
a	12998	13478	15133	16601	16656
b	16440	16477	14894	16127	16677
c	16315	16904	16769	17001	16345
d	10990	16533	16642	16820	12906
e	16525	16503	16412	16141	15826
f	12679	14849	16811	16054	15932
Mean	14325	15791	16110	16457	15724
SD.	2194	1342	864	405	1425

TABLE VII: Bound gaps (%) versus sample size.

	Sample size			
Experiment	2	4	6	8
a	0	0	19.88	9.36
b	0	0	17.11	13.41
c	0	0	0	18.08
d	0	0	18.02	122.29
e	0	0	0	25.60
f	0	0	17.06	12.98
Average	0	0	12.01	33.61

Table VII shows the gap between the upper bound and the best solution, after an hour of solve time. Clearly the gap increases with the number of scenarios.

Next, we turn our attention to testing our policies on "out-of-sample" experiments. We construct experiments based on data from peak periods of highest priced days in 2016, and test the resulting policies on out-of-sample scenarios. We report on experiments conducted to find the optimal consumption bid curve and reserve offers for a large consumer in the South Island.

We constructed 62 scenarios based on historical data. To define our sample space, we used the data of morning peaks (7–9 AM) of Tuesday to Thursday of alternate weeks in winter 2016 (Jun-Aug), this set of days include time periods with spike prices of electricity and reserve.

To assess the performance of the policies, we pick 35 out-of-sample time periods (from July and August 2016), and simulate the expected value (EV) policy and stochastic optimization (SO) policies (from sample size of 6 scenarios) for these time periods. In Table VIII, we first report on the expected, minimum, and maximum utility of the clairvoyant optimization. We also present the EV policy. In order to construct the EV policy, we compute an expected scenario (ES). The nodal demands for the ES are the averages over the 62 scenarios for each node. The ES offer stacks for each generator are computed as a five tranche stack that has the least Eucleadian distance (least square) from the offers in the 62 scenarios. The EV policy is the optimal action (demand and reserve supply) of the major consumer under the ES. We implement this on out-of-sample scenarios. Finally we proceed to simulating a group of SO policies, the same way as in Table VI.

TABLE VIII: Stochastic optimization policy simulation

Policy		exp	min	max
Clairvoyant		31087.06	20388.95	49170.30
EV		26581.37	12720.86	38548.52
SO	#			
	1	26977.19	14873.97	44515.63
	2	27777.41	16039.93	45501.42
	3	26422.86	12693.7	44515.60
	4	27801.57	14873.97	45501.42
	5	26977.19	14873.97	44515.63
	Ave.	27191.25	14671.12	44909.95

Note that the clairvoyant policy, optimizes each scenario individually, and therefore provides an absolute upper-bound on the performance on any policy. On average, the SO policies perform better than the EV policy. Moreover, with the expected value policy, the maximum utility attained over the 35 out-of-sample, periods is 38,548.52, where as the average over the five SO policies is 44909.95, and 49170.30 for the clairvoyant. This, demonstrates that the EV policy is not able to adapt its consumption level as readily as the SO policies, meaning that it cannot take advantage of favorable periods.

VII. Conclusions

In this paper we addressed the uncertainty in energy and reserve markets. Our stochastic model optimizes over a set of scenarios, delivering admissible optimal ILR offer and consumption bid curves for a large consumer of electricity. Our approach requires solving large MIPs due to stochasticity and the resulting monotonicity constraints. In order to solve them in a timely fashion, we introduced methods to enhance our formulation. Solution time can be improved by enforcing constraints that rely on the structure of offer and bid stacks.

We find that having more scenarios would be beneficial if we could solve the problems to optimality in a reasonable time frame. We only compromise because at this juncture, the solution times become unreasonable for practical use.

Our single period model has provided significant intuition for a large consumer's consumption and reserve offer strategy for a trading period. However, a large consumer also has the ability to shift consumption of electricity by utilizing flexibility in its production schedule. As the next step, we will solve the large consumer's problem over a time horizon (such as a month or a week) using a Markov decision processes approach.

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