A robust bi-objective optimization approach for operating a shared energy storage under price uncertainty

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Abstract

Energy storage is acknowledged to play an important role in modern energy technologies due to its potential to reduce operational costs, enhance the resilience, and level energy load for energy systems. Efficient energy storage management can achieve cost savings, also known as energy arbitrage, by charging at off-peak prices and discharging at peak prices. This arbitrage can be further boosted if allowing the energy storage to be shared by multiple users/buildings. However, since energy arbitrage relies on the variation of energy prices, it is hard to achieve this arbitrage if the prices are uncertain. To address this challenge, we present a robust optimization approach to fairly and efficiently operate an energy storage shared between two users under price uncertainty. This sharing strategy is formulated as a bi-objective mixed integer bilinear programming model. To facilitate solution efficiency, we propose a binary formulation for piecewise McCormick relaxations to approximate the bilinear model by a tractable linear model. A computational study demonstrates the effectiveness of our robust sharing strategy for managing energy storage sharing under price uncertainty. Also, it shows that the proposed binary formulation for piecewise McCormick relaxations reduces the runtime by around 80\% compared to the traditional unary formulation.

\textbf{Keywords:} energy storage sharing, robust optimization, piecewise McCormick relaxation, Nash bargaining solution, bi-objective mixed integer linear programming

1. Introduction

Energy markets are always striving to match real-time supply and demand of electricity. Demand Response (DR) is one effective approach for utility companies to level the overall load of the entire system. Their DR program is to guide consumers to change their energy consumption behavior by implementing the non-uniform energy price such as time-of-use and real-time price (Siano, 2014; Wang et al., 2015b; Nezamoddini and Wang, 2017). This leaves an opportunity for the end users equipped with the Energy Storage (ES) to obtain energy arbitrage by charging at off-peak prices and discharging at peak prices (Bradbury et al., 2014; Harsha and Dahleh, 2015; Weitzel and Glock, 2018). ES is able to provide other benefits as well such as backup energy for improving the resilience of the energy system and peak shaving for leveling the energy load.
ES also plays a significant role in the utilization of distributed energy generation in terms of balancing the energy generation and load, and increasing the profits and energy efficiency. ES can ensure the reliable generation of intermittent renewable energy, such as solar and wind power, under uncertain circumstances (Toledo et al., 2010; Zhao et al., 2015; van Ackooij et al., 2018). Also, ES is able to improve the efficiency and flexibility of the combined cooling, heating and power system with massive energy load (Dai et al., 2015; Wang et al., 2015a). Besides, ES is the core component of the electric vehicle to improve the economic performance for the vehicle-to-grid integration (Peterson et al., 2010; Kuang et al., 2017). Consequently, with the increasing penetration of ES (in particular, batteries), ES has been making the world aware of its importance in the future of smart grids and energy systems.

Given the massive investment costs and space limitations, installing ES for an individual or isolated user seems to be impossible. Hence, ES sharing for multiple users/buildings has become attractive to researchers in terms of utilizing ES in practice. One of the benefits for ES sharing is that it allows the users to exchange their stored energy, which leaves room to further augment the energy arbitrage. Recent research focuses on developing the sharing strategy to control ES so that it optimizes the cost savings for all users. Hu et al. (2012) develop a decentralized decision framework based on a memetic algorithm to achieve the tradeoff for energy storage sharing in an integrated building energy system. Wang et al. (2013) develop a dispatch strategy for the ES shared by domestic customers and network operators to meet the needs of both sides. Rahbar et al. (2016) propose an algorithm for the central controller to optimally operate a shared ES system integrated with renewable energy sources. Tushar et al. (2016) develop a noncooperative Stackelberg game based auction approach to determine how residential units share their ES capacity with the facility controllers of large buildings.

However, the existing studies for ES sharing often give priority to maximize cost savings for the entire system and hence the fairness for each user in the sharing system is not emphasized. To bridge this research gap, a contract balance strategy for the ES sharing problem is proposed in our previous paper (Dai and Charkhgard, 2018), and is demonstrated to achieve the tradeoff between fairness and efficiency for ES sharing in the presence of two users. This paper extends the contract balance strategy and naturally inherits its property of balancing fairness and efficiency for ES sharing.

While balancing fairness and efficiency is promising, the contract balance strategy is only studied under the deterministic setting (and only for two users). Evidently, in practice, it is inevitable that energy systems experience uncertainty in terms of energy generation, load, and price. Overall, the mainstream methods to operate energy systems under uncertainty are divided into two categories: stochastic optimization and robust optimization. Stochastic optimization relies on the distribution of uncertain parameters or probability of each possible scenario to handle the uncertainty in the energy system (Möst and Keles, 2010; Hu and Cho, 2014; Zhang et al., 2017). Robust optimization ensures the robustness of the energy systems under uncertainty by modeling the set-based uncertainty (Jiang et al., 2014; Wang et al., 2017; Zhang et al., 2018).

Overall, all sources of uncertainty are important. However, the issue of price uncertainty is becoming even more important because real-time pricing has been increasingly adopted by utility companies as one of the means to perform their DR programs. Since forecasting the real-time price for long time horizons is difficult, numerous papers have been published on the topic of handling real-time price uncertainty. Chen et al. (2012) compare the stochastic and robust approaches for DR management for residential appliances under uncertain real-time prices. Vojvodic et al. (2016)
develop a multistage stochastic model to optimize forward generation thresholds for operating a pumped-storage plant under uncertain real-time price and a novel heuristic approach is proposed to facilitate solving this large-scale multistage stochastic optimization problem. Krishnamurthy et al. (2018) propose a stochastic model to maximize the energy arbitrage for operating the ES under uncertain day-ahead and real-time prices. Interested readers are referred to (Wei et al., 2015; Díaz and Moreno, 2016; Najafi et al., 2016) for further details on maximizing profits under price uncertainty.

In light of the above, this paper intends to address the problem of operating a shared ES for two users/buildings under electricity price uncertainty. We propose a robust optimization based sharing strategy that takes into account fair ES sharing under uncertain real-time price and nominal energy load. This sharing strategy is a robust extension of our previously proposed contract balance strategy due to its capability of balancing the fairness and efficiency for ES sharing. Adopting robust optimization to handle the price uncertainty gives us two benefits. First, modeling set-based uncertainty robust optimization is more applicable than the traditional approaches seizing upon probability distributions. Second, the robust counterpart to a deterministic problem formulated as a min-max problem is usually transferred into a tractable form which decreases the complexity of solving it. Hence, based on the robust formulation of a mixed integer linear program (MILP) proposed by Bertsimas and Sim (2003), we formulate our robust contract balance strategy as a bi-objective mixed integer bilinear programming model. We also develop a novel distance-based measurement to evaluate the performance of robust and nominal solutions since bi-objective optimization always returns a set of efficient solutions instead of a unique solution. By conducting a comprehensive computational study, we demonstrate the effectiveness of our robust contract balance strategy to ensure the robustness and profitability of the solution under price uncertainty. In addition, we employ the Nash bargaining solution (Nash Jr, 1950) to guide the solution selection from the set of efficient solutions of this bi-objective problem for practical purposes. Experimental results illustrate that Nash bargaining solutions can achieve the balance distribution of ES sharing benefits to all users which strengthens their cooperation to share ES.

Another contribution of this work is that we propose a binary formulation based on piecewise McCormick relaxations to translate our bilinear formulation of the robust contract balance strategy into a bi-objective mixed integer linear program (BOMILP) to facilitate its solution. McCormick relaxation is an approach to approximate a bilinear term by replacing it with its McCormick envelope proposed in (McCormick, 1976). In order to increase the accuracy of the approximation, McCormick relaxation is formulated in a piecewise way by partitioning the domain of variables in the bilinear term (Cerisola et al., 2012; Castro, 2015). Our proposed binary formulation for piecewise McCormick relaxations is a time-efficient formulation. It is worth mentioning that our binary formulation reduces the runtime by approximately 80% compared to the traditional formulation for piecewise McCormick relaxations, whilst the qualitative difference in the results between both formulations is negligible.

The remainder of the paper is organized as follows. In Section 2, we introduce the essential concepts for BOMILPs and robust optimization. In Section 3, we present the robust formulation for the contract balance strategy. In Section 4, we explain the binary formulation for piecewise McCormick relaxations. In Section 5, we conduct a computational study. Finally, in Section 6, we give some concluding remarks.
2. Preliminaries

2.1. Essential concepts for BOMILP

In this section, we introduce some necessary notations and concepts related to BOMILP to facilitate presentation and discussion of the remaining sections. A BOMILP can be stated as follows:

\[
\min_{x \in \mathcal{X}} \{ z_1(x), z_2(x) \},
\]

where \( x := (x^1, x^2) \) represents an \( n \)-dimensional vector (where \( n = n_1 + n_2 \)), \( \mathcal{X} := \{ (x^1, x^2) \in \mathbb{Z}_+^{n_1} \times \mathbb{R}_+^{n_2} : A^1 x^1 + A^2 x^2 \leq b \} \) is the feasible set in the decision space, and \( z_i(x) = c_i^T x \) for each \( i \in \mathcal{N} \) represents a linear objective function. The image \( \mathcal{Y} \) of \( \mathcal{X} \) under vector-valued function \( z = \{ z_1, z_2 \} \) represents the feasible set in the objective/criterion space, i.e., \( \mathcal{Y} := z(\mathcal{X}) := \{ y \in \mathbb{R}^2 : y = z(x) \text{ for some } x \in \mathcal{X} \} \). It is assumed that \( \mathcal{X} \) is bounded, and all coefficients/parameters are rational, i.e., \( A^1 \in \mathbb{Q}^{m \times n_1}, A^2 \in \mathbb{Q}^{m \times n_2}, \text{ and } c_i \in \mathbb{Q}^{n_1+n_2} \) for all \( i \in \mathcal{N} \).

**Definition 1.** A feasible solution \( x' \in \mathcal{X} \) is called efficient or Pareto-optimal if there is no other \( x \in \mathcal{X} \) such that \( z_i(x) \leq z_i(x') \) for \( i \in \mathcal{N} \) and \( z(x) \neq z(x') \). If \( x' \) is efficient, then \( z(x') \) is called a nondominated point. The set of all efficient solutions \( x' \in \mathcal{X} \) is denoted by \( \mathcal{X}_E \). The set of all nondominated points \( z(x') \in \mathcal{Y} \text{ for some } x' \in \mathcal{X}_E \) is denoted by \( \mathcal{Y}_N \), and referred to as the nondominated frontier or the efficient frontier.

**Definition 2.** The point \( z^I \in \mathbb{R}^2 \) is the ideal point if \( z_i^I = \min_{(x,y) \in \mathcal{X}_E} z_i(x,y) \) for all \( i \in \mathcal{N} \).

The ideal point is often an imaginary point in the criterion space, i.e., there is no corresponding feasible solution for it in the decision space. Hence, bi-objective optimization is concerned with finding all nondominated points and generating the complete nondominated frontier. The nondominated frontiers of the BOMILPs may contain single points, segments, and half-open (or open) segments. Figure 1 is an illustration of the nondominated frontier of a BOMILP, where the ideal point is denoted by the square. In this study, we employ the Triangle Splitting Method (TSM) developed by Boland et al. Boland et al. (2015) to solve BOMILPs. TSM is proven to be effective in generating an exact representation of the nondominated frontier of a BOMILP.

![Figure 1: The nondominated frontier and (imaginary) ideal point of a BOMILP](image)

Once we obtained the entire nondominated frontier, we still need to select a nondominated point and its corresponding efficient solution from this frontier to implement. This process can
be accomplished by Optimizing Over the Efficient Set (OOES). The problem of OOES can be formulated as follows:
\[
\min_{x \in A_E} f(x) \text{ or } \max_{x \in A_E} f(x),
\] (2)
where \( f(x) \) can be a linear or nonlinear function.

2.2. Robust optimization

Robust optimization is a fundamental method to handle data uncertainty in an optimization problem. The idea of robust optimization is to hedge against the perturbation of the presupposed coefficients in the mathematical model. It is free from the exact distribution of the uncertain data or the probability of each possible scenario, which makes it outperform stochastic optimization in some real-world problems. Overall, robust optimization is a conservative approach to guarantee the robustness of the solution under uncertainty. We employ the robust optimization approach developed by Bertsimas and Sim (2003) in this work, since their method can transfer the robust counterpart of a MILP model into an equivalent formulation without min-max terms. We next give the description of their method.

Let \( x \) and \( c \) be both \( n \)-dimensional vectors, \( A \) be an \( m \times n \) matrix, and \( b \) be an \( m \)-dimensional vector. Furthermore, let \( N = \{1, 2, \ldots, n\} \) and \( M = \{1, 2, \ldots, m\} \). The nominal formulation for a MILP problem is given as
\[
\text{(MIP)} \quad \min c^T x \\
\text{s.t. } \begin{align*}
Ax &\leq b \\
x &\geq 0 \\
x_j &\in \mathbb{Z} \quad \forall j \in \{1, \ldots, n_1\}.
\end{align*}
\]
where \( x \) is a vector of \( n \) variables in which the first \( n_1 \) of them are integers and the rest of them, i.e., \( n_2 \), are continuous (so, \( n = n_1 + n_2 \)). We denote the above model by “(MIP)”.

Robust optimization does not rely on the distribution of uncertain parameters by modeling set-based data uncertainty. Let \( i \in M \) and \( j \in N \). In order to model the uncertainty of each entry \( a_{ij} \) in matrix \( A \), we make \( a_{ij} \) a random variable assuming values in \([a_{ij} - \hat{a}_{ij}, a_{ij} + \hat{a}_{ij}]\) where \( \hat{a}_{ij} \geq 0 \). Similarly, we make each entry \( c_j \) a random variable assuming values in \([c_j, c_j + \hat{c}_j]\) where \( \hat{c}_j \geq 0 \). Note that we limit ourselves to the possibility of the value of \( c_j \) increasing, since for minimization problems, this will result in a worse objective value.

One of the highlights of Bertsimas and Sim’s approach is that they allow the decision makers to adjust the level of conservatism. This level is measured by the number \( \Gamma_i \) where \( i \in M \cup \{0\} \). \( \Gamma_i \) for each \( i \in M \) assumes values in \([0, |J_i|]\) where \( J_i = \{j : \hat{a}_{ij} > 0\} \), and \( \Gamma_0 \) assumes values in \([0, |J_0|]\) where \( J_0 = \{j : \hat{c}_j > 0\} \). We separate \( J_i \) for each \( i \in M \cup \{0\} \) into two sets, \( S_i \) and \( \{t_i\} \), i.e., \( J_i = S_i \cup \{t_i\} \) and \( S_i \cap \{t_i\} = \emptyset \). If \( j \in S_i \), \( a_{ij} \) can increase or decrease by at most \( \hat{a}_{ij} \), and \( c_j \) can increase by at most \( \hat{c}_j \). If \( j \in \{t_i\} \), \( a_{ij} \) can increase or decrease by at most \( (\Gamma_i - |\Gamma_i|)\hat{a}_{ij} \), and \( c_j \) can increase by at most \( (\Gamma_0 - |\Gamma_0|)\hat{c}_j \). The parameters \( \Gamma_i \) restrict the number of uncertain parameters in the objective function and the constraints that are affected by the uncertainty. It can also be referred to as the degree of conservatism for a decision maker when being exposed to uncertainty.
With the above preliminaries, the robust counterpart for (MIP) can be formulated as

\[
(RC) \quad \min c^T x + \max \left\{ \sum_{j \in S_0} \hat{c}_j x_j + (\Gamma_0 - \lfloor \Gamma_0 \rfloor) \hat{c}_{t_0} x_{t_0} \right\}
\]

\[
\text{s.t.} \quad \sum_j a_{ij} x_j + \max \left\{ \sum_{j \in S_i} \hat{a}_{ij} x_j \right\} \leq b_i \quad \forall i \in M
\]

\[
x \geq 0 \quad x_j \in \mathbb{Z} \quad \forall j \in \{1, \ldots, n_1\},
\]

and we denote this formulation as “(RC)”. Obviously, solving (RC) is challenging due to its min-max structure. Based on Bertsimas and Sim’s approach, (RC) can be transformed to an equivalent MILP formulation which we denote as “(RE)” and is given as follows:

\[
(RE) \quad \min c^T x + z_0 \Gamma_0 + \sum_{j \in J_i} p_{0j}
\]

\[
\text{s.t.} \quad \sum_{j \in N} a_{ij} x_j + z_i \Gamma_i + \sum_{j \in J_i} p_{ij} \leq b_i \quad \forall i \in M
\]

\[
z_0 + p_{0j} \geq \hat{c}_j y_j \quad \forall j \in J_0
\]

\[
z_i + p_{ij} \geq \hat{a}_{ij} y_j \quad \forall i \in M, \forall j \in J_i
\]

\[
p_{ij} \geq 0 \quad \forall i \in M \cup \{0\}, \forall j \in J_i
\]

\[
y_j \geq 0 \quad \forall j \in N
\]

\[
z_j \geq 0 \quad \forall j \in N
\]

\[
x \geq 0
\]

\[
x_j \in \mathbb{Z} \quad \forall j \in \{1, \ldots, n_1\}.
\]

3. Robust optimization for energy storage sharing

The purpose of this paper is to develop a fair sharing strategy to handle uncertainty in the ES sharing system. As is well-known, ES is capable of cost savings for an energy system by providing the energy arbitrage. The energy arbitrage can be further boosted if allowing multiple users to share an ES and exchange the energy they stored. One concern when sharing ES is the fairness between all users. In our previous paper Dai and Charkhgard (2018), we proposed a contract balance strategy which is validated to achieve the fair energy distribution when maximizing the energy arbitrage for an ES sharing system. In that model, we assumed all parameters such as electricity price and demand of buildings to be nominal.

However, it is inevitable that electricity price (if real-time pricing is applied) or energy demand may deviate from our prediction in practice. Hence, our optimal solution obtained under deterministic assumptions may be infeasible due to the parameters’ perturbation or an increase in the operational costs outside of our tolerance, i.e., the operational costs of this solution are higher than without using ES. This is why we are motivated to develop the robust optimization based contract
Nomenclature

**Decision Variables**

\( b^i_t \) The amount of energy that building \( i \in N \) has purchased from the power grid in time period \( t \in T \) to be sent to the electrical energy storage. Note that \( b^i_t \times \eta^c \) is the real amount of energy that will be stored in the electrical energy storage in time period \( t \in T \) by building \( i \in N \).

\( d^i_t \) The amount of electricity reaching building \( i \in N \) from the electrical energy storage in time period \( t \in T \). Note that \( b^i_t / \eta^d \) is the real amount of energy discharged from the electrical energy storage by building \( i \in N \) in time period \( t \in T \).

\( e^i_t \) The amount of electricity reaching building \( i \in N \) from the power grid in time period \( t \in T \).

\( m_t \) The amount of electricity available in the electrical energy storage in time period \( t \in T \).

\( s^c_t \) A state variable that assumes the value of 1 if the electrical energy storage is charging in time period \( t \in T \), and 0 otherwise.

\( s^d_t \) A state variable that assumes the value of 1 if the electrical energy storage is discharging in time period \( t \in T \), and 0 otherwise.

**Parameters**

\( \bar{B}, \underline{B} \) The capacity/buffer of the electrical energy storage. The total amount of energy available at the electrical energy storage cannot be more than \( \bar{B} \)/less than \( \underline{B} \) in each time period.

\( \bar{C}, \underline{C} \) The maximum/minimum amount of energy that can be charged into the electrical energy storage in a given time period if the storage is at charging state.

\( \bar{D}, \underline{D} \) The maximum/minimum amount of energy that can be discharged from the electrical energy storage in a given time period if the storage is at discharging state.

\( \eta^c, \eta^d \) The charging/discharging efficiency coefficient of the electrical energy storage, i.e., \( \eta^c, \eta^d \in (0, 1] \).

\( N \) The index set of buildings, i.e., \( N := \{1, 2\} \).

\( T \) The index set of time periods of equal length.

\( L^i_t \) The electricity (load) demand of building \( i \in N \) in time period \( t \in T \).

\( r^i_t \) The electricity price of building \( i \in N \) in time period \( t \in T \).

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balance strategy for handling the uncertainty in the ES sharing system. In order to better illustrate our proposed strategy, we still consider (in this paper) the typical ES system with two buildings sharing one ES (as Dai and Charkhgard Dai and Charkhgard (2018) did in their paper).

Before discussing the robust contract balance strategy for handling the uncertainty in the ES sharing system, we note that only the uncertainty of electricity price is considered in this work. The following two reasons apply: first, the energy arbitrage is sensitive to the electricity price. This is because the energy arbitrage benefits from charging at off-peak price and discharging at peak price to satisfy the energy demand instead of electricity purchases from the market. Second, the contract balance strategy validates the fair energy sharing largely depending on the electricity
price. In this strategy, we compute the average price of the stored electricity for each building and use it to compute the payoff for the building purchasing stored electricity from the other buildings.

3.1. Deterministic formulation for the contract balance strategy

In order to facilitate the illustration of the robust contract balance strategy for handling price uncertainty, we first review the nominal version of this strategy presented in Dai and Charkhgard (2018). Overall, the underlying idea of the contract balance strategy is that users can freely use stored energy but they have to pay other users if they discharge more energy from the storage than what they charge into it (during the entire planning horizon). In fact, the assumption is that each user will never pay for the energy that other users have used. For example, suppose that user X discharges 9 kW extra energy from the storage during the entire planning horizon. Suppose further that at the end of the planning horizon, it turns out the cost (of this 9 kW energy) has been paid by user Y, e.g., user Y has paid $4 for it. Hence under this contract, user X should pay $4 to user Y at the end of the planning horizon.

In light of the above, we now present the nominal formulation. Note that in this formulation we still retain the bilinear terms. The parameters and decision variables used in the nominal formulation for the contract balance strategy are listed in the Nomenclature. The assumptions that ES is shared by two buildings and the power grid capacity is unlimited applies throughout. The standard formulation for the contract balance strategy is expressed as follows and we denote this formulation as “(D1)”.

\[
\begin{align*}
\text{(D1)} & \quad \min \sum_{t \in T} r^1_t (e^1_t + b^1_t) - \max\{\bar{r}^1 g^1, 0\} + \max\{\bar{r}^2 g^2, 0\} \quad (3) \\
& \quad \min \sum_{t \in T} r^2_t (e^2_t + b^2_t) - \max\{\bar{r}^2 g^2, 0\} + \max\{\bar{r}^1 g^1, 0\} \quad (4) \\
& \quad \text{s.t. } e^1_i + d^i_t = L^i_t \quad \forall i \in \mathcal{N}, \forall t \in \mathcal{T} \quad (5) \\
& \quad s^c_t + s^d_t \leq 1 \quad \forall t \in \mathcal{T} \quad (6) \\
& \quad C s^c_t \leq \sum_{i \in \mathcal{N}} b^i_t \eta^c \leq \bar{C} s^c_t \quad \forall t \in \mathcal{T} \quad (7) \\
& \quad D s^d_t \leq \sum_{i \in \mathcal{N}} d^i_t \eta^d \leq \bar{D} s^d_t \quad \forall t \in \mathcal{T} \quad (8) \\
& \quad \sum_{i \in \mathcal{N}} b^i_t \eta^c - \sum_{i \in \mathcal{N}} d^i_t \eta^d = m_t - m_{t-1} \quad \forall t \in \mathcal{T} \quad (9) \\
& \quad B \leq m_t \leq \bar{B} \quad \forall t \in \mathcal{T} \quad (10) \\
& \quad m_0 = m_{|\mathcal{T}|} = B \quad (11) \\
& \quad e^i_t, b^i_t, d^i_t, m_t \geq 0 \quad \forall i \in \mathcal{N}, t \in \mathcal{T} \quad (12) \\
& \quad s^c_t, s^d_t \in \{0, 1\} \quad \forall t \in \mathcal{T} \quad (13) \\
& \quad g^1, g^2 \in \mathbb{R} \quad (14) \\
& \quad \bar{r}^1, \bar{r}^2 \geq 0, \quad (15)
\end{align*}
\]

where \( g^i \) is introduced to measure the amount of energy exchanged between two buildings and \( \bar{r}^i \) indicates the average price of building \( i \) charging energy to the storage during the entire planning horizon.
These two decision variables can be computed by imposing the following constraints:

\[ g_i = \sum_{t \in T} \left( b_i^t - \frac{d_i^t}{\eta^c \eta^d} \right) \quad \forall i \in \mathcal{N} \tag{16} \]

\[ \sum_{t \in T} r_i^t b_i^t = \sum_{t \in T} \bar{r}_i^t b_i^t \quad \forall i \in \mathcal{N}. \tag{17} \]

Overall, objective functions (3)–(4) indicate the operational costs for both buildings, constraint (5) assures the energy demand is satisfied, and constraints (6)–(10) control the operation of ES. Specifically, constraint (6) ensures that the state of the electrical energy storage cannot be charging and discharging at the same time. Constraints (7) and (8) ensure that the amount of energy charged and/or discharged from the electrical energy storage is within the allowed range during any time period. Constraint (9) imposes the energy conservation in the electrical energy storage during any time period. Finally, constraint (10) ensures that the amount of energy available at the electrical energy storage is within the allowed range during any time period. Finally, for simplicity, we assume that \( m_0 = m_{|T|} = B \) and so we impose it as constraint (11).

Note that there are max functions in the objective functions (3) and (4). We can translate these objective functions into an equivalent form without the max functions by introducing two non-negative variables, \( \hat{g}_1 \) and \( \hat{g}_2 \), and one binary variable, \( \bar{y} \). Then we reformulate the model (D1) for the contract balance strategy as follows and we denote this formulation as “(D2)”: 

\[
\begin{align*}
\text{(D2)} \quad & \min \sum_{t \in T} r_1^t (c_1^t + b_1^t) - \bar{r}_1 \hat{g}_1^1 + \bar{r}_2 \hat{g}_2^1 \quad \text{min} \sum_{t \in T} r_2^t (c_2^t + b_2^t) - \bar{r}_2 \hat{g}_2^2 + \bar{r}_1 \hat{g}_1^2 \tag{18} \\
& \text{s.t. (5)–(17)} \\
& \quad g^1 \leq \hat{g}_1^1 \quad \text{(20)} \\
& \quad \hat{g}_1^1 \leq \bar{G} \bar{y} \quad \text{(21)} \\
& \quad \hat{g}_1^1 \leq g^1 + \bar{G} (1 - \bar{y}) \quad \text{(22)} \\
& \quad g^2 \leq \hat{g}_2^2 \quad \text{(23)} \\
& \quad \hat{g}_2^2 \leq g^2 + \bar{G} \bar{y} \quad \text{(24)} \\
& \quad \hat{g}_1^1, \hat{g}_2^2 \geq 0 \quad \text{(25)} \\
& \quad \bar{y} \in \{0, 1\} \quad \text{(26)} \\
\end{align*}
\]

where \( \bar{G} \) is an adequate upper bound of \( g^1 \) and \( g^2 \), e.g. \( \frac{1}{2} n \max \{ \bar{C}/\eta^c, \bar{D}/\eta^c \} \).

### 3.2. Uncertainty in the equality constraints

As is well-known, the idea of handling uncertainty is to capture the solution that can provide a backup to maintain the feasibility for worse scenarios, i.e., we need to modify the solution not to bind the inequality constraints under uncertainty. Obviously, we cannot provide a solution that successfully hedges against any parameter perturbation in the equality constraints due to uncertainty.
Unfortunately, we have equality constraint (17) with uncertain parameters in our contract balance strategy. Constraint (17) restricts that the average price $\bar{r}_i$ should be equal to the weighted average electricity price based on the amount of energy charged during each time period. Obviously, a slight variation on the electricity price $r_i^t$ will violate this constraint, i.e., our obtained average price lacks resilience. In order to handle this particular uncertainty, we change the equality constraint to an inequality constraint:

$$\sum_{t \in T} r_i^t b_i^t \leq \sum_{t \in T} \bar{r}_i b_i^t \quad \forall i \in N.$$  \hspace{1cm} (28)

Here, we allow the “average” price (more accurately, exchange price) $\bar{r}_i$ to be larger than the weighted average price. This transformation has two benefits. First, we enable the solution of $\bar{r}_i$ to provide backup, which allows us to obtain a feasible solution working well in all possible scenarios for the price uncertainty. Besides, allowing the exchange price $\bar{r}_i$ to be greater than the average electricity price is meaningful in real-world practice. In order to incentivize the building to store extra energy for selling to other buildings, it is necessary to guarantee that selling stored energy to other buildings is profitable, which requires the exchange price to be greater than the cost, the average price.

When we replace the equality constraint (17) with inequality constraint (28), however, another issue arises. The objective functions of our contract balance strategy are shown as (3) and (4). Since we allow $\bar{r}_i$ to be anything but no less than the average price, then if $\hat{g}_1$ is positive, we can keep increasing the value of $\bar{r}_i$ to reduce the operational cost of the first building and simultaneously the cost of the second building is increased to infinity. Hence, there will exist long tails on both sides of the nondominated frontier which is impractical. The building will participate in the cooperation to share ES only if the operational cost can be reduced by the energy arbitrage. Based on this requirement, we are able to provide an upper bound to the operational cost of both buildings as follows:

$$\sum_{t \in T} r_i^t(e_i^t + b_i^t) - \bar{r}_i \hat{g}_1 + \bar{r}_2 \hat{g}_2 \leq \sum_{t \in T} r_i^t L_i^1$$  \hspace{1cm} (29)

$$\sum_{t \in T} r_i^t(e_i^t + b_i^t) - \bar{r}_i \hat{g}_1 + \bar{r}_2 \hat{g}_2 \leq \sum_{t \in T} r_i^t L_i^2.$$  \hspace{1cm} (30)

We assume that the power grid is unlimited, which means the buildings can satisfy their electricity demand without ES. Then the right hand sides of constraints (29) and (30) indicate the operational cost that the building completely purchases electricity from the power grid to fulfill its electricity load without using ES. Thus, we establish a restriction to avoid $\bar{r}_i$ increasing to an irrational value. We give the formulation for this revised contract balance strategy as follows. To facilitate the illustration of linearizing the bilinear terms, we use equations (33) and (34) to represent the bilinear terms in the following formulation which we denote as “(D3)”.

$$\min z_1 := \sum_{t \in T} r_i^t(e_i^t + b_i^t) - v^1 + v^2$$  \hspace{1cm} (31)

$$\min z_2 := \sum_{t \in T} r_i^t(e_i^t + b_i^t) - v^1 + v^2$$  \hspace{1cm} (32)

s.t. (5)-(16), (20)-(30)
\[ v^i = r^i g^i \quad \forall i \in \mathcal{N} \]  
\[ w^i = r^i b^i \quad \forall i \in \mathcal{N} \]  
\[ \sum_{t \in \mathcal{T}} r^i b^i_t \leq \sum_{t \in \mathcal{T}} w^i_t \quad \forall i \in \mathcal{N} \]

where \( v^i \) is introduced to quantify the payoff for exchanging stored energy and \( w^i_t \) represents the regulated cost for charging energy from building \( i \in \mathcal{N} \) in time period \( t \in \mathcal{T} \).

### 3.3. Formulation for the robust contract balance strategy

In order to handle the uncertainty in the ES sharing system, we employ the robust optimization technique to generate the solution that works well even in the worst case scenarios. The formulation for the robust contract balance strategy is established following Bertsimas and Sim’s approach shown in models (RC) and (RE).

First, we set up the robust counterpart model (D3) for the revised contract balance strategy. Assume that \( r^i_t \) takes values in \([r^i_t, r^i_t + \delta^i_t]\), where \( \delta^i_t \) represents the maximum deviation from the nominal price coefficient, \( r^i_t \), for both objective functions and constraints. The robust counterpart formulation for (D3) is given as “(R1)".

(R1)

\[
\begin{align*}
\min \hat{z}_1 & := z_1 + \max_{\{S^1 \cup \{t_1\} \mid S^1 \subseteq \mathcal{T}, |S^1| \leq |\Gamma^1|, t_1 \in \mathcal{T} \setminus S^1\}} \left\{ \sum_{t \in S^1} \hat{r}^i_t (e^i_t + b^i_t) + (\Gamma^1 - [\Gamma^1]) \hat{r}^i_t (e^i_t + b^i_t) \right\} \\
\min \hat{z}_2 & := z_2 + \max_{\{S^2 \cup \{t_2\} \mid S^2 \subseteq \mathcal{T}, |S^2| \leq |\Gamma^2|, t_2 \in \mathcal{T} \setminus S^2\}} \left\{ \sum_{t \in S^2} \hat{r}^i_t (e^i_t + b^i_t) + (\Gamma^2 - [\Gamma^2]) \hat{r}^i_t (e^i_t + b^i_t) \right\} \\
\text{s.t.} & \quad (5) - (16), (20) - (27), (33), (34) \\
& \quad \sum_{t \in \mathcal{T}} r^i_t b^i_t + \max_{\{S^1 \cup \{t_1\} \mid S^1 \subseteq \mathcal{T}, |S^1| \leq |\Gamma^1|, t_1 \in \mathcal{T} \setminus S^1\}} \left\{ \sum_{t \in S^1} \hat{r}^i_t (e^i_t + b^i_t) + (\Gamma^1 - [\Gamma^1]) \hat{r}^i_t (e^i_t + b^i_t) \right\} \leq \sum_{t \in \mathcal{T}} w^i_t, \quad \forall i \in \mathcal{N} \\
& \quad z_i + \max_{\{S^i \cup \{t_i\} \mid S^i \subseteq \mathcal{T}, |S^i| \leq |\Gamma^i|, t_i \in \mathcal{T} \setminus S^i\}} \left\{ \sum_{t \in S^i} \hat{r}^i_t (e^i_t + b^i_t) + (\Gamma^i - [\Gamma^i]) \hat{r}^i_t (e^i_t + b^i_t) \right\} \leq \sum_{t \in \mathcal{T}} r^i_t L^i_t + \\
& \quad \max_{\{S^i \cup \{t_i\} \mid S^i \subseteq \mathcal{T}, |S^i| = |\Gamma^i|, t_i \in \mathcal{T} \setminus S^i\}} \left\{ \sum_{t \in S^i} \hat{r}^i_t L^i_t + (\Gamma^i - [\Gamma^i]) \hat{r}^i_t L^i_t \right\}, \quad \forall i \in \mathcal{N},
\end{align*}
\]

where \( \Gamma^i \) takes values in the interval \([0, |\mathcal{T}|]\). Constraint (38) is the robust counterpart of constraint (35), which relaxes the exchange price to be larger than the average price. Constraint (39) is introduced as the robust counterpart of constraints (29) and (30) to remove the irrational part of the nondominated frontier. The left hand side of constraint (39) is the objective value \( \hat{z}_i \). The right hand side of constraint (39) is the maximum cost that the building can achieve if completely
using the power grid to satisfy its load under fixed $\Gamma^i$. For each $i \in \mathcal{N}$, let

$$z^P_i := \sum_{t \in \mathcal{T}} r^i_t L^i_t + \max_{\{S \cup \{t^i\} | S \subseteq \mathcal{T}, |S| = |\Gamma^i|, t^i \in \mathcal{T} \setminus S\}} \left\{ \sum_{t \in S^i} \hat{r}^i_t L^i_t + (\Gamma^i - \lfloor \Gamma^i \rfloor) L^i_{t^i} \right\}.$$ 

Obviously, $z^P_i$ is constant for this robust counterpart formulation and can be computed easily in advance. Note that the purpose of constraint (39) is to cut the nondominated frontier at the rational point. In constraint (39), the operational costs of each building should be no larger than a rational amount, i.e., the operational cost each building can achieve if they completely purchase electricity from the power grid to fulfill their demand for the worst case scenario. If $\hat{z}_i \leq z^P_i$, then some optimistic decision makers are willing to participate in the ES sharing for cost saving purposes. Of course, some pessimistic decision makers would cooperate only if $\hat{z}_i \leq \lambda z^P_i$ where $\lambda$ assumes values in $(0, 1)$. Overall, if $\hat{z}_i > z^P_i$, no decision maker would share ES, since at that exchange price, they will always experience a higher operational cost than not using ES for all possible scenarios. $z^P_i$ is the threshold at which the maximum exchange price can be achieved for the buildings sharing ES and exchanging the stored energy.

Equivalently for transforming (RC) to (RE), we can present the BOMILP formulation for (R1) as follows and we denote this formulation as “(R2)’’.

\begin{align*}
\text{min } \bar{z}_1 & := z_1 + \theta^1_0 \Gamma^1 + \sum_{t \in \mathcal{T}} \beta^1_{0t} \quad (40) \\
\text{min } \bar{z}_2 & := z_2 + \theta^2_0 \Gamma^2 + \sum_{t \in \mathcal{T}} \beta^2_{0t} \quad (41) \\
\text{s.t. } & (5)-(16), (20)-(27), (33), (34) \\
& \theta^i_0 + \beta^i_{0t} \geq \hat{r}^i_{t^i} (e^i_t + b^i_t) \quad \forall i \in \mathcal{N}, \forall t \in \mathcal{T} \quad (42) \\
& \sum_{t \in \mathcal{T}} r^i_t b^i_t + \theta^i_1 \Gamma^i + \sum_{t \in \mathcal{T}} \beta^i_{1t} \leq \sum_{t \in \mathcal{T}} w^i_t \quad \forall i \in \mathcal{N} \quad (43) \\
& \theta^i_1 + \beta^i_{1t} \geq \hat{r}^i_{t^i} b^i_t \quad \forall i \in \mathcal{N}, \forall t \in \mathcal{T} \quad (44) \\
& z_i + \theta^i_0 \Gamma^i + \sum_{t \in \mathcal{T}} \beta^i_{0t} \leq z^P_i \quad \forall i \in \mathcal{N} \quad (45) \\
& \theta^i_0, \theta^i_1, \beta^i_{0t}, \beta^i_{1t} \geq 0 \quad \forall i \in \mathcal{N}, \forall t \in \mathcal{T}. \quad (46)
\end{align*}

4. Linearization technique for the bilinear terms

In this section, we introduce an approximation technique based on the so-called McCormick relaxation (see for instance McCormick (1976); Castro (2015)) to linearize the bilinear terms in our robust contract balance strategy. As is well-known, the McCormick relaxation is capable of linearizing a bilinear term with two bounded continuous variables using the so-called McCormick envelopes. However, the solution of the McCormick relaxation is an approximation of the original bilinear term, and the quality of this approximation is domain-dependent. Hence, we adopt a piecewise approach to the McCormick relaxation to capture a high-quality approximation of the bilinear term. The piecewise McCormick relaxation is done by partitioning the domain of a variable in the bilinear term into several sequential intervals and applying McCormick relaxations to the bilinear terms formed in every interval.
The common approach to execute the piecewise McCormick relaxation is unary formulation based. This unary formulation introduces a set of binary variables to indicate which partition of the bilinear terms is activated. For example, if the 4\textsuperscript{th} binary variable in the set is equal to 1, it indicates the value of the partitioned variable in the bilinear term should be chosen in the 4\textsuperscript{th} partition of its range and all the other binary variables in the set should be equal to 0. The issue for the unary formulation is that when we increase the partition numbers for high quality approximations, the formulation becomes computationally intractable.

To counter this defect of the unary formulation and improve the computational efficiency for large partition numbers, we introduce a binary formulation based piecewise McCormick relaxation to linearize the bilinear terms constructed by two continuous variables. The binary formulation employs an integer variable to indicate the index of the activated partition of the bilinear term and the value of this integer variable is represented by the binary numeral system. For example, if the 4\textsuperscript{th} partition is activated, then this integer variable is represented as $1_2 = 2$. In the following, we present the details of these two versions of piecewise McCormick relaxations.

### 4.1. Mathematical expressions for piecewise McCormick relaxations

For bounded continuous variables $x \in [X, \bar{X}]$ and $y \in [Y, \bar{Y}]$, and a variable $w$ representing the product of $x$ and $y$, consider the following bilinear set

$$
P(w) := \left\{(x, y, w) \in \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+ : w = xy; X \leq x \leq \bar{X}; Y \leq y \leq \bar{Y}\right\},$$

where $w$ in the notation $P(w)$ indicates that this set is constructed for the bilinear term $xy$.

The unary formulation based piecewise McCormick relaxation is expressed as follows. Assume that we separate variable $y$ into $M$ pieces. Then

$$
M(w) := \left\{(x, \tilde{x}, y, \tilde{y}, z, w) \in \mathbb{R}_+ \times \mathbb{R}_+^M \times \mathbb{R}_+ \times \mathbb{R}_+^M \times \{0, 1\}^M \times \mathbb{R}_+ : x = \sum_{i=1}^M \tilde{x}_i; \quad y = \sum_{i=1}^M \tilde{y}_i; \right.

w \geq \sum_{i=1}^M (Y_i \tilde{x}_i + X \tilde{y}_i - Y_i \tilde{X} z_i); \quad w \geq \sum_{i=1}^M (\bar{Y}_i \tilde{x}_i + X \tilde{y}_i - \bar{Y}_i \tilde{X} z_i); \quad w \leq \sum_{i=1}^M (\bar{Y}_i \tilde{x}_i + X \tilde{y}_i - \bar{Y}_i \tilde{X} z_i) - \bar{Y}_i X z_i; \quad w \leq \sum_{i=1}^M (Y_i \tilde{x}_i + X \tilde{y}_i - Y_i \tilde{X} z_i); \quad \sum_{i=1}^M z_i = 1; \quad X z_i \leq \tilde{x}_i \leq \bar{X} z_i, \quad \forall i \in\{1, \ldots, M\}; \quad Y_i z_i \leq \tilde{y}_i \leq \bar{Y}_i z_i, \quad \forall i \in\{1, \ldots, M\}\}.
$$

where $Y_i$ and $\bar{Y}_i$ are the lower and upper bounds for $y_i$ if the $i$-th piece is activated. Apply the following formula to compute $Y_i$ and $\bar{Y}_i$:

$$
Y_i := \bar{Y} + \frac{(\bar{Y} - Y) (i - 1)}{M},
\bar{Y}_i := Y + \frac{(\bar{Y} - Y) i}{M}.
$$

Based on Section 5.2.2 in the thesis of Rigterink Rigterink (2017), if the bilinear term is constructed by a continuous and an integer variable, we can conduct a binary expansion on its piecewise
participating in the bilinear terms as follows:

\[ B(w) := \left\{ (x, y, z, u, v) \in \mathbb{R}_+ \times \mathbb{Z}_+ \times \mathbb{R}_+ \times \{0, 1\}^K \times \mathbb{R}_+^K : w = \sum_{k=1}^{K} 2^{k-1} u_k; \sum_{k=1}^{K} 2^{k-1} z_k \leq \bar{Y}; \right. \]
\[ \left. u_k \geq X z_k, \quad v \geq X q; \quad v \geq D x + \bar{X} q - D \bar{X}; \quad v \leq \bar{X} q; \right. \]
\[ \left. u_k \geq X z_k, \quad \forall k \in \{1, \ldots, K\}; \quad u_k \geq x + \bar{X} z_k - \bar{X}, \quad \forall k \in \{1, \ldots, K\}; \quad u_k \leq x + X z_k \right. \]
\[ \left. - X, \quad \forall k \in \{1, \ldots, K\}; \quad u_k \leq \bar{X} z_k, \quad \forall k \in \{1, \ldots, K\} \right}\),

where \( K = \lceil \log_2 \bar{Y} \rceil + 1 \).

Since the binary expansion for piecewise McCormick relaxations only applies to mixed integer bilinear terms, we transfer \( P(w) \) into the following form:

\[ I(w) := \left\{ (x, p, q) \in \mathbb{R}_+ \times \mathbb{Z}_+ \times \mathbb{R}_+ : w = xY + D x p + x q; X \leq x \leq \bar{X}; p \leq M - 1; q \leq D \right\}, \]

where \( M \) is the number of pieces we want to separate the variable \( y \) into, and \( D = (\bar{Y} - Y)/M \) is the length of each piece on the \( y \)-axis. We employ the binary expansion formulation \( B \) to handle the mixed integer bilinear term \( x p \), and the standard McCormick relaxation to relax \( x q \). Hence, the binary formulation based piecewise McCormick relaxation for \( I(w) \) is given by the following formulation:

\[ C(w) := \left\{ (x, q, z, u, v) \in \mathbb{R}_+ \times \mathbb{R}_+ \times \{0, 1\}^K \times \mathbb{R}_+^K \times \mathbb{R}_+ : w = Y x + D \sum_{k=1}^{K} 2^{k-1} u_k + v; \right. \]
\[ \left. \sum_{k=1}^{K} 2^{k-1} z_k \leq M - 1; \quad v \geq X q; \quad v \geq D x + \bar{X} q - D \bar{X}; \quad v \leq \bar{X} q; \right. \]
\[ \left. u_k \geq X z_k, \quad \forall k \in \{1, \ldots, K\}; \quad u_k \geq x + \bar{X} z_k - \bar{X}, \quad \forall k \in \{1, \ldots, K\}; \quad u_k \leq x + X z_k \right. \]
\[ \left. - X, \quad \forall k \in \{1, \ldots, K\}; \quad u_k \leq \bar{X} z_k, \quad \forall k \in \{1, \ldots, K\} \right\}, \]

where \( K = \lceil \log_2 M \rceil \).

4.2. Piecewise McCormick relaxation for the robust formulation

In this section, we adopt the binary formulation based piecewise McCormick relaxation \( C(w) \) to handle the bilinear terms in the robust formulation \( R2 \) for the robust contract balance strategy.

Before giving a detailed formulation of this relaxation, we first define the domains of the variables participating in the bilinear terms as follows:

- \( b_i^t \in [\underline{A}, \bar{A}] \) for all \( i \in \mathcal{N} \) and \( t \in \mathcal{T} \). If \( s_t^i = 0 \), then \( b_i^t = 0 \), and if \( s_t^i = 1 \), then \( b_i^t \in [\bar{C}/\eta, \bar{C}/\eta] \). We denote the complete domain of \( b_i^t \) as \([\underline{A}, \bar{A}]\) where \( \underline{A} = 0 \) and \( \bar{A} = \bar{C}/\eta \).
- \( \bar{r}^i \in [\bar{R}^i, \bar{R}^i] \) for all \( i \in \mathcal{N} \) where \( \bar{R}^i := \min\{r^i_t : t \in \mathcal{T}\} \) and \( \bar{R}^i := \max\{r^i_t + \bar{r}^i : t \in \mathcal{T}\} \). Since \( \bar{r}^i \) occurs in each bilinear term, we conduct the partitioning on the domain of \( \bar{r}^i \). Furthermore, based on the comparison of piecewise numbers in Dai and Charkhgard (2018), we equally separate the domain of \( \bar{r}^i \) into 32 pieces, i.e \( M = 32 \).
\( \hat{g}^i \in [G, \bar{G}] \) for all \( i \in \mathcal{N} \) where \( G := 0 \) and \( \bar{G} := \frac{1}{2}|\mathcal{T}| \max \{\bar{C}/\eta, \bar{D}/\eta\} \). The upper bound can be obtained by assuming that one building only charges the electrical ES and the other only discharges. So, given that the storage cannot be at both charging and discharging states at the same time, the result follows.

Furthermore, we introduce the following variables to facilitate the binary formulation based piecewise McCormick relaxation for the robust formulation (R2):

- We select the variable \( \bar{r}^i \) for all \( i \in \mathcal{N} \) for partitioning due to its co-occurrence in all of the bilinear terms. Assume we partition \( \bar{r}^i \) into \( M \) pieces. Then we can rewrite \( \bar{r}^i \) in a mixed integer form by the following formulation \( \mathbf{I}(w) \): \( \bar{r}^i = R^i \hat{g}^i + D^i p^i + q^i \), where \( D^i = (\bar{R}^i - R^i)/M \), \( p^i \in \{o \in \mathbb{Z}_+ : o \leq M - 1\} \), and \( q^i \in \{s \in \mathbb{R}_+ : s \leq D^i\} \).

- We introduce binary variables \( y^i_k \) for all \( i \in \mathcal{N} \) and \( k \in \{1, \ldots, K\} \) to express the integer variable \( p^i \) in the binary numeral system, i.e., \( p^i = \sum_{k=1}^{K} 2^{k-1}y^i_k \), where \( K = \lceil \log_2(M-1) \rceil \).

- Based on formulation \( \mathbf{C}(w) \), we introduce continuous variables \( h^i_k \) and \( f^i \) to represent \( v^i = R^i \hat{g}^i + D^i \sum_{k=1}^{K} 2^{k-1}h^i_k + f^i \) for all \( i \in \mathcal{N} \), and continuous variables \( u^i_{tk} \) and \( l^i_t \) to represent \( w^i_t = R^i b^i_t + D^i \sum_{k=1}^{K} 2^{k-1}u^i_{tk} + l^i_t \) for all \( i \in \mathcal{N} \) and \( t \in \mathcal{T} \).

Hence, the binary formulation based piecewise McCormick relaxation for the robust model (R2) can be expressed as follows and we denote this formulation as “(R3)”: 

\[
(\text{R3}) \quad \begin{align*}
\min \ z_1 &= z_1 + \theta^1 \Gamma^1 + \sum_{t \in \mathcal{T}} \beta_{0t}^1 \\
\min \ z_2 &= z_2 + \theta^2 \Gamma^2 + \sum_{t \in \mathcal{T}} \beta_{0t}^2 \\
\text{s.t.} \quad (5)-(16), (20)-(27), (42)-(46) \\
(\hat{g}^i, q^i, v^i, h^i, f^i) &\in \mathbf{C}(v^i) \quad \forall i \in \mathcal{N} \\
(b^i_t, q^i, v^i, u^i_{tk}, l^i_t) &\in \mathbf{C}(w^i_t) \quad \forall i \in \mathcal{N}, \; t \in \mathcal{T}, \quad (49)
\end{align*}
\]

where \( \mathbf{C}(v^i) \) denotes the set representing the binary formulation based piecewise McCormick relaxation for \( v^i = \bar{r}^i \hat{g}^i \), and \( \mathbf{C}(w^i_t) \) for \( w^i_t = \bar{r}^i b^i_t \).

5. Computational Study

Case studies are presented in this section to demonstrate that 1) the binary formulation can significantly improve the computational efficiency of piecewise McCormick relaxations without quality loss when compared to the unary formulation, 2) the robust solution derived from our robust contract balance strategy can provide remarkable benefits in contrast to the deterministic solution in terms of price uncertainty handling for the ES sharing problem, and 3) it is capable of optimizing the nondominated frontier to obtain a solution with balanced cooperation benefits distribution.

We implement all formulations and the solution algorithm TSM in C++ with CPLEX 12.7 as the single-objective MILP solver. All computational experiments have been carried out on a Dell PowerEdge R630 with two Intel Xeon E5-2650 2.2 Ghz 12-Core Processors (30MB), 128GB RAM, the RedHat Enterprise Linux 6.8 operating system, and using a single thread.
The dataset for testing our robust optimization based contract balance strategy contains six classes. Each class includes the same 20 instances and different values for $\Gamma_i$ and $\hat{r}_i$. These 20 instances are generated by randomly drawing $r_i$ and $L_i$ for each $i \in \mathcal{N}$ and $t \in \mathcal{T}$ as uniformly distributed integers from the interval $[1, 20]$. The capacity of the battery is set to $\bar{B} = 2 \times \max \{L_i : \forall i \in \mathcal{N}, \forall t \in \mathcal{T}\} = 40$ and $B = 1$. Furthermore, we assume that $\bar{C} = \bar{D} = 0.2 \times \bar{B}$ and $\bar{C} = \bar{D} = 0.05 \times \bar{B}$. We form six different classes by changing the values of $\Gamma_i$ and $\hat{r}_i$ in the robust formulation. Specifically, let $\Gamma_i = \mu \times |\mathcal{T}|$ where $\mu \in [0, 1]$ and $\hat{r}_i = \upsilon \times r_i$ where $\upsilon \in [0, \infty)$. For class A, we set $\mu = 0.3$ and $\upsilon = 0.3$, for class B, $\mu = 0.6$ and $\upsilon = 0.3$, for class C, $\mu = 0.3$ and $\upsilon = 0.6$, and for class D, $\mu = 0.6$ and $\upsilon = 0.6$. The values of $\upsilon$ are randomly chosen from the interval $[0, 0.6]$ in class E with $\mu = 0.3$, and in class F with $\mu = 0.6$.

5.1. Efficiency of binary formulation for piecewise McCormick relaxation

This section presents the comparison between the binary and unary formulation for the piecewise McCormick relaxation. We employ TSM to solve the bi-objective MILP to generate their exact nondominated frontier.

![Figure 2: Runtime and IP ratios of unary to binary formulation](image)

Figure 2 presents the runtime ratio and IP number ratio of unary formulation to binary formulation. Note that the runtime indicates the time it takes the TSM to solve a bi-objective MILP and to generate its entire nondominated frontier, and IP number is the number of single objective IPs solved by TSM in this process. Let $T_B$ be the runtime of the binary formulation and $T_U$ of the unary one. The points in Figure 2 marked as squares denote the runtime ratio $R_T = T_U / T_B$. Let $I_B$ be the IP number of the binary formulation and $I_U$ of the unary one. The points in Figure 2 marked as diamonds denote the IP number ratio $R_I = I_U / I_B$. It is shown that the runtime ratio
$R_T$ ranges from 5 to 14, and its average is approximately 8. The results imply that our binary formulation is able to guarantee at least 80% runtime reduction, and on average 87.5% in runtime savings compared to the unary formulation. In addition, the IP number ratios are shown to be close to 1, which means the number of single objective IPs solved for both the binary and the unary formulation are almost the same. For more details about the comparison between binary and unary formulation for piecewise McCormick relaxation, see Table 1 in the Appendix. We can conclude that the reason why the binary formulation can significantly reduce the runtime compared to the
unary formulation is that the runtime to solve a single objective IP is significantly decreased.

In addition, one may consider the difference between the solutions of binary and unary formulations. Figure 3 compares the nondominated frontiers of binary and unary formulations for one instance in six classes. The blue line represents the nondominated frontier for the binary formulation, and the dashed red line the nondominated frontier for the unary formulation. It is clear that the frontiers of both the binary and the unary formulation overlap except for several endpoints on the frontier which differ. Furthermore, even though there exists some bending on the frontiers in Class F, the shapes of the frontiers of both the binary and the unary formulation are still almost identical. Hence, we have illustrated that both the binary and the unary formulation represent almost identical solutions, and their differences are negligible.

These experimental results confirm that directly solving the binary formulation for our robust contract balance strategy (R3) is more efficient than solving the unary formulation, and the solutions for both formulations are almost identical.

5.2. Effectiveness of the robust contract balance strategy

In order to validate the effectiveness of the robust optimization based contract balance strategy for managing the uncertain energy price, we compare the robust solution \( x^r \in \mathcal{X}_E^R \) (where \( \mathcal{X}_E^R \) is the efficient set for the robust model (R2)) with the deterministic solution \( x^d \in \mathcal{X}_E^D \) (where \( \mathcal{X}_E^D \) is the efficient set for the deterministic model (D3)). Note that these efficient sets are captured by solving the binary formulation based piecewise McCormick relaxation of (R2) and (D3).

Since \( x^r \) and \( x^d \) are both Pareto-optimal for (R2) and (D3), respectively, the way we conduct this comparison is to evaluate the performance of \( x^r \) for (D3) and of \( x^d \) for (R2). Let “(RtoD)” denote the deterministic model obtained from fixing some of the decision variables of (D3) to the value of the corresponding variables in \( x^r \), and let “(DtoR)” denote the robust model obtained from fixing some of the decision variables of (R2) to the value of the corresponding variables in \( x^d \). Thus, the performance of \( x^r \) for (D3) and of \( x^d \) for (R2) can be represented by the results of (RtoD) and (DtoR), respectively. The fixed variables in both (RtoD) and (DtoR) include \( e_i \), \( b_i \), \( d_i \), \( m_t \), \( s_{ct} \), \( s_{ct} \), \( g_i \), and \( \hat{g}_i \). Obviously, models (RtoD) and (DtoR) are still bi-objective optimization problems. Note that we do not apply piecewise McCormick relaxations to these two formulations due to the fact that one of the variables participating in the bilinear terms has been fixed, i.e., (RtoD) and (DtoR) are bi-objective MILPs and can directly be solved by TSM. Moreover, the frontier of (RtoD) or (DtoR) is a segment without any bending or discontinuity, because the sum of the operational costs of both buildings is fixed if the above variables are fixed.

When using \( x^r \) and \( x^d \) to formulate (RtoD) and (DtoR), we cannot apply every solution in \( \mathcal{X}_E^R \) and \( \mathcal{X}_E^D \) since they are infinite sets. Our approach is to select a finite subset of \( \mathcal{X}_E^R \) and \( \mathcal{X}_E^D \) and apply all solutions in this subset. To achieve this, we select \( I + 1 \) (where \( I > 0 \) is a user-defined parameter) points from the nondominated frontier of (D3) by vertically partitioning it into \( I \) equal segments with respect to the \( \bar{z}_1 \)-axis. The selected points are simply the endpoints of the segments. After picking the points, we construct a set, denoted by \( \mathcal{X}_D \), that contains an efficient solution corresponding to each of the points. Obviously, \( \mathcal{X}_D \subseteq \mathcal{X}_E^D \) is finite and this enables us to formulate (DtoR) for all \( x^d \in \mathcal{X}_D \). Similarly, we select \( I + 1 \) points from the nondominated frontier of (R2) by vertically partitioning it into \( I \) equal segments with respect to the \( \bar{z}_1 \)-axis. After picking the points, we construct a set, denoted by \( \mathcal{X}_R \), that contains an efficient solution corresponding to each of the points. Obviously, \( \mathcal{X}_R \subseteq \mathcal{X}_E^R \) is finite and this enables us to formulate (RtoD) for all \( x^r \in \mathcal{X}_R \). In summary, with the efficient solutions in \( \mathcal{X}_R \) and \( \mathcal{X}_D \), we obtain finite (RtoD) and (DtoR) models.
Figure 4 shows a comparison between the nondominated frontiers of (R2), (D3), and some (RtoD) and (DtoR) models for one instance in six classes. For the (RtoD) and (DtoR) models we set $I = 5$ and then we obtain six (RtoD) and six (DtoR) models. In the legend, “Robust Frontier” implies the frontiers of (R2), “Deterministic Frontier” the frontiers of (D3), “Robust to Deterministic” the frontiers of (RtoD), and “Deterministic to Robust” the frontiers of (DtoR).
First, we can see in Figure 4 that the frontiers of (DtoR) are always dominated by the frontier of (R2), which means the deterministic solution always increases the operational cost of both buildings in contrast to the robust solution when a certain level of robustness should be maintained under the price uncertainty. This is the first advantage of the robust solution \( x^r \): it guarantees the best outcome for robust model (R2). Additionally, as can be seen in Figure 4, the frontiers of (RtoD) are intuitively very close to the frontier of (D3). This implies that for the most optimistic scenario, i.e., the nominal scenario, operating the energy system following the robust solution can still achieve almost the same performance as the deterministic solution. We also find that the gap between the frontiers of (DtoR) and the frontiers of (R2) is significant. It indicates that the deterministic solution will result in a notable benefit loss in contrast to the robust solution when a certain robustness is being sought to handle price uncertainty. The gap between the frontiers of (DtoR) and (R2) is enlarged as well with an increased value of \( \hat{r}^d \) when comparing Figure 4(a) with 4(c) and 4(b) with 4(d). Thus, the second advantage of the robust solution \( x^r \) is uncovered: the performance of implementing \( x^r \) for deterministic model (D3) is better than implementing \( x^d \) for robust model (R2).

In light of the above observations, we compare the distance between the frontiers of (RtoD) and (D3), which is denoted as “RtoD Distance”, with the distance between the frontiers of (DtoR) and (R2), which is denoted as “DtoR Distance”, to interpret the benefit of the robust solution. Since the frontiers of the above models are not parallel, we cannot use the line-line distance to compute the DtoR Distance and RtoD Distance. Thus, we compute the distances with the following procedure.

For the RtoD Distance, we first select a set of points on the nondominated frontier of (D3) by vertically cutting the frontier where the intervals between each two adjacent cuts are identical in length. Similarly, we select a set of points on the frontier for each (RtoD) model. We compute the distances of a point \( y_n \) on the frontier of (RtoD) to all points selected on the frontier of (D3), and refer to the minimum of these distances as the distance of \( y_n \) to the frontier of (D3). Then the RtoD Distance can be represented by the average of the distances of all selected points on the frontiers for all (RtoD) models to the frontier of (D3). In detail, we set \( |X^R| = 101 \) (\( I = 100 \)), then 101 points are selected from the frontier of (R2) and 101 (RtoD) models are generated. We also select 101 points on the frontier for each (RtoD) model, which provide 101 \( \times \ 101 = 10201 \) points selected from all frontiers of the (RtoD) models. Then the RtoD Distance is calculated by taking the average of the distances of these 10201 points to the frontier of (D3). The DtoR Distance is evaluated analogously.

If the RtoD Distance is less than the DtoR Distance, i.e., the ratio of DtoR Distance to RtoD Distance is greater than 1, it validates the second advantage of the robust solution: solving the deterministic model in a robust setting is subject to larger benefit losses than solving the robust model in a deterministic setting. Figure 5 presents the ratios of DtoR Distance to RtoD Distance for all instances in six classes. The horizontal axis represents the ratio of DtoR Distance to RtoD Distance and its scales are repeated for the six classes. The vertical axis has two parts: the upper part denotes the frequency of the ratios being located in a given interval, and the lower part the index of instances. The histograms show the number of instances whose ratios belong to each of the following intervals: \([0.5, 1]\), \((1, 1.5]\), \((1.5, 2]\), \((2, 2.5]\), and \((2.5, 3]\). The scatter plot indicates the exact ratio for each instance in the six classes. As can be seen in Figure 5, approximately 80% of instances have a ratio of DtoR Distance to RtoD Distance greater than 1. This demonstrates the aforementioned second advantage of a robust solution for most instances. Comparing classes C and D with A and B, we find that when \( v \) is increased from 0.3 to 0.6, i.e., the range of energy price
variation is extended, the ratios of DtoR Distance to RtoD Distance are generally enlarged. This implies that if the perturbation of energy price is intensified, the benefits of a robust solution are further emphasized.

In summary, the experimental results justify the two advantages of robust solutions compared to deterministic solutions. Robust solutions carry out the best performance with a certain robustness of solutions guaranteed if there exists a perturbation of electricity price and its benefit loss is negligible when the prices show no difference compared to the predictions. In contrast, the deterministic solutions result in a clear benefit deterioration when compared to the robust solutions.

5.3. Unique robust solution selection from $\mathcal{X}_E^R$

This section explores how to select a desirable point on the nondominated frontier to compute its corresponding efficient solution in practice. We have illustrated the effectiveness of the robust solution generated by our robust contract balance strategy in the last section and we have shown that there exist infinite robust solutions in the efficient set $\mathcal{X}^R_E$ obtained by solving (R2). However, this begs the question how to select a unique robust solution from $\mathcal{X}^R_E$ in practice. One effective technique to select a solution from the efficient set is OOES (2).

We present two versions of the objective functions for OOES. One is to select the solution of the nondominated point with the minimal distance to the ideal point. The ideal point based OOES is formulated as

$$\min_{x^r \in \mathcal{X}_E^R} \| \bar{z}(x^r) - \bar{z}^I \|^2,$$

where $\bar{z}(x^r) = (\bar{z}_1(x^r), \bar{z}_2(x^r))$ and $\bar{z}^I$ is the ideal point for (R2). Since the ideal point $\bar{z}^I$ represents
the best outcome for both buildings, the point with minimal distance to \( \bar{z}^I \) can be described as the point that achieves the best tradeoff for both buildings seeking their individual best outcomes. The other one follows the Nash bargaining problem Nash (1950); Charkhgard et al. (2018). Here we investigate the benefits for both buildings to participate in the collaboration for ES sharing, so the disagreement point is naturally determined by assuming both buildings entirely use the electricity purchased from the power grid to satisfy their own demands, that is to say point \( (z^P_1, z^P_2) \). The Nash bargaining problem based OOES is formulated as

\[
\max_{x' \in X^N_H} (z^P_1 - \bar{z}_1(x'))(z^P_2 - \bar{z}_2(x')).
\]

In order to maintain the collaboration for both buildings sharing ES and exchanging stored energy, it is necessary for the efficient solution to fairly distribute the benefits of the cooperation. Here, the benefits are the operational cost savings for each building in contrast to the noncooperative outcome, which is quantified by \( z^P_i - \bar{z}_i(x') \) for each \( i \in \mathcal{N} \). We introduce the ratio of unbalanced benefits to the total cost savings for an efficient solution to examine its fairness for the collaborative benefits distribution. The ratio for efficient solution \( x' \) can be computed by

\[
\frac{|(z^P_1 - \bar{z}_1(x')) - (z^P_2 - \bar{z}_2(x'))|}{(z^P_1 - \bar{z}_1(x')) + (z^P_2 - \bar{z}_2(x'))}.
\]

Clearly, this ratio ranges between 0 and 1 and the smaller values correspond to fairer benefit distributions. If the ratio is 0, then a completely fair benefit distribution for both buildings is achieved.

![Figure 6: Ratios of unbalanced benefit to total cost saving for the solution of two types OOES](image)

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Figure 6 shows the ratios of unbalanced benefits to the total cost savings for both types of OOES for all instances in the six classes. The horizontal axis denotes the index of the instances in each class, and the vertical axis represents the ratios in percentages. The blue squares denote the ratios of the solutions obtained from the ideal point based OOES, and the red diamonds denote the Nash bargaining problem based OOES. First, Figure 6 shows that most ratios of Nash bargaining solution are 0 or very close to 0. This implies that Nash bargaining solutions are almost always able to accomplish an equal benefits distribution for both buildings. Comparing the ratios of ideal point based solutions and Nash bargaining solutions in Figure 6, we find that Nash bargaining solutions almost always provide the smaller ratio and for some of the instances their gaps are greater than 15%.

Given the above findings, we can conclude that adopting the Nash bargaining solutions selected from the efficient set to practice our robust contract balance strategy enables us to achieve the desirable fair distribution for the cooperative benefits and enhance the solidity of the cooperation between both buildings.

6. Conclusion

In this paper, we presented a robust optimization based sharing strategy to handle the energy price uncertainty for ES sharing. This robust sharing strategy was developed based on our previously proposed contract balance strategy due to its property to achieve the balance of fairness and efficiency for the ES sharing. The robust contract balance strategy is formulated as a bi-objective mixed integer bilinear model by applying the set-described price uncertainty. Moreover, in order to handle the bilinear terms in the model, we propose a novel binary formulation based piecewise McCormick relaxation to linearize the bilinear terms. A computational study demonstrates that our robust sharing strategy is able to ensure the robustness and to minimize the operational costs for the ES sharing system under price uncertainty. It is also worth mentioning that the binary formulation for piecewise McCormick relaxations is capable of reducing the runtime by approximately 80% when compared to the unary formulation, and the solutions of both formulations are almost identical. In addition, due to the infinitely many efficient solutions for the bi-objective model, we employ the Nash bargaining solution to guide the unique solution selection from the efficient set. We validated that the Nash bargaining solutions almost equally distribute the ES sharing benefits to both buildings, which enhances the cooperation of all buildings to share ES.

In future studies, we will develop a branch-and-bound algorithm to generate the Nash bargaining solutions for the ES sharing problem without capturing the entire nondominated frontiers, i.e., modeling our robust contract balance strategy as a bi-objective framework will no longer be a must. Our future study is motivated by the following: first, generating the exact nondominated frontiers for a bi-objective MILP is of significant computational complexity. Second, if we consider ES sharing by more than two users/buildings, to the best of our knowledge, there is no algorithm that generates the exact frontiers for a BOMILP with more than two objectives.

References


Appendix: Run time and IP numbers for binary and unary formulation

Table 1 shows the details of the run time and IP numbers for both the binary and unary formulation based piecewise McCormick relaxations.

In the table, “Time” indicates the run time when using TSM to solve the binary and unary formulation based piecewise McCormick relaxation of (R2), and “IPs” the number of single objective IPs solved. Additionally, “ave” denotes the average run time and IP numbers of all instances in the given class.
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11 48.5 235 287.3 236
12 182.7 205 1595.9 215
13 41.7 166 521.8 169
14 87.7 194 829.3 203
15 119.9 218 874.4 222
16 38.3 125 304.3 133
17 27.6 89 160.6 86
18 91.2 207 736.8 211
19 28.1 170 164.2 155

**B**
11 38.2 206 234.9 215
12 163.3 216 1240.0 215
13 267.7 191 915.2 198
14 59.8 189 615.8 193
15 54.1 184 490.4 190
16 55.8 180 280.3 179
17 18.3 107 201.9 122
18 84.2 132 726.1 174
19 115.3 219 988.9 243
20 114.9 267 785.0 275

**C**
11 37.5 195 241.7 198
12 146.3 167 1800.3 172
13 20.6 95 255.1 140
14 160.2 259 1101.8 261
15 29.2 84 262.0 85
16 150.2 221 1418.8 235
17 12.6 74 116.8 84
18 76.1 194 616.7 201
19 27.4 107 170.8 120
20 71.6 159 457.4 157

**D**
11 32.9 192 1087.2 194
12 132.9 192 177.9 198
13 200.0 205 1256.3 202
14 71.8 158 460.1 169
15 107.4 267 739.0 274
16 32.6 110 230.6 116
17 116.4 211 742.2 216
18 54.5 190 347.8 190
19 39.1 144 310.6 148
20 25.4 135 146.8 130

**E**
11 48.5 235 287.3 236
12 182.7 205 1595.9 215
13 41.7 166 521.8 169
14 87.7 194 829.3 203
15 119.9 218 874.4 222
16 38.3 125 304.3 133
17 27.6 89 160.6 86
18 91.2 207 736.8 211
19 28.1 170 164.2 155
20 114.9 267 785.0 275

**F**
11 38.2 206 234.9 215
12 163.3 216 1240.0 215
13 267.7 191 915.2 198
14 59.8 189 615.8 193
15 54.1 184 490.4 190
16 55.8 180 280.3 179
17 18.3 107 201.9 122
18 84.2 132 726.1 174
19 115.3 219 988.9 243
20 114.9 267 785.0 275

**Ave**
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68.2 173 515.6 179

**Bave**
61.5 146 506.6 151
74.3 168 607.3 174

**Cave**
78.5 196 558.1 201