The Value of Multi-stage Stochastic Programming in Risk-averse Unit Commitment under Uncertainty

Ali İrfan Mahmutoğulları, Shabbir Ahmed, Özlem Çavuş and M. Selim Aktürk

Abstract—Day-ahead scheduling of electricity generation or unit commitment is an important and challenging optimization problem in power systems. Variability in net load arising from the increasing penetration of renewable technologies have motivated study of various classes of stochastic unit commitment models. In two-stage models, the generation schedule for the entire day is fixed while the dispatch is adapted to the uncertainty, whereas in multi-stage models the generation schedule is also allowed to dynamically adapt to the uncertainty realization. Multi-stage models provide more flexibility in the generation schedule, however, they require significantly higher computational effort than two-stage models. To justify this additional computational effort, we provide theoretical and empirical analyses of the value of multi-stage solution for risk-averse multi-stage stochastic unit commitment models. The value of multi-stage solution measures the relative advantage of multi-stage solutions over their two-stage counterparts. Our results indicate that, for unit commitment models, value of multi-stage solution increases with the level of uncertainty and number of periods, and decreases with the degree of risk aversion of the decision maker.

Index Terms—Unit commitment, risk-averse optimization, stochastic programming.

I. INTRODUCTION

Unit commitment (UC) is a challenging optimization problem used for day-ahead generation scheduling given net load forecasts and various operational constraints [1]. The output schedule includes on-off status of generators and the production amounts, called economic dispatch [2], for every time step.

There has been a great deal of research on deterministic UC models where the problem parameters are assumed to be known exactly [3]. These models cannot capture variability and uncertainty. Common sources of uncertainty are departures from forecasts and unreliable equipment. The departures from forecasts generally stem from the variability in net load and production amounts, whereas unreliable equipment may result in generator and transmission line outages [2], [4]. The penetration of renewable energy has increased the volatility of power systems in recent years. The production amount of energy from wind and solar power are not controllable but can only be forecasted [5].

Robust optimization and stochastic programming are two common frameworks used to address the uncertainty in UC problems. In robust optimization models, it is assumed that the uncertain parameters take values in some uncertainty sets and the objective is to minimize the worst case cost (cf. [6], [7], [8], [9] and [10]). In stochastic programming models, the uncertainty is represented by a probability distribution (cf. [11], [12], [13], [14] and [15]). In two-stage stochastic programming UC models, the generation schedule is fixed for the entire day before the beginning of the day while dispatch is adapted to uncertainty as in [16], [17] and [18]. On the other hand, in multi-stage stochastic programming UC models both the generation schedule and dispatch are allowed to dynamically adapt to uncertainty realization at each hour (see for example, [15], [19] and [20]). Therefore, they incorporate multistage forecasting information with varying accuracy and express relation between time periods appropriately. However, in general, the multi-stage models are computationally difficult. A detailed comparison of two- and multi-stage models can be found in [21] and [22].

The computational challenge of multi-stage models motivates the question on whether the effort to solve them is worthwhile. In [23], this question is addressed for a risk-neutral stochastic capacity planning problem. In the present paper, we address this question for risk-averse UC (RA-UC) problems where the objective is a dynamic measure of risk. We provide theoretical and empirical analysis on the value of the multi stage solution (VMS) where VMS measures the relative advantage to solve the multi-stage models over their two-stage counterparts.

The rest of the paper is organized as follows: In Section II, we define the RA-UC problem and present two- and multi-stage stochastic models. In Section III, we define VMS and provide analytical bounds for it. In Section IV, we present results of computational experiments. In Section V, we discuss possible future extensions of the current work.

II. RISK-VERSE UNIT COMMITMENT PROBLEM

A. Deterministic UC formulation

We first present an abstract deterministic formulation of the UC problem. Let \( I \) be the number of generators and \( T \) be the number of periods. Also, let \( \mathcal{I} := \{1, \ldots, I\} \) and
Decision variables $u_{it}$ and $v_{it}$ represent the binary on/off status and production of generator $i$ in period $t \in T$, respectively. The bold symbols $u_t := (u_{1t}, u_{2t}, \ldots, u_{It})$ and $v_t := (v_{1t}, v_{2t}, \ldots, v_{I_t})$ are the vectors of status and production decisions in period $t \in T$, respectively. The vector $w_t$ denotes auxiliary variables associated with period $t \in T$. These variables are used model various operational constraints. The objective (1) is the sum of production, start-up and shut-down costs in all periods. The function $f_t(\cdot)$ represents the total cost in a period $t \in T$. Constraint (2) ensures satisfaction of the power demand. Constraint (3) enforces lower and upper production limits on the generators. Other operational restrictions are represented by constraints (4) and (5). The temporal relationship between consecutive periods such as start-up, ramp-up, shut-down and ramp-down restrictions are modeled by the set constraint (5). Domain restrictions of the decision variables are given by constraint (6). A concrete version of the above abstract formulation is presented in Appendix A.

### B. Uncertainty and Risk models

In the deterministic formulation above, net load values are assumed to be known exactly. This is a restrictive assumption in practice. We assume that the net load is random and denoted by a random variable $\tilde{d}_t$ in period $t \in T$ from a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Here $\Omega$ is a sample space equipped with sigma algebra $\mathcal{F}$ and probability measure $\mathbb{P}$. An element of the sample space $\Omega$ is called as a scenario (or a sample path) and represents a possible realization of the net load values in all periods. The sequence of sigma algebras $\{\emptyset, \Omega\} = \mathcal{F}_1 \subseteq \mathcal{F}_2 \subseteq \cdots \subseteq \mathcal{F}_T = \mathcal{F}$ is called as a filtration and it represents the gradually increasing information throughout the decision horizon $1, 2, \ldots, T$. The set of $\mathcal{F}_t$–measurable random variables is denoted by $\mathcal{Z}_t$ for $t \in T$. The random demand $\tilde{d}_t$ in period $t$ is $\mathcal{F}_t$–measurable, that is $\tilde{d}_t \in \mathcal{Z}_t$ for $t \in T$. Note that since $\mathcal{F}_1 = \{\emptyset, \Omega\}$ by definition, $\mathcal{Z}_1 = \mathbb{R}$ and the demand in the first period is deterministic.

To extend the deterministic UC model to this uncertainty setting, we have that the decisions in period $t$ depend on realization of the history of net load process $\tilde{d}_t := (\tilde{d}_1, \ldots, \tilde{d}_t)$ up to period $t$. Therefore, we use the $\mathcal{F}_t$–measurable vectors $\tilde{u}_t(\tilde{d}_t), \tilde{v}_t(\tilde{d}_t)$ and $\tilde{w}_t(\tilde{d}_t)$ to represent status, production and auxiliary decisions in period $t \in T$, respectively. The total cost at period $t$ is also $\mathcal{F}_t$–measurable, i.e., $f_t(\tilde{u}_t(\tilde{d}_t), \tilde{v}_t(\tilde{d}_t), \tilde{w}_t(\tilde{d}_t)) \in \mathcal{Z}_t$. We use conditional risk measures in order to quantify the risk involved in a random cost at period $t + 1$ based on the available informations at period $t$ for $t \in T \setminus \{T\}$. The mapping $\rho_t : \mathcal{Z}_{t+1} \rightarrow \mathcal{Z}_t$ is called a conditional risk measure if it satisfies the following four axioms of coherent risk measures (the subscript $t$ is suppressed for notational brevity):

(A1) Convexity: $\rho(Z + (1 - \alpha)W) \leq \alpha \rho(Z) + (1 - \alpha)\rho(W)$ for all $Z, W \in \mathcal{Z}$ and $\alpha \in [0, 1]$.

(A2) Monotonicity: $Z \geq W$ implies $\rho(Z) \geq \rho(W)$ for all $Z, W \in \mathcal{Z}$.

(A3) Translational Equivariance: $\rho(Z + c) = \rho(Z) + c$ for all $c \in \mathbb{R}$ and $Z \in \mathcal{Z}$.

(A4) Positive Homogeneity: $\rho(cZ) = c\rho(Z)$ for all $c > 0$ and $Z \in \mathcal{Z}$.

The objective of the risk averse UC (RA-UC) problem is to minimize the risk involved with the cost sequence $\{Z_t\}_{t=1}^T$ where $Z_t := f_t(\tilde{u}_t(\tilde{d}_t), \tilde{v}_t(\tilde{d}_t), \tilde{w}_t(\tilde{d}_t))$ is a shorthand notation for the total cost in period $t \in T$. Thus, as in [25], [26], we define the dynamic coherent risk measure $\rho : \mathcal{Z}_1 \times \mathcal{Z}_2 \times \cdots \times \mathcal{Z}_T \rightarrow \mathbb{R}$ by using nested composition of the conditional risk measures $\rho_1(\cdot), \rho_2(\cdot), \ldots, \rho_{T-1}(\cdot)$, that is,

$$\rho(Z_1, Z_2, \ldots, Z_T) := Z_1 + \rho_1(Z_2 + \cdots + \rho_{T-1}(Z_T) \cdots)$$

is the risk associated with this cost sequence. Due to translational equivariance property of conditional risk measures, we have an alternative representation of the dynamic coherent measure of risk $\rho(\cdot)$ as

$$\rho \left( \sum_{t=1}^T Z_t \right) := \rho(Z_1, Z_2, \ldots, Z_T)$$

where $\rho = \rho_1 \circ \rho_2 \circ \cdots \circ \rho_{T-1} : \mathcal{Z} \rightarrow \mathbb{R}$ is called as a composite risk measure and $\mathcal{Z} := \mathcal{Z}_T$. The composite risk measure $\rho(\cdot)$ satisfies the coherence axioms (A1)-(A4). Therefore, $\rho(\cdot)$ is a coherent risk measure as shown in [26, Eqn. 6.234].

### C. Two-stage and Multi-stage models

We consider two different models for the RA-UC problem. In the two-stage model, the on/off status decisions are fixed at the beginning of the day and production (or dispatch) decisions are adapted to uncertainty in the random demand. On the other hand, in the multi-stage model, both the status and production decisions are fully adapted to uncertainty in net load. In order to clarify the distinction between two models, the decision dynamics in the two- and multi-stage models are depicted as in Fig. 1 and Fig. 2, respectively.
Observe minimizes the value of the dynamic coherent risk measure. Hence, the multi-stage model of the RA-UC problem, states that only production and auxiliary decisions depend on future realizations in all periods if \( \Omega \) is finite. However, the number of binary variables in TS is proportional to \( N \times T \) where \( N \) is the number of possible demand realizations in all periods and \( T \) is the number of periods. Therefore, it is important to figure out if the additional effort to solve MS is worthwhile. We define the VMS in order to quantify the relative advantage of the multi-stage solution over their two-stage counterparts.

### Value of the Multi-stage Solution

Although an optimal solution of MS provides more flexible day-ahead schedule with respect to different realizations of parameters, the number of binary variables in MS is proportional to \( N \times T \) where \( N \) is the number of possible demand realizations in all periods and \( T \) is the number of periods. However, the number of binary variables in TS is proportional to \( T \times I \). Since \( N \gg T \) for any non-trivial problem, computational difficulty of MS is significantly more than TS. Therefore, it is important to figure out if the additional effort to solve MS is worthwhile. We define the VMS in order to quantify the relative advantage of the multi-stage solution over their two-stage counterparts.

#### Definition 1

The value of multi-stage solution (VMS) is the difference between the optimal values of TS and MS, that is, \( VMS = z^{TS} - z^{MS} \) where \( z^{TS} \) and \( z^{MS} \) are the optimal values of TS and MS, respectively.

Since an optimal solution of MS provides more flexibility in status decisions with respect to uncertain net load realizations, we have \( z^{TS} \geq z^{MS} \) and therefore \( VMS \geq 0 \). Next we provide theoretical bounds on the VMS under some assumptions.

#### Assumption 1

There exists a generator \( j^* \in I \) such that \( q_j \leq d_t \leq \bar{q}_j \), with probability 1 and with no minimum start up and shut down time for each \( t \in T \).

#### Assumption 2

There exists an upper bound \( d_{t}^{\text{max}} \in \mathbb{R}_+ \) on the net load values such that \( 0 \leq d_t \leq d_{t}^{\text{max}} \) with probability 1 for each \( t \in T \).

#### Assumption 3

The production cost of the each generator \( i \in I \) is linear and stationary, and there are no start-up and shut-down costs. In this case the total cost function is of the form \( f_i(u_i, v_i, w_i) = \sum_{i \in I} (a_i u_{it} + b_i v_{it}) \) for some positive coefficients \( a_i \) and \( b_i \) for all \( i \in I \).

Assumption 1 ensures that TS and MS always have at least one feasible solutions and therefore both problems have complete recourse. Assumption 2 states that the net load in...
Theorem 1. Under Assumptions 1, 2 and 3 we have that
\[ \alpha_* D^{\max} - \alpha^* \rho(\bar{D}) \leq \text{VMS} \leq \alpha^* D^{\max} - \alpha_* \rho(\bar{D}). \]
where
\[ \alpha_* := \min_{i \in \mathcal{I}} \{ a_i + b_i \eta \} / \max_{i \in \mathcal{I}} \{ \eta_i \} \] and
\[ \alpha^* := \max_{i \in \mathcal{I}} \{ a_i + b_i \eta \} / \min_{i \in \mathcal{I}} \{ \eta \} \]
are cost related problem parameters.

Proof. Assumption 1 implies that both TS and MS are feasible. Since the net loads are bounded due to Assumption 2, both models have at least one optimal solution.

Let \( \{ \hat{u}_t, \hat{v}_t, \hat{w}_t \}_{t \in \mathcal{T}} \) be an optimal policy obtained by solving the multi-stage model MS. By Assumption 3, we have
\[ f_i(\hat{u}_t(d_t), \hat{v}_t(d_t), \hat{w}_t(d_t)) = \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} a_i \hat{u}_t(d_t) + b_i \hat{v}_t(d_t) \]
for a realization \( d_1, d_2, \ldots, d_T \) of the random net load process \( \tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_T \), let \( [\hat{u}_t, \hat{v}_t, \hat{w}_t] := [\hat{u}_t, \hat{v}_t, \hat{w}_t]([d_t]) \) be the optimal status and production decisions for \( t \in \mathcal{T} \). Then, we have
\[
\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} a_i u_{it}^* + b_i v_{it}^* \geq \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} a_i u_{it} + b_i q_i \]
where the first, third and fifth inequalities follow from feasibility. Since \( \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} a_i u_{it}^* + b_i v_{it}^* \geq a_* \sum_{t \in \mathcal{T}} d_t \) for any sample path \( d_1, d_2, \ldots, d_T \), we have \( \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} a_i \hat{u}_t(d_t) + b_i \hat{v}_t(d_t) \geq a_* \sum_{t \in \mathcal{T}} \hat{d}_t \) due to the monotonicity axiom (A2) and positive homogeneity axiom (A4), we get
\[
z^{MS} = \rho \left( \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} a_i \hat{u}_t(d_t) + b_i \hat{v}_t(d_t) \right) \geq \rho(a_* \hat{D}) = \alpha_* \rho(\bar{D}). \]
Next, we consider a feasible policy \( \{ \hat{u}_t, \hat{v}_t, \hat{w}_t \}_{t \in \mathcal{T}} \) to the multi-stage model where \( \hat{u}_{j-t}(\tilde{d}_t) = 1, \hat{v}_{j-t}(\tilde{d}_t) = \tilde{d}_t \) and all other status and generation variables are set to zero for a sample path \( d_1, d_2, \ldots, d_T \). The feasibility of the solution is guaranteed by Assumption 1. Then,
\[
z^{MS} \leq \rho \left( \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} a_i \hat{u}_t(d_t) + b_i \hat{v}_t(d_t) \right) \]
where the first inequality follows from feasibility, the second inequality follows from Assumption 1 and the third equality follows from (21) and (22).

The claim of the theorem follows from (21) and (22).

If the generators are almost identical and lower and upper production limits are close enough, we have \( a_* \approx \alpha \approx \alpha^* \). Then, we have
\[
VMS \approx \alpha (D^{\max} - \rho(\bar{D})). \]
Note that \( 0 \leq \rho(\bar{D}) \leq D^{\max} \) and the approximation (23) implies that the VMS increases with \( D^{\max} \) and therefore variability in the net load. However, for fixed variability, the VMS decreases with \( \rho(\bar{D}) \) and therefore the degree of risk aversion.

Assume that the net load in period \( t \in \mathcal{T} \) is \( \tilde{d}_t = \bar{d}_t + \mathcal{U}(-\Delta, \Delta) \) where \( \bar{d}_t \) is a deterministic value and \( \mathcal{U}(-\Delta, \Delta) \) is an error term uniformly distributed between \( -\Delta \) and \( \Delta \) for some \( \Delta \in \mathbb{R}^+ \). Also assume that the composite risk measure \( \rho(\cdot) \) is obtained using conditional mean-upper semi deviation as given in (7). Then,
\[
VMS \approx \alpha (D^{\max} - \rho(\bar{D})) \approx \alpha \left( \sum_{t=1}^{T} d_t^{\max} - \rho \left( \sum_{t=1}^{T} \bar{d}_t \right) \right) \]
where the second equality follows from definitions of \( d_t^{\max} \), \( \bar{d}_t \) and evaluation of mean-upper semi deviation risk measure \( \rho(\cdot) \). The approximation in (24) suggests that the VMS increases with the number of periods \( T \) and the variability in
the net load $\Delta$. However, VMS decreases with the degree of risk aversion $\lambda$.

**IV. COMPUTATIONAL EXPERIMENTS**

The analytical result of the previous section rely on restrictive assumptions to simplify the structure of the RA-UC problem. In order to see how the VMS behave in the absence of these assumptions, we conduct a set of computational experiments next.

We consider a power system with 10 generators in the computational experiments. We use the data set presented in [1] with some modifications. We also consider a random net load process with eight scenarios where the power demand at each hour is subject to uncertainty. The scenario tree depicting the random process is given Fig.3. A similar scenario tree structure is used in [28].

![Scenario tree](image)

Fig. 3: Scenario tree

The test data is presented in Appendix B. We use the base demand values presented in Table IV to generate random demands. A variability parameter $\epsilon$ is used to control the dispersion of demand across all scenarios. Demand values for each scenario are presented in Table V. All other parameters are set to the values presented in Table VI. A PC with two 2.2GHz processors and 6 GB of RAM is used in the computational experiments.

The quadratic production cost functions $\{g_i(\cdot)\}_{i \in I}$ are approximated by a piecewise linear cost function with four pieces of equal lengths. We use a conditional mean-upper semi deviation risk measure (7) in each period. The conditional risk measures $\rho_1(\cdot), \rho_2(\cdot), \ldots, \rho_{T-1}(\cdot)$, the dynamic coherent risk measure $\rho(\cdot)$ and the composite risk measure $\rho(\cdot)$ are defined accordingly.

We model and solve the two-stage model TS and the multi-stage model MS for five different values of variability parameter $\epsilon$ and six different values of the penalty parameter $\lambda$. For each $\epsilon$ and $\lambda$ pair, we calculate VMS in terms of difference of optimal values, that is,

$$VMS = z^{TS} - z^{MS},$$

and in terms of percentage

$$VMS(\%) = \frac{z^{TS} - z^{MS}}{z^{MS}}.$$

The results on the VMS are presented in Fig.4.

Fig.4 verifies our analytical findings on VMS. We observe an increase in VMS with the uncertainty in net load values. The VMS and hence importance of the multi-stage model increases as the dispersion among the scenarios increases. As expected, the day-ahead schedule obtained by solving the multi-stage model is more adaptive and provides more flexibility in case of high variability of problem parameters. We also observe decrease in the VMS with the level of risk aversion. In parallel with the analytical results in Theorem 1, higher risk aversion leads lower VMS. Hence, the importance of the multi-stage model decreases as risk aversion increases.

We also consider a rolling horizon policy obtained by solving two-stage approximations to the multi-stage problem in each period and fixing the decisions at that stage with respect to the optimal solution of the two-stage model. In order to measure the quality of the rolling horizon policy, we calculate the gap between the value of the rolling horizon policy and the optimal value of MS. The gap value GAP is calculated in terms of difference of objective values

$$GAP = z^{RH} - z^{MS},$$

and in terms of percentage

$$GAP(\%) = \frac{z^{RH} - z^{MS}}{z^{MS}}.$$

where $z^{RH}$ is the value of the rolling horizon policy. Note that since rolling horizon provides a feasible policy to the multistage problem that is at least as good as that of TS, we have that $0 \leq GAP \leq VMS$. The results are presented in Fig.5.

We present the solution times for each TS and MS instance at Table I and Table II, respectively. The required time to obtain the rolling horizon policy is also presented in Table III.

**TABLE I: Solution times of TS (in seconds)**

<table>
<thead>
<tr>
<th>$\epsilon$</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>7.5</td>
<td>10.4</td>
<td>9.6</td>
<td>7.7</td>
<td>7.2</td>
<td>7.2</td>
</tr>
<tr>
<td>0.2</td>
<td>4.2</td>
<td>3.8</td>
<td>3.5</td>
<td>4.0</td>
<td>3.7</td>
<td>3.2</td>
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<tr>
<td>0.3</td>
<td>12.2</td>
<td>10.9</td>
<td>9.5</td>
<td>8.1</td>
<td>7.8</td>
<td>6.0</td>
</tr>
<tr>
<td>0.4</td>
<td>7.9</td>
<td>3.8</td>
<td>4.1</td>
<td>4.0</td>
<td>3.3</td>
<td>2.7</td>
</tr>
<tr>
<td>0.5</td>
<td>8.8</td>
<td>5.4</td>
<td>6.3</td>
<td>4.8</td>
<td>4.8</td>
<td>4.6</td>
</tr>
</tbody>
</table>

**TABLE II: Solution times of MS (in seconds)**

<table>
<thead>
<tr>
<th>$\epsilon$</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1004.2</td>
<td>1280.0</td>
<td>1255.2</td>
<td>1489.7</td>
<td>1789.6</td>
<td>2009.1</td>
</tr>
<tr>
<td>0.2</td>
<td>328.3</td>
<td>381.6</td>
<td>400.4</td>
<td>444.7</td>
<td>324.6</td>
<td>393.8</td>
</tr>
<tr>
<td>0.3</td>
<td>480.0</td>
<td>1042.4</td>
<td>435.8</td>
<td>780.0</td>
<td>453.8</td>
<td>358.5</td>
</tr>
<tr>
<td>0.4</td>
<td>192.9</td>
<td>674.5</td>
<td>529.4</td>
<td>323.0</td>
<td>328.6</td>
<td>279.8</td>
</tr>
<tr>
<td>0.5</td>
<td>85.7</td>
<td>147.5</td>
<td>116.6</td>
<td>119.0</td>
<td>118.5</td>
<td>113.1</td>
</tr>
</tbody>
</table>

**TABLE III: Required time to obtain the rolling horizon policy (in seconds)**

<table>
<thead>
<tr>
<th>$\epsilon$</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
</tr>
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<tbody>
<tr>
<td>0.1</td>
<td>16.6</td>
<td>15.1</td>
<td>14.7</td>
<td>13.6</td>
<td>14.9</td>
<td>12.8</td>
</tr>
<tr>
<td>0.2</td>
<td>8.0</td>
<td>9.0</td>
<td>9.0</td>
<td>8.7</td>
<td>8.0</td>
<td>8.5</td>
</tr>
<tr>
<td>0.3</td>
<td>15.1</td>
<td>17.3</td>
<td>15.1</td>
<td>15.2</td>
<td>14.6</td>
<td>11.4</td>
</tr>
<tr>
<td>0.4</td>
<td>9.0</td>
<td>10.4</td>
<td>8.3</td>
<td>9.1</td>
<td>7.7</td>
<td>7.8</td>
</tr>
<tr>
<td>0.5</td>
<td>10.2</td>
<td>9.6</td>
<td>9.0</td>
<td>12.3</td>
<td>9.7</td>
<td>9.5</td>
</tr>
</tbody>
</table>

In all instances, the rolling horizon policy performs much better than the policy obtained by solving the two-stage
problem with a small increase in computational effort. The GAP (%) of rolling horizon policy is 0.12% on the average (with maximum 0.32%) whereas the VMS (%) is 1.42% on the average (with maximum 3.20%). Thus, the rolling horizon policy obtained by using two-stage approximations to the multi-stage solution can provide enough flexibility in generation schedule to obtain a near-optimal schedule in RA-UC problems with a reasonable computational effort.

The computational effort to solve the MS model is much larger than that of the TS model and the rolling horizon policy in all instances. The higher the demand variability leads higher VMS while decreasing the solution times as an additional benefit.

V. CONCLUSION

Recent improvements in the renewable power production technologies have motivated the stochastic unit commitment problems, since these models can explicitly address the variability in net load. Multi-stage models provide completely flexible schedules where all decisions are adapted to the uncertainty. However, these models require high computational effort, and therefore, their two-stage counterparts are used to obtain approximate policies. In order to justify the additional effort to solve the multi-stage model rather than its two-stage counterpart, we define the VMS and provide analytical and computational results on it. These results reveal that, for RA-UC problems, the VMS decreases with the degree of risk aversion, and increases with the level of uncertainty and number of time periods.

Performance of the rolling horizon policies obtained by two-stage approximations of the multi-stage models are promising. As a future research direction, it would be interesting to consider the rolling horizon policies in instances with more complicated random net load processes. However, in that case, the number of two-stage models to be solved would be large and their solution would require significant computation time. Theoretical analysis of the value of rolling horizon policies is also an important future step.
APPENDIX A

DETERMINISTIC UNIT COMMITMENT FORMULATION

Indexes and Sets

- \( t \): Period index, \( i \): Generator index,
- \( T \): Number of periods, \( I \): Number of generators,
- \( T' \): Set of periods, \( I' \): Set of generators,

Parameters

\( a_i \): Fixed cost of running generator \( i \in I \),
\( g_i(\cdot) \): Production cost function of running generator \( i \in I \),
\( b_i, c_i \in \mathbb{R}_+ \),
\( S_{UI} \): Start-up cost of generator \( i \in I \),
\( S_D \): Shut-down cost of generator \( i \in I \),
\( q_i \): Minimum production amount of generator \( i \in I \),
\( q_i' \): Maximum production amount of generator \( i \in I \),
\( d_i \): Net load in period \( t \in T \),
\( M_i \): Minimum up time of generator \( i \in I \),
\( L_i \): Minimum down time of generator \( i \in I \),
\( V_i' \): Start up rate of generator \( i \in I \),
\( V_i \): Ramp up rate of generator \( i \in I \),
\( B_i' \): Shut down rate of generator \( i \in I \),
\( B_i \): Ramp down production limit of generator \( i \in I \).

Variables

\( u_{it} \): Status of generator \( i \in I \) in period \( t \in T \),
\( v_{it} \): Production amount of generator \( i \in I \) in period \( t \in T \),
\( y_{it} \): Start up of generator \( i \in I \) in period \( t \in T \),
\( z_{it} \): Shut down of generator \( i \in I \) in period \( t \in T \),

\[ u_{it} \in \{0, 1\}, v_{it} \in \mathbb{R}_+, y_{it}, z_{it} \in \{0, 1\} \]

\[ u_{it}, y_{it}, z_{it} \in \{0, 1\}, v_{it} \geq 0, \forall t \in T, \forall i \in I. \]

The objective (25) is total fixed, production, start up and shut down costs. Constraints (26), (27), (28) and (29) are minimum up time, minimum down time, start up and shut down constraints, respectively. The ramp/start up rate constraint is given in (30). Similarly, (31) is the ramp/shut down rate constraint.

APPENDIX B

COMPUTATIONAL EXPERIMENT DATA

**TABLE IV: Demand Data (MW = megawatt)**

<table>
<thead>
<tr>
<th>Period (or hour)</th>
<th>1-6</th>
<th>7-12</th>
<th>13-18</th>
<th>19-24</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>700</td>
<td>750</td>
<td>850</td>
<td>950</td>
</tr>
<tr>
<td>2</td>
<td>1150</td>
<td>1200</td>
<td>1300</td>
<td>1400</td>
</tr>
<tr>
<td>3</td>
<td>1400</td>
<td>1300</td>
<td>1200</td>
<td>1050</td>
</tr>
<tr>
<td>4</td>
<td>19</td>
<td>20</td>
<td>21</td>
<td>22</td>
</tr>
<tr>
<td>5</td>
<td>67</td>
<td>65</td>
<td>60</td>
<td>56</td>
</tr>
<tr>
<td>6</td>
<td>1200</td>
<td>1400</td>
<td>1200</td>
<td>1100</td>
</tr>
</tbody>
</table>

**TABLE V: Scenario Data**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Probability</th>
<th>Period (or hour)</th>
<th>1-6</th>
<th>7-12</th>
<th>13-18</th>
<th>19-24</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.125</td>
<td>( d_t )</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.125</td>
<td>( d_t' )</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.125</td>
<td>( d_t'' )</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.125</td>
<td>( d_t''' )</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.125</td>
<td>( d_t'''' )</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.125</td>
<td>( d_t''''' )</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE VI: Generator Data (MW = megawatt)**

<table>
<thead>
<tr>
<th>( i )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_i ) (S/h)</td>
<td>1000</td>
<td>970</td>
<td>700</td>
<td>680</td>
<td>450</td>
</tr>
<tr>
<td>( b_i ) (S/MWh)</td>
<td>16.19</td>
<td>17.26</td>
<td>16.6</td>
<td>16.5</td>
<td>19.7</td>
</tr>
<tr>
<td>( c_i ) (S/MWh)</td>
<td>682.5</td>
<td>682.5</td>
<td>195</td>
<td>195</td>
<td>243</td>
</tr>
<tr>
<td>( q_i ) (MW)</td>
<td>225</td>
<td>225</td>
<td>30</td>
<td>30</td>
<td>37.5</td>
</tr>
<tr>
<td>( V_i' ) (MW)</td>
<td>337.5</td>
<td>337.5</td>
<td>45</td>
<td>45</td>
<td>56.25</td>
</tr>
<tr>
<td>( B_i' ) (MW)</td>
<td>405</td>
<td>405</td>
<td>54</td>
<td>54</td>
<td>67.5</td>
</tr>
<tr>
<td>( M_i ) (h)</td>
<td>8</td>
<td>8</td>
<td>5</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>( L_i ) (h)</td>
<td>8</td>
<td>8</td>
<td>5</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

**TABLE VII: Scenario Data**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Probability</th>
<th>Period (or hour)</th>
<th>1-6</th>
<th>7-12</th>
<th>13-18</th>
<th>19-24</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.125</td>
<td>( d_t )</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.125</td>
<td>( d_t' )</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.125</td>
<td>( d_t'' )</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.125</td>
<td>( d_t''' )</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.125</td>
<td>( d_t'''' )</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.125</td>
<td>( d_t''''' )</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
REFERENCES


Shahbirm Ahmed is the Anderson-Interface Chair and Professor in the H. Milton Stewart School of Industrial & Systems Engineering at the Georgia Institute of Technology. His research interests are in stochastic and discrete optimization. Dr. Ahmed is a past Chair of the Stochastic Programming Society and serves on the editorial board of several journals. His honors include the INFORMS Computing Society Prize, the National Science Foundation CAREER award, two IBM Faculty Awards, and the INFORMS Dantzig Dissertation award. He is a Fellow of INFORMS.

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