Assessing the Hazard-Decision Simplification Cost in Multistage Stochastic Hydrothermal Scheduling

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Abstract

Stochastic dual dynamic programming (SDDP) is a method largely used to solve multistage long-term hydrothermal scheduling (LTHS) problems. Motivated by computational benefits, the hazard-decision (HD) implementation scheme assumes a post-decision state formulation, where the state variables represent the system condition immediately after here-and-now decisions of a given stage. Thus, the decision vector of the SDDP subproblem is a function of the uncertainty realization in that stage. However, most of the LTHS models currently in use by system operators directly associate the here-and-now decisions of a given stage to the SDDP subproblem variables, thereby relying on the simplifying assumption that all variables are decided per scenario. Hence, such applications do not distinguish here-and-now from wait-and-see decisions. The objectives of this work are threefold: to raise awareness of this implementation issue; to demonstrate that it may lead to time-inconsistent policies, thereby exhibiting reduced performance in out-of-sample tests; and to propose an augmented-state dynamic equation to efficiently solve the decision-hazard LTHS problem through the classical HD SDDP implementation. A cost increase of 12\% is estimated, and distortions on the thermoelectric generation and spot prices are observed due to the HD simplification within a case study based on realistic data from the Brazilian power system.

Keywords: OR in energy, stochastic dual dynamic programming, hazard-decision vs decision-hazard, multistage long-term hydrothermal scheduling, time consistency.
1. Introduction

A key concept of economic theory is the opportunity cost of limited resources. In power-system economics, such costs play a major role in the market design and infrastructure planning activities to short-term operations and bidding strategies in day-ahead markets. Therefore, an appropriate opportunity-cost assessment strategy has profound impacts on the economic, environmental, and social dimensions. In this context, one of the primary concerns in the operation planning of hydrothermal power systems, or systems with high storage capacity, is the intertemporal management of limited generation resources (Pereira and Pinto, 1991; Zambelli et al., 2013; Brigatto et al., 2017; Street et al., 2017; Papavasiliou et al., 2018). Herein, we consider the centralized long-term hydrothermal scheduling (LTHS) problem, targeting the total expected operating cost minimization. Notwithstanding, the reported developments may encompass a broad range of similarly structured problems. More objectively, we focus on a key implementation aspect that impacts the calculation of the cost-to-go function within the stochastic dual dynamic programming (SDDP) technique (Pereira and Pinto, 1991): the post-decision state formulation of the dynamic model (Powell, 2011).

Motivated by computational benefits, in the post-decision state formulation (Powell, 2011) used in SDDP, the state variables are assumed to represent the system condition immediately after here-and-now decisions1 of a given stage $t$ to be made. Therefore, at each stage $t$, the decision vector of the SDDP subproblem is a function of the uncertainty realization in that stage and the initial state of the system. However, most of the LTHS models currently in use and related publications associate all decisions of a given stage to the SDDP subproblem variables (as done in Pereira and Pinto (1991); Maceiral et al. (2018); Street et al. (2017); Brigatto et al. (2017); de Matos et al. (2015); Shapiro (2011); Shapiro et al. (2013)), including those that should have been considered here-and-now decisions. As per Pereira and Pinto (1991), this simplifying assumption, also known as hazard-decision (HD) hypothesis, assumes that all variables of a given stage are decided per scenario. Hence, such applications and studies do not distinguish here-and-now from wait-and-see decisions within a given stage, and can be classified as one-step-ahead anticipative processes.

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1Here-and-now decisions are those that must be made under uncertainty, or before the observation of the uncertainty factors Birge and Louveaux (2011).
The HD policy is not implementable in practice as no decision is actually delivered in the first-stage. To deliver implementable policies (with here-and-now decisions) relying on the HD simplification assumption, in practice, a deterministic version (based on the average scenario) of the dynamic model formulation is used only at the first period (stage). Therefore, the implementation of a HD policy relies on an heuristic that can be suboptimal by two reasons: 1) because it uses a deterministic-based heuristic to solve the first-stage problem and 2) it makes use of planned decisions that are one-step-ahead anticipative (per scenario), thereby passing an optimistic view of the future for the first-stage problem. Most of the previously reported SDDP-based works rely on both simplifications.

The discrepancy between the model used to define implemented and planned decisions is a source of time-inconsistency gap (sub-optimality) as reported in Brigatto et al. (2017). Notwithstanding, it is worth mentioning that the post-decision state formulation plays a crucial role in modern SDDP algorithms. Within this formulation scheme, the cost-to-go function assessment in the backward pass relies on the solution of an expected-value optimization problem that can be decomposed and efficiently solved per scenario through parallelization procedures. We refer the interested reader to Ding et al. (2019).

In real decision processes however, the HD premise is not valid for all decision variables. For instance, nuclear and coal-fired plants may take several hours to start up and shut down. Besides that, independent system operators (ISO) also lack the information of which scenario they are in during the intra-stage operation. As a consequence, within the energetic LTHS problem, the decision of dispatching thermoelectric units to save water for the next stage is performed before observing the inflows. In contrast, hydros are fast resources capable of addressing most of the variability imposed by uncertainties. In this context, slow thermoelectric generation amounts are the natural choice for here-and-now decisions of each stage. On the other hand, hydros and fast thermal units generation are assumed wait-and-see variables and defined to meet load under each inflow scenario.2

Interestingly, the previously described HD information issue associated with the use of the SDDP by the Brazilian system operator and planner to

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2Due to electrical constraints, depending on the characteristics of each country, this decisions will be ex-post validated in more detailed models.
solve LTHS problem is raised in Kligerman (Campinas, São Paulo, Brazil, 2002) and discussed in Zambelli et al. (2013). However, this criticism is also valid for other systems worldwide relying on studies based on the aforementioned HD simplification. In Zambelli et al. (2013), a predictive open-loop feedback control approach is used as an alternative to the HD-based SDDP scheme to solve the LTHS problem. In Zambelli et al. (2011), the performance of the two methods is compared through out-of-sample scenarios for the same problem and setup. Notwithstanding, to the best of the authors’ knowledge, the described HD information issue has not been addressed to the LTHS problem solved through the SDDP algorithm nor its time-inconsistency cost quantified.

The framework where here-and-now decisions are used to consider the nonanticipative structure of the decision process within a dynamic problem can be termed as decision hazard (DH). The consideration of an explicit DH process into the SDDP procedure introduces additional complexities as it assumes a pre-decision state formulation (Powell and Topaloglu, 2014). In this case, some of each stage variables (e.g., a subset of thermal unit generation) are defined as nonanticipative, and must be decided at the beginning of the stage. These variables are considered here-and-now decisions for the related stages. The remainder of the variables is called wait-and-see variables and are decided after, or during, the uncertainty realization of the related stage. Thus, at each stage, the decision process under the DH scheme involves a two-stage stochastic problem comprising here-and-now decisions, whose level of information regarding the uncertainty realization is limited, and wait-and-see corrective actions performed under the perfect information of each scenario realization. The consideration of two-stage stochastic models prevents the direct decomposition of each stage subproblem per scenario.

In this work, we propose solving the DH multistage LTHS problem by means of the traditional post-decision state implementation of the SDDP. To do that, the state of the dynamic model is extended to consider the here-and-now decisions as part of the state variables. By means of this reformulation of the state variables, power system agents using SDDP can better approximate in the planning stage the actual decision process implemented in practice. As shown in our case study, one of the benefits of considering such a feature in the planning stage is an improved performance for the implemented actions in terms of cost savings, lower expensive thermal generation, and reduced spot-price volatility.

Methodologically, we leverage the fact that the SDDP approximates the
cost-to-go function using a series of Benders decomposition steps to decom- 
pose the two-stage stochastic subproblems characterizing the DH scheme. 
Hence, we propose that, at each stage, all here-and-now variables of the DH 
dynamic problem should be fixed and defined as new state variables, giving 
rise to an augmented-state dynamic model (ASDM) formulation for the 
dynamic equations accounting for the DH scheme. In this context, the here- 
and-now decisions of a given stage are co-optimized with the second-stage 
variables of the previous stage.

It is worth mentioning that the idea of expanding the state-space to ad-
dress constraints to pre-determine two months in advance the dispatch of thermal units committed with liquefied-natural-gas contracts has been ad- dressed in Diniz and Maceira (2013), while the same approach has been used to account for reserve capacity sales within a competitive environment Helseth et al. (2016). Nevertheless, the thrust of this paper is to use an ASDM to address and quantify the impact of a more subtle problem, namely, the HD simplification assumption largely adopted in current industry implemen-
tations of the SDDP used to solve the LTHS problem.

The contributions of this work are threefold:

1. Raise the awareness of the HD information issue as a source of time-
inconsistency, as per Brigatto et al. (2017), in the SDDP algorithm. In this paper we address the problem of a central coordinator that mini-
mizes the dispatch cost of the system. However, the whole discussion and implementation techniques are directly extensible to the case of an agent maximizing revenues to devise the optimal hydro-scheduling within a competitive market.

2. Provide a computational methodology to quantify, through out-of-sample tests, the time-inconsistency (sub-optimality) gap generated by a HD information issue and the value of incorporating the DH scheme in the planning stage. We provide a practical method based on Brigatto et al. (2017) that allows agents to isolate and quantify the expected cost only due to the HD simplification and the expected benefit of considering the DH approach.

3. Propose the use of an augmented-state dynamic equation model to efficiently solve the DH multistage LTHS problem by the classical post-
decision state implementation of the SDDP algorithm.

This paper is organized as follows. In section 2, both the HD and DH frameworks are introduced, and the new ASDM is presented. In section 3,
the performance metrics are introduced to compare different policies based on the HD and DH schemes. In Section 4, a case study based on the Brazilian power system is presented and the final conclusions are addressed in section 5.

2. Hazard-Decision vs Decision-Hazard

In this section, we briefly explain the classical formulation of the LTHS problem based on the HD simplification. Subsequently, we propose a pre-decision-state DH dynamic model and show how this decision scheme affects the SDDP implementation. Then, we introduce the ASDM to account for here-and-now variables within the post-decision-state HD modeling framework, thereby allowing the decomposition of the DH dynamic model into scenario-independent (decomposable) subproblems. Finally, we adapt the SDDP method to the proposed ASDM.

For didactic purposes, we assume a compact model for the system dispatch constraints. In such a model, the power flow (direct current model – DC) and reservoirs storage (system state) transition equations are explicitly represented. The remaining operative constraints such as the maximum and minimum bounds, Kirchhoff’s Voltage Law, etc., are represented through a polyhedral set $\mathcal{X}_t$ (we refer to Brigatto et al. (2017) for further details). Furthermore, for simplicity and didactic purposes, we develop the dynamic models based on a stagewise-independent random process for the inflows as per Brigatto et al. (2017); Street et al. (2017). Nevertheless, the consideration of autoregressive models can be straightforwardly considered without affecting the developments devised in this work (Shapiro, 2011). Hence, a discrete sample space, $\Omega_t$, is considered for each stage $t \in T$ from where scenarios $\omega \in \Omega_t$ are sampled based on their probabilities, $p_\omega$. $T$ represents the set of stages, or periods, considered in the LTHS problem.

The SDDP procedure assumes that the state of the system is known at the beginning of period $t$ and aims to evaluate the cost-to-go function through the minimal expected cost of operating the system from $t$ until the end of the horizon. In our application, the state of the system at the beginning of $t$ is composed of the stored volume in each reservoir at the end of the previous period, $t - 1$, and is represented by the vector $v_{t-1}$. Finally, according to Pereira and Pinto (1991), the evaluation of the cost-to-go function for each stage, $t$, and state, $v_{t-1}$, of the system, i.e., the assessment of $Q_t(v_{t-1})$, is based on the expectation of the immediate and future operational costs.
2.1. Hazard-Decision Approach and Problem Formulation

In the HD scheme, the immediate cost is obtained by the inner product between the vector of unitary costs, \(c_t\), and the dispatched thermoelectric generation vector, \(g_{t,\omega}\), i.e., \(c_t^\top g_{t,\omega}\), for each inflow scenario, \(w_{t,\omega}\). The future cost is represented by the cost-to-go function for the next stage, \(Q_{t+1}(v_{t,\omega})\), evaluated at the resulting state of the system at the end of period \(t\), i.e., \(v_{t,\omega}\). The aforementioned description leads to the well-known dynamic programming formulation for the HD multistage stochastic model used in LTHS planning:

\[
Q_t(v_{t-1}) = \min_{\{X_{t,\omega}\}_{\omega \in \Omega_t}} \sum_{\omega \in \Omega_t} p_{\omega} \left[ c_t^\top g_{t,\omega} + Q_{t+1}(v_{t,\omega}) \right] 
\]

\[s.t.\]
\[
Af_{t,\omega} + Bg_{t,\omega} + Pu_{t,\omega} = d_t \quad \forall \omega \in \Omega_t 
\]
\[
v_{t,\omega} = v_{t-1} - H(u_{t,\omega} + s_{t,\omega}) + w_{t,\omega} \quad \forall \omega \in \Omega_t 
\]
\[
(g_{t,\omega}, u_{t,\omega}, s_{t,\omega}, f_{t,\omega}, v_{t,\omega}) \in X_t \quad \forall \omega \in \Omega_t. 
\]

Throughout this work, \(X_{t,\omega} = \{g_{t,\omega}, u_{t,\omega}, s_{t,\omega}, f_{t,\omega}, v_{t,\omega}\}\) represent the set of decision vectors containing, respectively, the generation of each thermoelectric unit, the water discharged into the turbines of each hydroelectric unit, the water spilled by each hydroelectric unit, the power flow transferred through each transmission line, and the volume of water stored in each reservoir of the system at the end of period \(t\) and scenario \(\omega\).

The expression (1) comprises the minimization of the immediate and future expected costs, as previously described in this section. The expression (2) considers the nodal energy balance equation, where \(A\) represents the network incidence matrix, \(B\) accounts for the total thermal generation in each bus of the system, \(P\) considers the productivity of each hydroelectric unit to account for the total hydro generation in each bus, and \(d_t\) is the vector of the nodal net demand of period \(t\) in each bus of the system. The expression (3) accounts for the state transition and mass conservation functions, where \(H\) (matrix with only +1, 0, −1 elements) translates the cascading topology by assigning the total water discharged and spilled by each hydroelectric unit to the corresponding downstream reservoirs. Finally, the expression (4) accounts for the operational constraints, such as the Kirchhoff’s Voltage Law, and bounds Brigatto et al. (2017).

According to the HD scheme, largely adopted in LTHS literature (see Pereira and Pinto (1991); de Matos and Finardi (2012); Shapiro (2011);
Shapiro et al. (2013); Soares et al. (2017); Brigatto et al. (2017); Street et al. (2017), and Ding et al. (2019)), operative decisions are made under the perfect information of the water inflow scenario at period $t$. Under this hypothesis, the cost-to-go function defined in (1)-(4) can be decomposed into scenario-based functions as follows:

$$Q_t(v_{t-1}) = \sum_{\omega \in \Omega} p_\omega Q_t(v_{t-1}, w_{t,\omega}), \quad (5)$$

where function $Q_t(v_{t-1}, w_{t,\omega})$ represents the optimal value for the parcel of the objective function of (1)-(4) associated with inflow scenario $w_{t,\omega}$. Hence, $Q_t(v_{t-1}, w_{t,\omega})$ can be expressed through the following deterministic linear optimization problem:

$$Q_t(v_{t-1}, w_{t,\omega}) = \min_{X_{t,\omega}} c_t^T g_{t,\omega} + Q_{t+1}(v_{t,\omega}) \quad (6)$$

s.t. $Af_{t,\omega} + Bg_{t,\omega} + Pu_{t,\omega} = d_t \quad (7)$

$v_{t,\omega} = v_{t-1} - H(u_{t,\omega} + s_{t,\omega}) + w_{t,\omega} \quad (8)$

$\left(g_{t,\omega}, u_{t,\omega}, s_{t,\omega}, f_{t,\omega}, v_{t,\omega}\right) \in X_t. \quad (9)$

Because $Q_t(v_{t-1}, w_{t,\omega})$ is a convex and piecewise linear function of $v_{t-1}$ for all values of $w_{t,\omega}$, a lower approximation, based on the maximum within a set of supporting planes, can be built. The SDDP methodology relies on successive improvements of such a lower approximation by means of the inclusion of new supporting planes, known as Benders’ cuts (Pereira and Pinto, 1991), in each iteration of the method. Within this setting, for each iteration $i$ of the method, the lower approximation for the conditional cost-to-go function can be devised through the following linear optimization problem:

$$\tilde{Q}_t^{(i)}(v_{t-1}, w_{t,\omega}) = \min_{\alpha, X_{t,\omega}} c_t^T g_{t,\omega} + \alpha \quad (10)$$

s.t. $Af_{t,\omega} + Bg_{t,\omega} + Pu_{t,\omega} = d_t \quad (11)$

$v_{t,\omega} = v_{t-1} - H(u_{t,\omega} + s_{t,\omega}) + w_{t,\omega} \quad (\tilde{\pi}_t^{(i)}) \quad (12)$

$\alpha \geq \tilde{Q}_t^{(k)}(v_t^{(k)}) + \tilde{\pi}_t^{(k),T} (v_{t,\omega} - v_t^{(k)})$, $\forall k \in K^{(i)} \quad (13)$

$\left(g_{t,\omega}, u_{t,\omega}, v_{t,\omega}, s_{t,\omega}, f_{t,\omega}\right) \in X_t. \quad (14)$

Problem (10)-(14) is a linear programming (LP) problem that approximates (6)-(9) from below. The difference between those two models lies in the sec-
ond term of the objective function, which relates to the cost-to-go function representation. In (10)-(14), the cost-to-go function is replaced by an auxiliary variable, $\alpha$, representing the maximum among all the Benders’ cuts, as per (13), found until iteration $i-1$. Thus, $K^{(i)}$ is the set of indexes of the Benders’ cuts found until iteration $i$.

As per Pereira and Pinto (1991), the forward iteration of the SDDP algorithm finds good candidate points of the state, $\{v_t^{(k)}\}_{t \in T, k \in K^{(i)}}$, for which the cost-to-go function should be better approximated. Those points are obtained in the forward step along the SDDP iterations, where a system simulation is performed by solving the approximate dynamic problem (10)-(14) for a Monte Carlo simulation of the inflow scenarios, $\{w_{t,\omega_k}\}_{t \in T, k \in K^{(i)}}$. Following the convergence procedure used in Brigatto et al. (2017) and Street et al. (2017), an evaluation step, based on a forward step with a large number of samples, can be performed and the expected cost of the current approximate policy can be assessed for a user-defined confidence level. If the convergence criterion is not achieved, new Benders’ cut coefficients, $\tilde{Q}^{(k)}_{t+1}(v_t^{(k)})$ and $\tilde{\pi}^{(k)}_{t+1}$, can be obtained through the backward step. Such a step is aimed to perform an approximation improvement on the cost-to-go function representation given by the Benders’ cuts. The calculation of the new cuts over the newly generated state points provides tighter approximations for the true dynamic model (1)-(4). Both coefficients are obtained by the solution of problem (10)-(14) in a reverse, or backward, temporal direction, from $t = T$ to $t = 1$. The first coefficient can be calculated as the expected value of the objective function among all inflow scenarios, i.e.,

$$\tilde{Q}^{(k)}_{t+1}(v_t^{(k)}) = \sum_{\omega \in \Omega_{t+1}} p_\omega \tilde{Q}^{(k)}_{t+1}(v_t^{(k)}, w_{t+1,\omega}),$$

(15)

whereas the second, a subgradient vector, can be obtained as the expected value of the dual vectors of constraint (12), i.e.,

$$\tilde{\pi}^{(k)}_{t+1} = \sum_{\omega \in \Omega_{t+1}} p_\omega \tilde{\pi}^{(k)}_{t+1,\omega}.$$

(16)

Based on the dynamic model presented in this section, the SDDP algorithm as described in Pereira and Pinto (1991); Shapiro et al. (2013); Street et al. (2017) or Ding et al. (2019) can be implemented and a solution for the HD multistage LTHS problem obtained. We refer to Brigatto et al. (2017)
and Street et al. (2017) for further details on the HD SDDP implementation and stopping criterion used in this work.

2.2. Decision-Hazard Approach

In this work, we illustrate the DH idea by allowing the dispatch of a subset of thermoelectric units within set $J$ to be considered as here-and-now decision in the dynamic programming formulation (1)-(4). This can be accounted for into the optimization model through the consideration of nonanticipativity constraints for the here-and-now decisions. We propose using an additional decision vector, namely, $g^o_t$, whose components, $\{g^o_t[j]\}_{j \in J}$, contain the here-and-now dispatch variable, for each unit $j \in J$, that should be made at the beginning of stage $t$. Hence, to implement the nonanticipative constraints in the dynamic model, a new set of constraints is used to ensure that the decision vector component $g_{t,\omega}[j]$ remains constant across the scenarios $\omega \in \Omega_t$, and equal to $g^o_t[j]$ for all the components associated with the here-and-now dispatch variables, i.e., $j \in J$. This leads to the following new formulation for the cost-to-go function:

$$Q_t(v_{t-1}) = \min_{g^o_t, \{X_t,\omega\} \in \Omega_t} \sum_{\omega \in \Omega_t} p_\omega \left[ c^T_t g_{t,\omega} + Q_{t+1}(v_{t,\omega}) \right]$$

s.t.  
$$Af_{t,\omega} + Bg_{t,\omega} + Pu_{t,\omega} = d_t \quad \forall \omega \in \Omega_t$$

$$v_{t,\omega} = v_{t-1} - H(u_{t,\omega} + s_{t,\omega}) + w_{t,\omega} \quad \forall \omega \in \Omega_t$$

$$g_{t,\omega}[j] = g^o_t[j] \quad \forall j \in J, \omega \in \Omega_t$$

$$(g_{t,\omega}, u_{t,\omega}, s_{t,\omega}, f_{t,\omega}, v_{t,\omega}) \in X_t \quad \forall \omega \in \Omega_t.$$

Model (17)-(20) is a two-stage stochastic programming model that extends the HD formulation (1)-(4) to consider a DH stochastic dynamic problem. Expression (20) accounts for the nonanticipativity constraints implying that the current stage scenario realization cannot be anticipated before the dispatch decision for the units in $J$ is made.

Although the SDDP approach could be adapted to consider a cost-to-go function defined by a two-stage stochastic program with nonanticipative constraints, (17)-(20), the backward problem would not be suitable for scenario decomposition as per expression (5). In addition, the forward iteration would require the solution of a full two-stage stochastic problem to define the final state at each stage, $\{v_{t}^{(k)}\}_{k=1,...,i-1}$. The previously described implementation issues associated with the consideration of nonanticipativity constraints
in (17)-(21) constitute a major incompatibility with the primary feature that allows the state-of-the-art SDDP procedure to solve large-scale problems in a reasonable time. It is important to highlight that many different acceleration strategies can be devised within the HD implementation of the SDDP procedure by exploiting the scenario-separability property. This feature allows the current state-of-the-art LP warm-start techniques to be used to solve similar problems only differing from the right-hand side, which is precisely our case.

To overcome the aforementioned implementation issues associated with the scenario-separability property not observed in the DH dynamic model (17)-(21), we propose that all here-and-now variables, \( \{g^o[j]\}_{j \in J} \), of each stage subproblem (a two-stage stochastic problem modeling the new DH scheme) be fixed and defined as state variables of the current stage. This approach yields an ASDM formulation for the dynamic equations. Additionally, the decision of those variables is passed to the previous stage as second-stage (scenario-dependent) variables. Hence, in any given stage \( t \), the vector of here-and-now variables of the next stage for each scenario branch, i.e., \( \{g^o_{t+1,\omega}\}_{\omega \in \Omega_{t+1}} \), will also be part of the decision variables. Thus, the new ASDM is as follows:

\[
Q_t(v_{t-1}, g^o_t) = \min_{\{g^o_{t+1,\omega}, X_t, \omega \in \Omega_t\}} \sum_{\omega \in \Omega_t} p_{t,\omega} \left[ c^T_t g_{t,\omega} + Q_{t+1}(v_{t,\omega}, g^o_{t+1,\omega}) \right] \tag{22}
\]

s.t.

\[
A f_{t,\omega} + B g_{t,\omega} + P u_{t,\omega} = d_t \quad \forall \omega \in \Omega_t \tag{23}
\]

\[
v_{t,\omega} = v_{t-1} - H(u_{t,\omega} + s_{t,\omega}) + w_{t,\omega} \quad \forall \omega \in \Omega_t \tag{24}
\]

\[
g_{t,\omega}[j] = g^o_{t,\omega}[j] \quad \forall j \in J, \omega \in \Omega_t \tag{25}
\]

\[
(g_{t,\omega}, g^o_{t+1,\omega}, u_{t,\omega}, s_{t,\omega}, f_{t,\omega}, v_{t,\omega}) \in X'_t \quad \forall \omega \in \Omega_t. \tag{26}
\]

It is noteworthy that model (22)-(26) differs from (17)-(21) in the following aspects: 1) unlike (20), \( g^o_t \) is a right-hand-side vector of coefficients defined by the state of the system (it is not a decision vector of period \( t \)); 2) new decision vectors, \( \{g^o_{t+1,\omega}\}_{\omega \in \Omega_{t+1}} \), accounting for the newly added state variables are considered in the model and the associated bounds accounted for in the extended version of the polyhedral set \( X'_t \) in (26).

The newly proposed ASDM (22)-(26) is a scenario-decomposable dynamic model, thereby suitable for state-of-the-art SDDP implementations. In this sense, following the same steps found in the previous section, prob-
lem (22)–(26) can be decomposed per scenario as follows:

\[ Q_t(v_{t-1}, g_t^o) = \sum_{\omega \in \Omega_t} p_\omega Q_t(v_{t-1}, g_t^o, w_{t,\omega}). \]  

(27)

Thus, in each iteration \( i \) of the SDDP method, \( Q_t(v_{t-1}, g_t^o, w_{t,\omega}) \) can be approximated from below by the following augmented-state dynamic equation:

\[ \tilde{Q}_t^{(i)}(v_{t-1}, g_t^o, w_{t,\omega}) = \min_{\alpha, g_{t+1,\omega}, \pi_{t,\omega}} \left[ c_t^\top g_{t,\omega} + \alpha \right] \]

s.t. \[ Af_{t,\omega} + B g_{t,\omega} + P u_{t,\omega} = d_t \]

(28)

\[ v_{t,\omega} = v_{t-1} - H(u_{t,\omega} + s_{t,\omega}) + w_{t,\omega} \] \( : (\pi_{t,\omega}^{(i)}) \)

(29)

\[ g_{t,\omega}[j] = g_t^{o(k)}[j], \quad \forall j \in J \] \( : (\gamma_{t,\omega}^{(i)}) \)

(30)

\[ \alpha \geq \tilde{Q}_{t+1}^{(k)}(v_t^{(k)}, g_{t+1}^{(k)}) + \tilde{\pi}_{t+1}^{(k)}(v_t - v_t^{(k)}) + \tilde{\gamma}_{t+1}^{(k)}(g_{t+2,\omega}^{o(k)} - g_{t+1}^{o(k)}), \quad \forall k \in K^{(i)} \]

(31)

\[ (g_{t,\omega}, g_{t+1,\omega}, u_{t,\omega}, s_{t,\omega}, f_{t,\omega}, v_{t,\omega}) \in \mathcal{X}_t'. \]

(32)

Model (28)-(33) is an augmented-state version of the dynamic equation (10)-(14) and can be used to solve the DH version of the LTHS problem using the SDDP algorithm, also allowing for all the features explored in the previously reported works. However, it is worth mentioning that the ASDM requires a new Benders’ cut expression, (32), accounting for the new vector of state variables. Hence, a new vector of coefficients, namely, \( \tilde{\gamma}_{t+1}^{(k)} \), is used to represent the subgradient of the cost-to-go function for the newly added dimensions. Within this new framework, the new Benders’ cuts coefficients can be calculated following the same rationale used in the previous section, i.e.,

\[ \tilde{Q}_{t+1}^{(k)}(v_t^{(k)}, g_{t+1}^{o(k)}) = \sum_{\omega \in \Omega_{t+1}} p_\omega \tilde{Q}_{t+1}^{(k)}(v_t^{(k)}, g_{t+1}^{o(k)}, w_{t+1,\omega}) \]

(34)

\[ \pi_{t+1}^{(k)} = \sum_{\omega \in \Omega_{t+1}} p_\omega \tilde{\pi}_{t+1}^{(k)} \quad \text{and} \quad \gamma_{t+1}^{(k)} = \sum_{\omega \in \Omega_{t+1}} p_\omega \tilde{\gamma}_{t+1}^{(k,\omega)} \]

(35)

For completeness, it is important to clarify the procedure to obtain a lower bound and first-stage here-and-now decisions within the proposed ASDM for
the DH scheme. For a given vector of initial storage amount $v_0$, the lower bound, $LB^{(i)}$, for the SDDP method and the first-stage vector of here-and-now decisions, $g_1^{o(i)}$, can be found by solving the following simplified version of (28)-(33):

$$LB^{(i)} = \min_{\alpha, g_1^o} \alpha$$

s.t. $\alpha \geq \tilde{Q}_1^{(k)}(v_0, g_1^{o(k)}) + \gamma_{t+1}^{(k)}(g_1^o - g_1^{o(k)}), \forall k \in K^{(i)}$  \hspace{1cm} (37)

$$g_1^o \in \text{proj}_{g_1^o}(\mathcal{X}_1').$$  \hspace{1cm} (38)

Model (36)-(38) solves the approximate version of the first-stage problem, $\min_{g_1^o} Q_1(v_0, g_1^o)$, where the Benders’ cuts only consider the components related to $g_1^o$, because $v_0$ is known. Finally, proj$_{g_1^o}(\mathcal{X}_1')$ considers the bound constraints for $g_1^o$ through the projection of $\mathcal{X}_1'$ onto the dimensions associated with this vector.

It is worth mentioning that the proposed ASDM increases the dimension of the state according to the number of here-and-now variables, $|J|$, which in our case is a subset of the thermoelectric units. However, for a more practical and efficient implementation of the ASDM, a moderate increase in the state can be achieved by adding to the state only the aggregated (total) thermoelectric generation for each electrical subsystem$^3$ in $s \in S$. In such case, vector $g_t^{(o)}$, whose components $\{g_t^o[s]\}_{s \in S}$ account for the total generation within each subsystem, can be defined with a reduced dimension (because, in general, $|S| << |J|$). To adapt model (28)-(33) to such a case, one is only required to replace (25) by

$$\sum_{j \in J_s} g_{t,\omega}[j] = g_t^o[s], \forall s \in S,$$  \hspace{1cm} (39)

where $J_s$ represents the subset of units of $J$ that belongs to subsystem $s$. Hence, by replacing (25) and (31) with (39), the dispatch within each subsystem will naturally follow the minimum cost order, where cheaper generators are prioritized.

$^3$We refer to Diniz and Maceira (2013) for similar aggregation scheme.
3. Performance measures

As argued in the introduction section, the implementability of a policy relies on decision models that deliver here-and-now nonanticipative decisions (implementable actions). To measure the isolated impact of a policy that simplifies the DH scheme, we propose two quantitative metrics: 1) the time-inconsistency GAP (TIGAP) owing to the HD simplification, which estimates the additional expected cost incurred by the inconsistency between planned and implemented decisions Brigatto et al. (2017), and 2) the value of the DH policy (VDHP), which resembles, within our context, the well-known concept of value of the stochastic solution (VSS) Birge and Louveaux (2011). Hence, three policies must be simulated and their expected costs estimated. They are introduced and explained in the following paragraphs.

1) **HD planning policy (HDplan)** – this policy is simulated based on model (1)-(4) assuming the HD hypothesis, i.e., the assessment of its expected cost assumes that the system operator has the perfect information of the uncertainty realization within stage $t$ when deciding vector $g^{(o)}_t$. The expected operational cost evaluated under such policy, namely, $C^{(HDplan)}$, constitutes an optimistic estimate for the expected operational cost throughout the study horizon. Therefore, this cost constitutes a lower-bound reference used to assess the value of the one-step-ahead perfect information. Although not implementable in practice, such policy is largely used in both academic (Pereira and Pinto (1991); Shapiro (2011); de Matos and Finardi (2012); Philpott and De Matos (2012); Brigatto et al. (2017); Street et al. (2017)) and industry studies (Maceiral et al. (2018); Shapiro et al. (2013)) owing to its simplicity.

2) **HD implementation policy (HDimp)** – the HD policy is one-step ahead anticipative, thereby not implementable in practice. To generate implementable decisions based on the HD cost-to-go function, we emulate the current industry practice adopted in Brazil, where first-stage here-and-now decisions are made based on a deterministic model that uses the expected value of inflows for the first period. Hence, at the beginning of each simulated inflow scenario, a first-stage vector of here-and-now decisions is obtained using model (6)-(9) for the expected inflow. Subsequently, the first-stage vector of here-and-now decisions is made fixed by means of a constraint in this model, and wait-and-see variables are obtained by solving the constrained
version of the model for each scenario of the current stage\textsuperscript{4}. The state is updated accordingly, and the process is repeated until the end of the horizon. This policy yields a hybrid and time-inconsistent policy because the used HD cost-to-go function did not consider the here-and-now decisions based on the expected scenario. Hence, the future planned decisions will differ from the actual implemented ones. As the HD implementation policy delivers suboptimal values for here-and-now variables, based on the expected value, the expected cost estimate for the HD implementation policy, namely, $C^{(HDimp)}$, should be greater than or equal to the expected cost given by the HD planning policy, i.e., $C^{(HDimp)} \geq C^{(HDplan)}$ Birge and Louveaux (2011). To accurately assess the cost of an inconsistent policy, the out-of-sample evaluation step should follow the procedure derived in Brigatto et al. (2017), where the cost-to-go functions are first derived based on forward iterations using the implementation model and backward iterations using the HD simplified model.

3) DH policy – this policy is simulated based on model (17)-(21), which can be efficiently implemented based on (22)-(26) as per section 2.2. It constitutes an improvement with regard to the HD implementation policy as it incorporates into the cost-to-go function the actual model that will be used in the future. Therefore, assuming that the implementation model is (17)-(21), the expected cost estimate for this policy, namely, $C^{(DH)}$, lies in between the cost estimate for the two previously introduced policies, i.e., $C^{(HDplan)} \leq C^{(DH)} \leq C^{(HDimp)}$.

The evaluation of each one of the three aforementioned policies depends on the different dynamic models used. However, in any of the three cases, each policy-expected cost estimate can be evaluated based on the sample average dispatch cost obtained as the result of a final forward step simulation within each policy configuration. Therefore, if $g^{(p)}_{t,\omega}$ is the dispatch decision obtained by policy $p \in \{HDplan, HDimp, DH\}$ at period $t$ and scenario $\omega$, each of the aforementioned expected cost estimates can be calculated by the

\textsuperscript{4}In this constrained model, only wait-and-see decisions are optimized and defined for each scenario, whereas the vector of here-and-now decisions is fixed and equal to the value obtained with the deterministic model.
following formula:
\[
C(p) = \frac{1}{N} \sum_{t \in T} \sum_{\omega \in \hat{\Omega}_t^N} c_t^T g_{t,\omega}^{(p)}.
\] (40)

In (40), \(\hat{\Omega}_t^N\) represents a set of \(N\) independently generated scenarios, randomly sampled from \(\Omega_t\), and \(g_{t,\omega}^{(p)}\) represents the optimal dispatch decisions, for each period \(t\), sampled scenario \(\omega \in \hat{\Omega}_t^N\), and policy \(p \in \{\text{HDplan, HDimp, DH}\}\).

Based on the cost of the three aforementioned policies, the evaluation metrics can be assessed as follows:

\[
TIGAP = C^{(\text{HDimp})} - C^{(\text{HDplan})},
\] (41)

\[
VDHP = C^{(\text{HDimp})} - C^{(\text{DH})}.
\] (42)

The TIGAP is a measure of the expected regret, or frustration, in terms of the additional expected cost incurred when adopting modeling simplifications in the planning stage. Hence, this metric can be used to test the planning model bias, and can be regarded as a measure of how optimistic the planning stage is with regard to the implementation model within a controlled experiment (where no other effect is considered). Finally, VDHP is a measure of the expected benefit (in terms of cost savings) by considering the nonanticipativity constraints in the planning stage, which, ultimately, relates to the benefit of an improved assessment of the opportunity cost of the water.

4. Case study

The objectives of this case study are threefold: 1) to raise the awareness of the HD issue using the TIGAP metric, 2) quantify the potential benefits of a DH scheme through the VHDP improvement index, and 3) demonstrate that the proposed ASDM is an effective model for solving the DH multi-stage LTHS problem. Hence, we used realistic data from the Brazilian power system, which currently adopts a similar expected-value-based approach as described for the HD implementation policy. The methods were developed with Julia Language version 0.5.1 using the Xpress solver. All computational experiments were executed on an Intel(R) Core(TM) i7-3960X CPU @ 3.3 GHz with 64 GB of RAM memory.

To achieve objectives 1 and 2 of this case study, we devised a controlled experiment based on the following hypotheses: 1) constraints (18)-(21) are
always feasible for the real system and therefore, all decisions simulated in
the implementation step, for both the HD implementable policy and DH
policy, are assumed to be feasible; II) to isolate the HD simplification effect
from the potential inconsistency due to the different time step with which
planning and operation are carried out, the whole study is conducted on
the same granularity\(^5\). This assessment framework allows us to isolate the
negative effects of the HD hypothesis without introducing bias from other
simplifications. In this context, the present case study can be regarded as an
extension of the issues raised in Brigatto et al. (2017). Finally, to achieve the
third objective of this case study, we compare the solution time of two SDDP-
based algorithms for solving the DH LTHS problem. The first algorithm relies
on the implementation of the SDDP algorithm with a subproblem relying
directly on the full two-stage DH model (17)-(21), hereinafter referred to as
fullDH-SDDP. In contrast, the proposed approach based on the augmented-
state DH version of the SDDP model (22)-(26) is hereinafter referred to as
ASDH-SDDP.

The Brazilian energetic model adopted in this work comprises four dis-
tinct subsystems: Southeast (SE), South (S), Northeast (NE), and North
(N). The hydroplants are aggregated into four different equivalent reservoirs,
each of which located in one of the subsystems. The system comprises 111
thermal plants distributed throughout the subsystems, from which the 55
cheaper and slower generators are considered to be dispatched as here-and-
now resources (2 nuclear, 11 coal, and 42 gas-fired). Thus, the dispatch
decision for the hydros and faster (and more expensive) thermoelectric units
are considered as wait-and-see recourse actions. Table 1 summarize the ther-
molectric data, where the total maximum and minimum generation levels
\(G_{\text{max}}\) and \(G_{\text{min}}\) are expressed in terms of average MW per month. The
complete dataset can be found in Street et al. (2017).

\(^5\)We assume that the only difference between the model of the system used in the
planning step only differs from the model used when implementing decisions is the HD
hypothesis.
### Table 1: Thermoelectric data per subsystem.

<table>
<thead>
<tr>
<th></th>
<th>Here-and-now generators</th>
<th>Wait-and-see generators</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of Units</td>
<td>Total $G_{max}$</td>
</tr>
<tr>
<td>SE</td>
<td>24</td>
<td>8,788</td>
</tr>
<tr>
<td>S</td>
<td>10</td>
<td>2,498</td>
</tr>
<tr>
<td>NE</td>
<td>8</td>
<td>2,860</td>
</tr>
<tr>
<td>N</td>
<td>13</td>
<td>2,451</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>55</strong></td>
<td><strong>16,597</strong></td>
</tr>
</tbody>
</table>

The planning horizon considered in this study encompasses 5 years, in a monthly basis. Although omitted in the model, a present value factor related to a discount rate of 0.5% per month (i.e., $1.005^{-1}$) is applied. Following previous works (see Street et al. (2017); Brigatto et al. (2017) and references therein), the inflow scenarios utilized in this study are based on monthly observations for the four subsystems from 1991 to 2015, amounting to 25 observations per month. A periodic autoregressive model of order one was also implemented according to Maceira et al. (2002) and Shapiro (2011) to incorporate the inter-temporal effects of water inflow. The simulation is based on a bootstrap procedure of the residuals following the finds of Castro et al. (2015).

We adopted the same parameters used in Brigatto et al. (2017) and Street et al. (2017), namely, 1000 initial forward-and-backward iterations\(^6\) respectively, followed by a convergence test based on intermediate evaluations steps at each 100 iterations. Each convergence check is based on a t-test for difference of means applied to the sampled average cost estimated in two consecutive evaluation steps and based on the respective lower bound differences. The sampled average cost used in the t-test are estimated based on a simulation with 3000 newly generated scenarios. The convergence of the lower bound is tested for a 1% gap. We refer the reader to the appendix of Brigatto et al. (2017) for the description of the stopping criterion used in this paper.

Regarding the first two objectives of this case study, Table 2 presents the

---

\(^6\)We consider one sample per forward iteration and all the residuals as scenarios in each backward iteration. Within this strategy, each forward step uses the information of all local approximations built on previous steps, which is an effective strategy when parallel computing is not used.
cost for the three policies used to devise the proposed evaluation metrics: (41) and (42).

<table>
<thead>
<tr>
<th></th>
<th>$C^{(HD_{\text{plan}})}$</th>
<th>$C^{(HD_{\text{imp}})}$</th>
<th>$C^{(DH)}$</th>
<th>TIGAP</th>
<th>VDHP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{TIGAP}$</td>
<td>12,467.7</td>
<td>13,960.4</td>
<td>13,222.3</td>
<td>1,492.7</td>
<td>738.1</td>
</tr>
<tr>
<td>$\text{VDHP}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The results depict a non-negligible impact of disregarding nonanticipativity constraints on the system operation. Based on the TIGAP metric, the expected regret (additional cost) for the system operator quantifying the next five years' expected cost based on an HD-based planning tool is MM$ 1,492.7$, i.e., approximately 12%. The adoption of a consistent DH policy, meanwhile, provides an expected reduction of MM$ 738.14$ (5.3%) with regard to the cost of the HD implementation policy, which represents approximately 50% of the inconsistency gap. It is worth emphasizing that the VDHP represents the expected over-cost, or sub-optimality cost, due to the adoption of a HD simplification in the planning step. Moreover, both the TIGAP and VDHP evaluation metrics are based on the sample average statistics, suitable for a t-test to verify the statistical significance. For a sample size of 3,000 simulated inflows, the t-test rejected the null hypothesis, where the true mean is assumed equal to zero, with p-values lower than 0.1%. Therefore, the results in Table 2 are statistically significant.
Figure 1: here-and-now thermal dispatch - SE system

Figure 2: Second-stage thermal dispatch - SE system
The relevant operational differences among the three policies, \{HDplan, HDimp, DH\}, are depicted in Figures 1 and 2, where the average first- and second-stage thermoelectric dispatches are presented, respectively. Additionally, in Figure 3, the average spot prices for the primary subsystem (SE) are presented for the three policies. In these figures, the red dots represent the HD planning policy, the dotted black lines refer to the HD implementation policy, and the continuous black lines are associated with results of the DH policy.

Figure 1 reveals a relevant structural increase in the expected dispatch of the here-and-now thermoelectric units for the DH policy in comparison to the HD-based ones. Notwithstanding, the primary difference between HD planning and the HD implementation policies are observed in the second-stage dispatch levels depicted in Figure 2. The unrealistic flexibility within the anticipative HD planning policy artificially reduces the second-stage dispatch levels (most expensive units), which would not be achievable within an implementable policy accounting for here-and-now decisions. As a consequence of this implementation issue, spot price discrepancies are observed and follow a similar pattern observed in the second-stage dispatch levels, as shown in Figure 3. Finally, it is worth mentioning that the DH policy mitigates these
two issues, while yielding an expected overall operational cost saving of 5.3% with regard to the HD implementation scheme.

Finally, regarding the third objective of this case study section, Table 3 compares the computational times for solving the DH multistage LTHS problem with the two DH-based algorithms: the fullDH-SDDP and ASDH-SDDP, with the computing time required to solve the HD planning policy. As shown, the incorporation of nonanticipativity constraints increases the total computational burden with regard to the HD reference as per the results for both DH-based algorithms. However, the relevant increase in the computational time, of 21.2 times, exhibited by the fullDH-SDDP algorithm can be significantly mitigated to a factor of 4.9 (approximately 23.1% of the fullDH-SDDP time), by the proposed ASDH-SDDP.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Time until convergence</th>
<th>Increasing factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>HD reference</td>
<td>1,758</td>
<td>1.0</td>
</tr>
<tr>
<td>fullDH-SDDP</td>
<td>37,181</td>
<td>21.2</td>
</tr>
<tr>
<td>ASDH-SDDP</td>
<td>8,680</td>
<td>4.9</td>
</tr>
</tbody>
</table>

5. Conclusions

The idea of isolating a given source of inconsistency that is due to a modeling simplification in the planning step and quantify its effects through the time inconsistency GAP within a controlled experiment is proposed in Brigatto et al. (2017). With that information, one can rank the simplifications and tackle them in order of relevance (GAP of inconsistency). In this work, we raised the awareness of the anticipativity issue associated with the HD hypothesis Pereira and Pinto (1991) largely adopted in LTHS applications (Shapiro (2011); Maceira et al. (2002); Maceiral et al. (2018); de Matos et al. (2015); Philpott and Guan (2008); Street et al. (2017); Brigatto et al. (2017)), and proposed an ASDM to efficiently address the DH multistage version of this problem.

Computational experiments with our illustrative Brazilian power system case study shows that the implemented actions based on a HD scheme may significantly deviate from the planned decisions. Within the limitations of the hypothesis used in our controlled case study, a cost increase of approximately 12% was observed. Additionally, more volatile and increased levels
of expensive thermoelectric corrective generation were observed when implementing the HD policy. The spot price distortions followed the same pattern. As per our results, the proposed DH scheme could mitigate such effects. Although, the 12% cost increase can not be directly generalized for other instances, it illustrates the potential negative effects due to this source of inconsistency. Thus, it is highly recommended that ISOs, relying on HD implementations, conduct specific studies based on official models and data to accurately quantify the effect of this simplification. Finally, the proposed ASDM could reduce to 23.1% the additional computational burden, inherent to the more complex DH policy, compared to the direct extension of the SDDP algorithm to consider the nonanticipative DH problem.

As future works, we highlight the importance of the evaluation of the cost of other sources of inconsistency. For instance, the impact of short-term uncertainty, reservoir aggregation, nonlinearities of unit commitment constraints, and the time-step granularity with which here-and-now decisions are revised are also relevant features that have been simplified in long-term planning models. Additionally, the investigation of decomposition techniques to improve the performance of the full DH model (fullDH-SDDP) and the consideration of new non-Gaussian stochastic processes into the SDDP algorithm constitute promising avenues for future research. Finally, further computational tests should be carried out to study the performance of the augmented-state DH formulation for more complex systems.

References


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