On Mixed Integer Programming Formulations for the Unit Commitment Problem

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We provide a comprehensive overview of mixed integer programming formulations for the unit commitment (UC) problem. UC formulations have been an especially active area of research over the past twelve years, due to their practical importance in power grid operations, and this paper serves as a capstone for this line of work. We additionally provide publicly available reference implementations of all formulations examined. We computationally test existing and novel UC formulations on a suite of instances drawn from both academic and real-world data sources. Driven by our computational experience from this and previous work, we contribute some additional formulations for both generator production upper bounds and piecewise linear production costs. By composing new UC formulations using existing components found in the literature and new components introduced in this paper, we demonstrate that performance can be significantly improved – and in the process, we identify a new state-of-the-art UC formulation.

Key words: Unit commitment, mixed integer programming, mathematical programming formulations

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Nomenclature

Indices and Sets

$g \in \mathcal{G}$	Thermal generators.
$l \in \mathcal{L}_g$	Piecewise production cost intervals for generator g: $1, \ldots, L_g$.
$s \in \mathcal{S}_g$	Start-up categories for generator g, from hottest (1) to coldest (S_g).
$t \in \mathcal{T}$	Hourly time steps: $1, \ldots, T$.
$[t, t') \in \mathcal{X}_g$	Feasible intervals of non-operation for generator g with respect to its minimum down-time, i.e., $[t, t') \in \mathcal{T} \times \mathcal{T}$ such that $t' \geq t + DT_g$, including times (as necessary) before and after the planning horizon $\mathcal{T}$.
$n \in \mathcal{N}$	Set of buses: $1, \ldots, N$.
$g \in \mathcal{G}_n$	Thermal generators at bus n.
$k \in \mathcal{K}$	Set of branches (lines): $1, \ldots, K$.
$k \in \delta^+(n)$	Lines to bus n, $\subseteq \mathcal{N}$.

$k \in \delta^-(n)$ Lines from bus n, $\subseteq \mathcal{N}$.

Parameters

B_k Susceptance of branch k.

C_g^l Cost coefficient for piecewise segment l of generator g (\$/MWh).

$\overline{C}_g^l$ Cost of generator g producing $\overline{P}^l$ MW of power (\$/h).

C_g^R Cost of generator g running and operating at minimum production $\underline{P}_g$ (\$/h).

C_g^s Start-up cost in category s for generator g (\$).

C_g^w Shut-down cost for generator g (\$).

C_{LP} Penalty cost for failing to meet or exceeding load (\$/MWh).

C_{RP} Penalty cost for failing to meet reserve requirement (\$/MWh).

$D_n(t)$ Load (demand) at time t at bus n (MW).

D_g Number of hours generator g is required to be off at $t=1$ (h).

DT_g Minimum down time for generator g (h).

F_k Transmission capacity of branch k (MW).

$k(n)$ From bus for branch k.

$k(m)$ To bus for branch k.

$p_g(0)$ Power output of generator g at time 0, $\in [0, \overline{P}_g]$ (MW).

$\overline{P}_g$ Maximum power output for generator g (MW).

$\overline{P}_g^l$ Maximum power available for piecewise segment l for generator g (MW).

$\underline{P}_g$ Minimum power output for generator g (MW).

$R(t)$ System-wide spinning reserve at time t (MW).

RD_g Ramp-down rate for generator g (MW/h).

RU_g Ramp-up rate for generator g (MW/h).

SD_g Shut-down rate for generator g (MW/h).

SU_g Start-up rate for generator g (MW/h).

TC_g Time offline after which generator g goes absolutely cold, i.e., enters state S_g.

$\underline{T}_g^s$ Time offline after which the start-up category s is used ($\underline{T}_g^1 = DT_g$, $\underline{T}_g^{S_g} = TC_g$).

$u_g(0)$ State of generator g at time 0, $\in \{0,1\}$.

U_g Number of hours generator g is required to be on at $t=1$ (h).

UT_g Minimum run time for generator g (h).

$\underline{W}_n(t)$ Aggregate renewables generation that is must-take at bus n at time t (MW).

$\overline{W}_n(t)$ Aggregate renewables generation available at bus n at time t (MW).

Variables

$p_g(t)$	Power generated by generator g at time t (MW), ≥ 0.
$p'_g(t)$	Power generated above minimum by generator g at time t (MW), ≥ 0.
$\overline{p}_g(t)$	Maximum power available from generator g at time t (MW), ≥ 0.
$\overline{p}'_g(t)$	Maximum power available above minimum from generator g at time t (MW), ≥ 0.
$p_{W,n}(t)$	Aggregate renewable generation used at bus n at time t (MW), ≥ 0.
$p^l_g(t)$	Power from piecewise interval l for generator g at time t (MW), ≥ 0.
$r_g(t)$	Spinning reserves provided by generator g at time t (MW), ≥ 0.
$u_g(t)$	Commitment status of generator g at time t, $\in \{0,1\}$.
$\tilde{u}_g(t)$	Commitment transition status of generator g at time t, $\in \{0,1\}$.
$v_g(t)$	Start-up status of generator g at time t, $\in \{0,1\}$.
$w_g(t)$	Shut-down status of generator g at time t, $\in \{0,1\}$.
$c^p_g(t)$	Production cost over $\underline{P}_g$ for generator g at time t (\$), ≥ 0.
$c^{SU}_g(t)$	Start-up cost for generator g at time t (\$), ≥ 0.
$c^{SD}_g(t)$	Shut-down cost for generator g at time t (\$), ≥ 0.
$\delta^s_g(t)$	Start-up indicator in category s for generator g at time t, $\in \{0,1\}$.
$x_g(t,t')$	Indicator arc for shut-down at time t, start-up at time t', uncommitted for $i \in [t,t')$, for generator g, $\in \{0,1\}$, $[t,t') \in \mathcal{X}_g$.
$\theta_n(t)$	Voltage angle at bus n at time t (radians).
$f_k(t)$	Power flow on branch k from bus $k(n)$ to $k(m)$ at time t (MW).
$s_n(t)$	Load mismatch at bus n at time t (MW).
$s_R(t)$	Reserve shortfall at time t (MW), ≥ 0.

1. Introduction

Unit commitment (UC) is a foundational problem in real-world power systems operations and planning. The UC problem is to determine an operating schedule – which specifies unit on/off statuses and associated production levels – for a fleet of thermal power generators to meet forecasted load at minimum total production cost, while satisfying a range of physical (e.g., Kirchhoff's laws) and operational (e.g., production ramping limits) constraints. Variants of UC are used in power systems operations for day-ahead market clearing, and day-ahead and intra-day reliability processes. Simplified versions of these UC variants also see widespread use in power systems planning, specifically in production cost models. Beginning with the PJM independent system operator in 2005, power systems operators in the United States have transitioned from solving UC via heuristics based on Lagrangian relaxation to formulating UC as mixed-integer (linear) programs (MIPs) (O'Neill 2017) that are solved via commercial branch-and-cut engines such as CPLEX (International Business Machines Corporation 2018) and Gurobi (Gurobi Optimization, Inc. 2018). The cost savings associated with this transition are estimated at $5 billion USD annually in the United States alone (O'Neill 2017). For a recent high-level overview on UC and its importance in power systems operations and planning, we refer to Anjos et al. (2017).

The typical real-world day-ahead UC problem is solved at an hourly time resolution over 36 to 48 hours, and considers hundreds to thousands of thermal generators and transmission systems with up to tens of thousands of buses and lines. Because the size of the transmission system can make solving the UC problem prohibitively difficult in practice, typically only a small subset of transmission constraints are considered in day-ahead UC variants (Chen et al. 2016). While system operators commonly have up to three hours to determine their day-head UCs, much of that time is consumed verifying the input data (e.g., market bids and offers), performing post-solve feasibility checks (e.g., considering AC power flow), re-solving after the integration of feasibility cuts, performing a pricing run, and finally publishing the results. Consequently, it is strongly desirable in practice to have a UC solution available in 10 to 15 minutes.

The importance of UC in practice and the size of real-world power systems has led to significant interest from both the research and practitioner communities in developing improved mathematical programming formulations of UC, in order to reduce branch-and-cut solve times. It is well-known that the choice of a MIP formulation for any given optimization problem can have a significant impact on practical computational difficulty, and UC is no exception. Consequently, there has been a significant research focus over the past twelve years in developing improved UC MIP formulations, i.e., focusing on "tighter" formulations with improved linear programming (LP) relaxations in order to reduce the tree exploration time associated with branch-and-cut. In this context, this paper makes the following contributions:

- We catalog existing formulations in the literature for the UC problem as originally defined by Carrion and Arroyo (2006), who additionally introduced the base MIP UC formulation that we consider herein. Improvements to this base formulation have been the subject of many subsequent papers, and such improvements are closely tracked by industry.

- We computationally examine the performance of 41 different UC formulations on 68 instances considering three generator fleets of varying size. This analysis carefully quantifies the performance of UC formulations reported in the literature and identifies the present state-of-the-art.

- We make publicly available reference implementations for all of the UC formulations considered, expressed in the Pyomo (`https://www.pyomo.org`) algebraic modeling language (Hart et al. 2011, 2017), as part of the EGRET software package (`https://github.com/grid-parity-exchange/Egret`). The load and generator data for all test instances considered is part of the IEEE PES Power Grid Lib - Unit Commitment benchmark library (`https://github.com/power-grid-lib/pglib-uc`). The release of these reference implementations will allow the research community to consider additional combinations that we have not considered on their own test instances. The availability of tested MIP models and instances for UC will significantly reduce the time required to examine alternative novel UC formulations, ensures that accurate and controlled computational experiments can be conducted across the research community, and provides a performance baseline against which the efficacy of novel UC formulations can be assessed.

- Driven by computational experience from Damcı-Kurt et al. (2016), Knueven et al. (2018a), and preliminary work for this paper, we introduce two formulation enhancements for UC. One is a set of strong variable upper bound inequalities for ramping-constrained generators. The other is a tightening of the piecewise linear production costs. The latter allows us to derive, in an online supplement (Knueven et al. 2018d), a convex hull result for a generator with piecewise linear production costs, start-up and shut-down ramping constraints, and minimum up-time/down-time constraints.

- We introduce two novel UC formulations, one of which represents new combinations of components from existing UC formulations, while the other draws on both new components and existing formulations. We demonstrate that the latter formulation significantly improves on the performance of any previously reported UC formulation, establishing a new state-of-the-art.

In all, this paper puts forth a new benchmark formulation for the UC problem. While we do not consider various extensions of UC, e.g., stochastic or robust variants, different reserve products, or improvements to the network model, nearly every extension has at its core formulations of the generation units, the variants of which is the primary focus of this paper. We therefore believe the results herein are broadly relevant to extensions of the UC problem.

The remainder of this paper is organized as follows. We begin in Section 2 by introducing the basic UC model and key issues in its formulations. In Section 3, we detail the key formulation variants introduced for UC to date. Our computational experiments are based on several complete UC formulations introduced by researchers over the past twelve years; these are documented in Section 4, placed in the notational context of Section 3. We additionally consider some new UC formulations, described in Section 5. Our computational experiments are summarized in Section 6, and we conclude with a summary of our findings and contributions in Section 7.

2. Overview

We formulate the general UC problem as follows:

$$\min \sum_{g\in\mathcal{G}} \sum_{t\in\mathcal{T}} c_g(t) \tag{1a}$$

$$\text{s.t.} \sum_{g\in\mathcal{G}} A_g(p_g, \overline{p}_g, u_g) + N(s) = L \tag{1b}$$

$$(p_g, \overline{p}_g, u_g, c_g) \in \Pi_g \qquad \forall g \in \mathcal{G}. \tag{1c}$$

Here c_g is the cost vector associated with $p_g, \overline{p}_g, u_g$, such that the objective function (1a) is to minimize system operation cost. The vectors p_g, $\overline{p}_g$, and u_g represent the feasible generation schedule, maximum power available, and the on/off status for generator g, respectively. The matrix $A_g(p_g, \overline{p}_g, u_g)$ determines how the generator interacts with the system requirements, which are written in matrix form as equation (1b). Here the variable s denotes other potential decision variables involving the operation of the system. Finally, constraint (1c) defines the feasible region for each generator's schedule and the cost associated with that schedule, with the set Π_g representing the description of the feasible operating region and associated cost for each generator $g \in \mathcal{G}$. Because generators usually have a discrete component to their operation (they are either on and operating above some minimum level $\underline{P}$, or off), the set Π_g is often non-convex. In reality, the system constraints (1b) are non-convex due to non-linearities associated with modeling AC power flow physics; in practice this is often approximated using a linear set of constraints. For the purposes of this paper we will assume the system constraints are linear in nature, and consider only a minor variation with regards to how reserve requirements are modeled.

Most of the research for MIP formulations of UC has focused on developing better descriptions of the non-convex set Π_g. We will refer to $\text{conv}(\Pi_g)$ as the *generator polytope*. Tighter descriptions of Π_g tend to improve the LP relaxation bound of (1), which often reduces computation times by reducing the (implicit) enumeration necessary to find and prove an optimal solution. However, in general, tighter descriptions of Π_g tend to use more variables and/or constraints. Consequently, which description to use in practice is a balance between tightness and compactness. Further, modern commercial MIP solvers have sophisticated cut generation and pre-solve routines which can serve to tighten a formulation automatically.

Common constraints for Π_g are minimum/maximum generation levels when on, production ramping limits, minimum up times and down times, piecewise linear convex production costs, and time-dependent start-up costs. Most of the UC literature involves finding a *locally ideal* or *locally tighter* formulation for a subset of the constraints in Π_g, that is, just considering one (or two) of the types of constraints above and deriving a result for that (or those) constraint type(s).

Regarding the minimum up-time/down-time constraints, Lee et al. (2004) showed that a convex hull description of these constraints alone is exponential in the space of the u variables, though the constraints can be separated in linear (in T) time. Malkin (2003) and Rajan and Takriti (2005) showed these same minimum up-time/down-time constraints have an $\mathcal{O}(T)$ convex hull description when one adds the start-up variable $v(t)$, indicating the generator is switched on at time t. Subsequent work has shown the up-time/down-time constraints provided by Malkin (2003) and Rajan and Takriti (2005) to be computationally beneficial (Ostrowski et al. 2012, Morales-España et al. 2013a), and are usually one of the core components of a modern UC model.

Frangioni et al. (2009) demonstrated how to tighten the piecewise linear production costs using *perspective cuts*. These serve to accurately model the production costs not just in the space of the p variables, but also in the space of the (p, u) variables. The formulation given is the convex hull of the piecewise linear production costs with the addition of basic minimum/maximum power constraints. Sridhar et al. (2013) gave similar results for a broad class of potential piecewise linear formulations, and Wu (2016) confirmed this result for the SOS2-type formulation and gives an additional locally ideal reformation of the piecewise linear constraints from Carrion and Arroyo (2006). Chen and Wang (2017) gave yet another locally ideal formulation which is a modification of the SOS2-type formulation. In this paper, we will present another formulation which is locally ideal for the piecewise production cost, generation limits, start-up and shut-down ramps, and the minimum up-time and down-time constraints. An extension of the result from Knueven et al. (2018c) will show this is also tight when time-dependent start-up costs are added.

Morales-España et al. (2013a) presented a tightening of the generation limits when there are start-up and shut-down ramping limits, which are the maximum the generator can produce when turning on and turning off, respectively. A follow-up paper, Gentile et al. (2017), showed that a slight modification of this formulation is locally ideal for the generation limits, start-up/shut-down ramp limits, and minimum up/down times.

Morales-España et al. (2013a) also introduced and computationally evaluated a reformulation for time-dependent start-up costs, which had been typically modeled as in Nowak and Römisch (2000). Brandenberg et al. (2017) gave a convex hull description for start-up costs as modeled in Nowak and Römisch (2000) with an exponential number of constraints, but provided a linear separation algorithm. Knueven et al. (2018c) proved the formulation from Morales-España et al. (2013a) is tighter than that of Nowak and Römisch (2000) and introduced a new formulation that is as tight, but smaller, than a locally ideal formulation for time-dependent start-up costs. Knueven et al. (2018c) further showed that any formulation ideal in the up-time/down-time constraints of Malkin (2003) and Rajan and Takriti (2005) will have an integer optimal solution when the start-up cost formulation introduced therein is added. Queyranne and Wolsey (2017) gave a convex hull description for time-dependent start-up costs which requires $\mathcal{O}(T^2)$ constraints. Finally, Silbernagl et al. (2016) introduced a different model for start-up costs that is a restriction of the model considered in Nowak and Römisch (2000), but which explicitly models power plant temperatures. This model provides a more accurate representation of start-up costs using only a linear number of variables and constraints. Silbernagl (2016) showed this formulation for start-up costs has an $\mathcal{O}(T^2)$ convex hull description.

UC ramping constraints are perhaps the most studied. Ostrowski et al. (2012) introduced strengthening inequalities for ramping, some of which are shown to be facets of certain projections of Π_g. Damcı-Kurt et al. (2016) defined the convex hull for the two-period ramp-up polytope and the two-period ramp-down polytope, as well as several classes of facet-defining inequalities for the ramp-up and ramp-down constraints, respectively. Pan and Guan (2016) gave a convex hull description of a generator for three time periods with minimum up/down time constraints, production limits, ramping limits, and start-up/shut-down limits, and facet-defining inequalities for this polytope when $T > 3$. Pan and Guan (2017) extended this result for general T in a few special cases, and gave convex hull descriptions for the ramp-up polytope (that is, minimum up/down time constraints, production limits, ramp-up limits, and start-up limits) and ramp-down polytope for general T. However, all of these formulations require an exponential number of constraints when expressed with the p, u, v (and/or w) variables.

As was the case with the minimum up-time/down-time constraints, an extended formulation may prevent such exponential blow-up in the number of constraints. Frangioni and Gentile (2015a,b) and Knueven et al. (2018a) showed the generator polytope conv(Π_g) can be expressed with $\mathcal{O}(T^3)$ variables and constraints. Guan et al. (2018) gave two different extended formulations for the generator polytope, and also introduced a representation of the generator polytope in the stochastic case. However, all of the extended formulations for the whole of conv(Π_g) tend to be too large for direct incorporation into a UC MIP model.

While a perfect formulation for an individual generator exists, it is not practical to use within a MIP formulation for UC. The question then is an engineering one–what MIP formulation is good in practice? In other words, how can we develop a MIP formulation that typically performs well when solving UC with a modern commercial MIP solver. This challenge has often been described in the literature as a trade-off between tightness and compactness. A "tight" formulation for a generator is either a convex hull description for Π_g or a set of equations that are thought to provide a good approximation for conv(Π_g). In contrast, a "compact" formulation is one that describes Π_g without using "too many" variables and constraints. The thought is that a tight formulation will provide a better bound for a MIP solver to exploit, at the cost of increasing the size of the LP relaxation. In contrast, a compact formulation will result in faster node solve times, at the cost of weakening the lower bound given by the LP relaxation.

However, this paradigm is not an entirely accurate presentation of the design landscape. Modern commercial MIP solvers have both sophisticated heuristics and cut generation routines, which focus on finding better incumbent solutions and lower bounds, respectively. Consequentially, many UC problems can be solved to a reasonable optimality gap at the root LP node or with only minimal enumeration. Differences in UC formulation may affect the cut generation routines or the heuristics in ways that are difficult to anticipate. For example, it has been observed that tighter formulations often improve the performance of MIP heuristics (Rothberg 2018). For these reasons, it is important to evaluate UC formulations empirically, as it is difficult to know a priori the effect of making a formulation tighter or more compact on solve times.

3. Formulations

Throughout much of this section, we discuss formulations for constraints and variables describing Π_g. When describing the formulation for a single generator, we will drop the subscript g for clarity and ease of presentation. Most of the constraints described here could be applied to every generator $g \in \mathcal{G}$; we will explicitly point out when this is not the case. Similarly, most constraints can be applied to every time period, so we will often forgo the qualifier $\forall t \in \mathcal{T}$, and put an additional qualifier on the time step t if necessary. In general, the variables $v(t)$ and $w(t)$ for $t < 1$ should be based on historical data, and for $t > T$ they can be assumed to be 0.

3.1. State Variables and Minimum Up-Time/Down-Time Constraints

3.1.1. Three- (and Two-) Binary Formulations Garver (1962) first proposed using three binary variables to represent the state of the generator: $u(t)$ if the generator is on at time t, $v(t)$ if the generator is turned on at time t, and $w(t)$ if the generator is turned off at time t. Garver (1962) also first formulated the logical constraints, which relate these three binary variables:

$$u(t) - u(t-1) = v(t) - w(t) \qquad t \in \mathcal{T}. \tag{2}$$

Common to all models is the need to enforce the initial up-time and down-time constraints based on generator g's history. As some generators can have long minimum up-times and down-times, these are formulated as:

$$\sum_{i=1}^{\min\{U,T\}} u(i) = \min\{U, T\} \tag{3a}$$

$$\sum_{i=1}^{\min\{D,T\}} u(i) = 0. \tag{3b}$$

Any reasonable formulation of these constraints should enable a MIP solver to, via pre-processing, eliminate the involved variables. Note that if U or D are 0 these sums become empty, thus eliminating these constraints.

A now common way to formulate minimum up-time and down-time is via the turn-on/turn-off inequalities proposed by Malkin (2003) and Rajan and Takriti (2005):

$$\sum_{i=t-UT+1}^{t} v(i) \le u(t) \qquad t \in \{UT, \dots, T\} \tag{4}$$

$$\sum_{i=t-DT+1}^{t} w(i) \le 1 - u(t) \qquad t \in \{DT, \dots, T\}. \tag{5}$$

Malkin (2003) and Rajan and Takriti (2005) independently showed that these constraints are facets of the convex hull of the up-time/down-time polytope, which together with (2) and variable bounds is an ideal formulation for up-time/down-time. A more recent and general proof can be found in Queyranne and Wolsey (2017), which extended this result to minimum up-times and down-times that vary across the planning horizon, as well as maximum up-times and down-times. One may use equation (2) to eliminate either the v or w variables while not losing strength. Typically the w variables are projected out as there are costs usually associated with start-up but not shut-down. The resulting logical and minimum down-time constraints, respectively, are:

$$v(t) \ge u(t) - u(t-1) \qquad \forall t \in \mathcal{T} \tag{6}$$

$$\sum_{i=t-DT+1}^{t} v(i) \le 1 - u(t-DT) \qquad t \in \{DT, \dots, T\}. \tag{7}$$

While not often used, this version of the logical and minimum down-time constraints was tested computationally in Yang et al. (2017) which completely projects out the shut-down variables w. A version of equation (7) was also computationally tested in Atakan et al. (2018). Note that in what follows, if only the two-binary variables $u(t)$ and $v(t)$ are used, we can always use the substitution provided by (2) to eliminate $w(t)$ from the expression.

3.1.2. State Transition Variables Atakan et al. (2018) suggested replacing the variable $u(t)$ with a state-transition variable $\tilde{u}(t)$ which encodes whether the generator g remains operational at time t. The state transition variables for start-up and shut-down are equivalent to Garver's (1962). They also introduced the mathematical relationship

$$u(t) = \tilde{u}(t) + v(t), \tag{8}$$

which allows for the transformation of constraints for Garver's (1962) three-binary variables to those for the state-transition variables.

In particular, the logic constraint (2) becomes

$$\tilde{u}(t) - \tilde{u}(t-1) = v(t-1) - w(t) \qquad t \in \mathcal{T}. \tag{9}$$

Atakan et al. (2018) used constraints (4) and (7) with this transformation applied for minimum up-time/down-time:

$$\sum_{i=t-UT+1}^{t-1} v(i) \le \tilde{u}(t) \qquad t \in \{UT, \dots, T\} \tag{10}$$

$$\sum_{i=t-DT}^{t} v(i) \le 1 - \tilde{u}(t-DT) \qquad t \in \{DT, \dots, T\}. \tag{11}$$

The historical stay-on and turn-on statuses at $t = 0$, $\tilde{u}(0)$ and $v(0)$, can be inferred from initial conditions. Similarly, constraints such as (3) can be formulated to enforce the initial up-time or down-time.

There are many other potential ways to formulate the minimum up-time/down-time constraints using two or three binary variables, but the ones presented above are most common in the MIP UC literature. Hedman et al. (2009) gave an overview of other possible formulations. We detail additional formulations for up-time and down-time from Dillon et al. (1978), Takriti et al. (2000), and Carrion and Arroyo (2006) in the online supplement (Knueven et al. 2018d, Section B.1).

3.2. Power and Reserve Variables

There are two suggestions in the literature for formulating the power output of a generator: (1) introducing $p_g(t)$ to represent the power produced by generator g at time t, and (2) introducing $p'_g(t)$ to represent the power produced above $\underline{P}_g$ by generator g at time t. The former formulation has been more common, though the latter was suggested by Garver

(1962) and has recently been promoted by Morales-España et al. (2013a). The two variable choices are equivalent mathematically through the linear relationship:

$$p_g(t) = p'_g(t) + \underline{P}_g u_g(t). \tag{12}$$

For the reserve contribution, again the literature suggests two possible approaches. One, introduced by Carrion and Arroyo (2006), is to model the maximum power available for generator g at time t, which we will call $\overline{p}_g(t)$. The other, introduced by Morales-España et al. (2013a), is to model the reserves contributed by generator g at time t, which we refer to as $r_g(t)$. We now introduce a third possibility, which is to have a variable for the maximum power available above minimum for generator g at time t, referred to as $\overline{p}'_g(t)$. Here, we have the following linear relationships:

$$\overline{p}_g(t) = \overline{p}'_g(t) + \underline{P}_g u_g(t) \tag{13}$$

$$\overline{p}'_g(t) = p'_g(t) + r_g(t) \tag{14}$$

$$\overline{p}_g(t) = p_g(t) + r_g(t). \tag{15}$$

As is the case in power formulations, three variable choices are mathematically equivalent. We further note the relationships:

$$p_g(t) \le \overline{p}_g(t) \tag{16}$$

$$p'_g(t) \le \overline{p}'_g(t), \tag{17}$$

both of which can be appropriately translated using the equations above.

The linear relationships provided by equations (12)–(15) allow for the translation of constraints involving one group of variables into constraints involving another group of variables through simple substitution. In the proceeding, we will use the constraints as they were presented by the original authors, applying the transformations directly when they allow for cancellations that reduce the number of non-zeros or simplify the constraint. Practically, formulations with $p'_g(t)$ are more difficult to implement if $\underline{P}_g$ and $\overline{P}_g$ are functions of time.

3.3. Generation Limits and Upper Bounds

The simplest generation limits, when stated in terms of the p and $\overline{p}$ variables, were given by Carrion and Arroyo (2006):

$$\underline{P}_g u_g(t) \leq p_g(t) \leq \overline{p}_g(t) \leq \overline{P}_g u_g(t). \tag{18}$$

Additionally, to ensure total capacity is not exceeded as the generator g is shutting down, Arroyo and Conejo (2000) defined the constraint:

$$\overline{p}_g(t) \leq \overline{P}_g(u_g(t) - w_g(t+1)) + SD_g w_g(t+1). \tag{19}$$

Morales-España et al. (2013a) proposed using the following start-up and shut-down ramping limits to tighten the variable upper bounds:

$$p'_g(t) + r_g(t) \leq (\overline{P}_g - \underline{P}_g)u_g(t) - (\overline{P}_g - SU_g)v_g(t) - (\overline{P}_g - SD_g)w_g(t+1) \quad g \in \mathcal{G}^{>1} \tag{20}$$

$$p'_g(t) + r_g(t) \leq (\overline{P}_g - \underline{P}_g)u_g(t) - (\overline{P}_g - SU_g)v_g(t) \quad g \in \mathcal{G}^{1} \tag{21a}$$

$$p'_g(t) + r_g(t) \leq (\overline{P}_g - \underline{P}_g)u_g(t) - (\overline{P}_g - SD_g)w_g(t+1) \quad g \in \mathcal{G}^{1}, \tag{21b}$$

where $\mathcal{G}^{>1} := \{g \in \mathcal{G} \mid UT_g > 1\}$ and $\mathcal{G}^1 := \{g \in \mathcal{G} \mid UT_g = 1\}$. With the $\overline{p}$ variables, constraint (20) can be expressed equivalently as

$$\overline{p}_g(t) \leq \overline{P}_g u_g(t) - (\overline{P}_g - SU_g)v_g(t) - (\overline{P}_g - SD_g)w_g(t+1) \quad g \in \mathcal{G}^{>1}, \tag{22}$$

and (21) can be translated similarly. Note that (21b) is the same expression as (19), through (13) and (14). Damcı-Kurt et al. (2016) showed that (21a) and (21b) are facets of the two-period ramp-up and ramp-down polytopes, respectively. Here the two-period ramp-up and ramp-down polytopes are simply the generator polytope for $T = 2$ when only considering the ramp-up and ramp-down constraints, respectively.

For $g \in \mathcal{G}^1$, when $SU_g \neq SD_g$, constraints (21) can be tightened (Gentile et al. 2017) via

$$p'_g(t) + r_g(t) \leq (\overline{P}_g - \underline{P}_g)u_g(t) - (\overline{P}_g - SU_g)v_g(t) - [SU_g - SD_g]^+ w_g(t+1) \quad g \in \mathcal{G}^1 \tag{23a}$$

$$p'_g(t) + r_g(t) \leq (\overline{P}_g - \underline{P}_g)u_g(t) - (\overline{P}_g - SD_g)w_g(t+1) - [SD_g - SU_g]^+ v_g(t) \quad g \in \mathcal{G}^1, \tag{23b}$$

where $[\cdot]^+ := \max\{\cdot, 0\}$. Note that when $UT_g = 1$, it is possible for $v_g(t) = 1$ and $w_g(t+1) = 1$. This modification ensures that in this case both constraints are tight. Gentile et al.

(2017) also gave a convex hull result using these tightened constraints in conjunction with (20). In particular, if $r_g(t)$ is fixed to zero and $UT_g > 1$, then (2), (4), (5), and (20), along with the basic $[0,1]$ bounds on the binary variables, provide a convex hull description for these constraints (i.e., minimum up-time/down-time, start-up/shut-down ramps, and generation limits) for a single generator. If $UT_g = 1$, then (2), (4), (5), and (23), along with variable bounds, is a convex hull description for a single generator with these constraints. In either case, adding ramping constraints, piecewise linear production costs, and/or time-dependent start-up costs will result in a description that, in general, is no longer a convex hull description.

3.4. Ramping Constraints

Ramping constraints were given by Arroyo and Conejo (2000), as follows:

$$\overline{p}(t) - p(t-1) \le RUu(t-1) + SUv(t) \tag{24}$$

$$p(t-1) - p(t) \le RDu(t) + SDw(t). \tag{25}$$

These ramping constraints ensure that a generator's power output not vary excessively over a certain time period. When the p' variables are used along with (20) or (21), Morales-España et al. (2013a) demonstrated these constraints can be encoded more simply as:

$$p'(t) + r'(t) - p'(t-1) \le RU \tag{26}$$

$$p'(t-1) - p'(t) \le RD. \tag{27}$$

3.4.1. Strengthening Ramping Ostrowski et al. (2012) introduced strengthened inequalities for ramping, under certain assumptions on the generator. Two ramp-up inequalities are proposed. The first holds for $g \in \mathcal{G}$ with $RU > SD - \underline{P}$ and $UT \ge 2$

$$\begin{aligned} p(t) - p(t-1) \le RUu(t) - \underline{P}w(t) - (RU - SD + \underline{P})w(t+1) & \\ + (SU - RU)v(t) & \qquad t \in \{1, \ldots, T-1\}, \end{aligned} \tag{28}$$

and the other when $RU > SD - \underline{P}$, $UT \ge 2$, and $DT \ge 2$:

$$\begin{aligned} p(t) - p(t-2) \le 2RUu(t) - \underline{P}w(t-1) - \underline{P}w(t) & \\ + (SU - RU)v(t-1) + (SU - 2RU)v(t) & \qquad t \in \{2, \ldots, T-2\}. \end{aligned} \tag{29}$$

Though not mentioned in Ostrowski et al. (2012), the $p(t)$ variable in (28) and (29) can be replaced with $\overline{p}(t)$ without loosing validity. Ostrowski et al. (2012) also proposed three ramp-down inequalities. The first is analogous to (28), and holds for $g \in \mathcal{G}$ with $RD > SU - \underline{P}$ and $UT \geq 2$

$$\begin{aligned} p(t-1) - p(t) \leq {} & RDu(t) + SDw(t) - (RD - SU + \underline{P})v(t-1) \\ & - (RD - \underline{P})v(t) \qquad t \in \mathcal{T}, \end{aligned} \tag{30}$$

the second, when $RD > SU - \underline{P}$, $UT \geq 3$, and $DT \geq 2$

$$\begin{aligned} p(t-1) - p(t) \leq {} & RDu(t+1) + SDw(t) + RDw(t+1) \\ & - (RD - SU + \underline{P})v(t-1) - (RD + \underline{P})v(t) \\ & - RDv(t+1) \qquad t \in \{1, \ldots, T-1\}, \end{aligned} \tag{31}$$

and the last, again when $RD > SU - \underline{P}$, $UT \geq 3$, and $DT \geq 2$:

$$\begin{aligned} p(t-2) - p(t) \leq {} & 2RDu(t) + SDw(t-1) + (SD + RD)w(t) \\ & - 2RDv(t-2) - (2RD + \underline{P})v(t-1) \\ & - (2RD + \underline{P})v(t) \qquad t \in \{2, \ldots, T-2\}. \end{aligned} \tag{32}$$

Ostrowski et al. (2012) showed that (28)–(32) are facets for projections of the generator polytope onto the time periods and variables involved in these equations.

Damcı-Kurt et al. (2016) suggested a different formulation for the ramp-up and ramp-down inequalities:

$$p(t) - p(t-1) \leq (SU - \underline{P} - RU)v(t) + (\underline{P} + RU)u(t) - \underline{P}u(t-1) \tag{33}$$

$$p(t-1) - p(t) \leq (SD - \underline{P} - RD)w(t) + (\underline{P} + RD)u(t-1) - \underline{P}u(t). \tag{34}$$

Damcı-Kurt et al. (2016) demonstrated that these inequalities are facets of the two-period ramp-up and ramp-down polytopes, respectively. Validity is not affected if $p(t)$ is replaced with $\overline{p}(t)$ in (33). These inequalities are sparser when expressed using the $p'(t)$ variables:

$$\overline{p}'(t) - p'(t-1) \leq (SU - \underline{P} - RU)v(t) + RUu(t) \tag{35}$$

$$p'(t-1) - p'(t) \leq (SD - \underline{P} - RD)w(t) + RDu(t-1). \tag{36}$$

These inequalities are as sparse as (24) and (25) while having some guarantee on their strength. We detail a two-period ramp-up and ramp-down inequalities from Damcı-Kurt et al. (2016) in the online supplement (Knueven et al. 2018d, Section B.3). Damcı-Kurt et al. (2016) also gave two additional exponential classes of two-period ramp-up and ramp-down inequalities; however, such inequalities require separation and hence will not be covered here. Similarly, Pan and Guan (2016) provided several exponential classes of two- and three-period ramping inequalities which are valid and facet-defining for the generator polytope, but as they also require separation, we will not consider them here.

3.4.2. The Generator Polytope Frangioni and Gentile (2015a,b) and Knueven et al. (2018a) introduced a perfect formulation for a generator with ramping constraints, and Guan et al. (2018) introduced three additional extended formulations for a ramping-constrained generator. While extended formulations for the generator polytope have been an interesting topic of research, like the exponential classes of ramping inequalities from (Damcı-Kurt et al. 2016, Pan and Guan 2016, 2017), they are probably not appropriate for direct incorporation into the UC MIP because the additional variables and/or constraints become too burdensome. Thus we will not consider them for our computational comparisons to follow as they should be implemented in a more advanced way, such as using them as a separator for cut generation. However, we discuss these results in the online supplement (Knueven et al. 2018d, Section B.4).

3.5. Variable Upper Bounds with Ramp Limits

When ramping constraints are added, there maybe be an exponential number of variable upper bound inequalities in the three- or two-binary space that are tight (i.e., they describe facets of the convex hull of the generator polytope), even when $SU = SD$ and $RU = RD$ (Damcı-Kurt et al. 2016, Pan and Guan 2016). However, computational experience with separating cuts from the generator polytope suggests that variable upper bound inequalities are often important in practice (Damcı-Kurt et al. 2016, Knueven et al. 2018a).

For notational ease, let $T^{RU} = \left\lfloor \frac{\overline{P}-SU}{RU} \right\rfloor$ and $T^{RD} = \left\lfloor \frac{\overline{P}-SU}{RD} \right\rfloor$, such that T^{RU} (T^{RD}) is the number of time periods the generator spends ramping to go from SU ($\overline{P}$) to $\overline{P}$ (SD). Ostrowski et al. (2012) proposed the following tighter upper bounds on $p(t)$ based on the ramp-down trajectory of the generator:

$$p(t) \leq \overline{P}u(t) - \sum_{i=0}^{K^{SD}(t)} (\overline{P} - (SD + iRD))w(t+1+i), \tag{37}$$

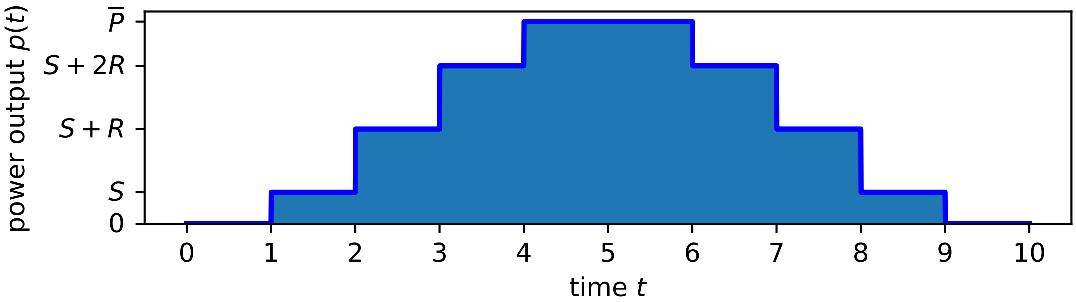

Figure 1 **Feasible region defined by inequalities** (39) **for a generator that turns on at** $t=1$ **and turns off at** $t=9$ **with** $UT=6$, $R=RU=RD$, $S=SU=SD$, $\underline{P}=0$, **and** $T^{RU}=T^{RD}=2$.

where $K^{SD}(t)=\min\{T^{RD},UT-1,T-t-1\}$. This constraint enforces the maximum limit on the generator as it is shutting down. If $w(t+2)=1$, and the generator remains off past $t+K(t)$, then the upper bound on $p(t)$ is $SD+RD$ – reflecting generator g's ramp-down and shut-down constraints.

Pan and Guan (2016) introduced several variable upper bounds, the following linear class of which is based on the ramp-up and shut-down trajectory of the generator:

$$\overline{p}(t)\leq\overline{P}u(t)-(\overline{P}-SD)w(t+1)-\sum_{i=0}^{\min\{UT-2,T^{RU}\}}(\overline{P}-SU-iRU)v(t-i). \tag{38}$$

This constraint has a nearly symmetric effect to that of (37), enforcing the maximum limit on the generator as it is starting up. This inequality additionally includes the shut-down limit, so it can also be seen as a generalization of (20). Because this inequality strictly involves the ramp-up trajectory with the shut-down limit, we can replace $p(t)$ with $\overline{p}(t)$ as we have done in (38).

In both cases, care must be taken with the minimum up-time UT. In particular, the minimum up-time must be such that if one of the v or w variables in inequalities (37) or (38) is 1, this implies $u(t)=1$. As with (20), it must also be the case that at most one of the v or w variables in each inequality can be 1 to maintain validity. Finally, these type of inequalities need only “look back” (38) or “look forward” (37) enough time steps to capture T^{RU} or T^{RD}. In the online supplement we detail how (37) is derived from (Ostrowski et al. 2012, Eq. (19)) and how (38) is derived from (Pan and Guan 2016, Eq. (28)), and we also present some additional variable upper bounds (Knueven et al. 2018d, Section B.5).

By combining the insights from (37) and (38), one can derive similar polynomial classes of variable upper bounds. As an example, provided $UT \geq T^{RU} + T^{RD} + 2$, the following inequality:

$$p(t) \leq \overline{P}u(t) - \sum_{i=0}^{T^{RU}} (\overline{P} - (SU + iRU))v(t-i) - \sum_{i=0}^{T^{RD}} (\overline{P} - (SD + iRD))w(t+1+i) \tag{39}$$

is valid for all $t \in [T^{RU}, T - T^{RD}]$. The logic of (39) is similar to that of (37) and (38). Because $UT \geq T^{RD} + T^{RD} + 2$, at most one of the v's or w's on the right-hand side of (39) is 1. When one of these variables is 1, the bound on $p(t)$ is adjusted downward to take into account the maximum power allowed by that ramp-up or ramp-down trajectory. Note that when $UT < T^{RU} + T^{RD} + 2$, one can truncate the sums in (39) to ensure that at most one v or w is 1 and any v or w being 1 implies $u(t) = 1$, given the minimum up-time UT. Figure 1 demonstrates how the inequalities (39) encode the ramping trajectory for a ramping-constrained generator. We discuss an extension to when $UT = T^{RU} + T^{RD} + 1$ in the online supplement (Knueven et al. 2018d, Section B.5).

By combining some of the inequalities and ideas above we can create a set of strong variable upper bound inequalities which are complementary. Consider again inequality (38). If $UT - 2 < T^{RU}$, we may not cover the entire start-up ramping trajectory. In this case, we can augment (38) by shifting the terms

$$\overline{p}(t) \leq \overline{P}u(t) - \sum_{i=0}^{\min\{UT-1, T^{RU}\}} (\overline{P} - SU - iRU)v(t-i), \tag{40}$$

which with (38) covers an additional time period. Given (38) and (40), we notice the shut-down ramp trajectory is not considered. Here we can use a modified version of (39). Let $K^{SD}(t)$ be as in (37) and $K^{SU}(t) = \min\{T^{RU}, UT - 2 - [K^{SU}(t)]^+, t-1\}$, then

$$p(t) \leq \overline{P}u(t) - \sum_{i=0}^{K^{SD}(t)} (\overline{P} - (SD + iRD))w(t+1+i) - \sum_{i=0}^{K^{SU}(t)} (\overline{P} - (SU + iRU))v(t-i), \quad \text{if } K^{SD}(t) > 0 \tag{41}$$

where $K^{SD}(t)$ and $K^{SU}(t)$ serve to include as many v's and w's as possible given UT and the beginning and ending time period. If $K^{SU}(t) < T^{RU}$, we add the term $-[(SU + (K^{SU}(t)+1)RU)) - (SD + T^{RD}RD)]^+ v(t - (K^{SU}(t)+1))$ for additional tightening similar to (23b). Here we give preference to having more w's so as to correctly capture the shut-down ramp trajectory, given that the start-up ramp trajectory is covered by (38) or (40). Notice that if we are not near the beginning or end time periods and $UT \geq T^{RU} + T^{RD} + 2$, then (41) is exactly (39). If $K^{SD}(t) \leq 0$, this inequality should be omitted as it is dominated by (38).

3.6. Piecewise Linear Production Costs

For the purposes of this paper we will assume production costs (the marginal cost per MW of power) are piecewise linear and convex. While in reality the production costs follow a nonlinear (often quadratic) curve, in most electricity markets in the United States generators bid in piecewise offers. Further, we suspect that the general results in this paper would hold true for various potential MIQP formulations of UC where the marginal production cost is represented using a convex quadratic function. Frangioni et al. (2009) gave a method for solving a MIQP formulation for UC as a MILP using cutting-planes.

Garver (1962) included piecewise production costs in his original formulation. For each segment $l \in \mathcal{L}$, and $t \in \mathcal{T}$, the variable $p^l(t)$ represents the contribution to output from piecewise segment l. Like Garver (1962), we will assume throughout that $\overline{P}^l > \underline{P}$ for every l, and for convenience we define $\overline{P}^0 = \underline{P}$. (A simple preprocessing of the data will ensure this is the case.) Garver's (1962) formulation can then be written as:

$$p^l(t) \leq (\overline{P}^l - \overline{P}^{l-1})u(t) \qquad l \in \mathcal{L}, \tag{42}$$

$$\sum_{l\in\mathcal{L}} p^l(t) = p'(t) \tag{43}$$

$$\sum_{l\in\mathcal{L}} C^l p^l(t) = c^p(t). \tag{44}$$

In actuality, Garver (1962) opted to use equation (43) to substitute out the p' variables, which do not explicitly appear in his model. However, with the basic limits on the $p'(t)$ variables, $p'(t) \leq (\overline{P} - \underline{P})u(t)$, this formulation is locally ideal: equations (42), (43), and $p'(t) \leq (\overline{P} - \underline{P})u(t)$ along with the unit bounds on $u(t)$ are a convex hull description for these constraints (Wu 2016). This is essentially one of the production cost formulations from Wu (2016), except with (12) applied to (43). Finally, we substitute $\sum_{l\in\mathcal{L}} C^l p^l(t)$ for $c^p(t)$ in the objective function whenever $c^p(t)$ appears in and only in an equality constraint.

Carrion and Arroyo (2006) suggested a slightly simpler formulation, replacing (42) with:

$$p^l(t) \leq (\overline{P}^l - \overline{P}^{l-1}) \qquad l \in \mathcal{L}. \tag{45}$$

Notice that (45) is the same as (42), except the binary on/off variable is not in the former. Because of this, this formulation does not have the locally ideal property (Sridhar et al. 2013).

Knueven et al. (2018b) tightened the bounds on $p^l(t)$ with the start-up and shut-down variables using the start-up and shut-down ramp:

$$p^l(t) \le (\overline{P}^l - \overline{P}^{l-1})u(t) - C^v(l)v(t) - C^w(l)w(t+1) \qquad l \in \mathcal{L}, \tag{46}$$

where:

$$C^v(l) := \begin{cases} 0 & \overline{P}^l \le SU \\ \overline{P}^l - SU & \overline{P}^{l-1} < SU < \overline{P}^l \\ \overline{P}^l - \overline{P}^{l-1} & \overline{P}^{l-1} \ge SU \end{cases}, \qquad C^w(l) := \begin{cases} 0 & \overline{P}^l \le SD \\ \overline{P}^l - SD & \overline{P}^{l-1} < SD < \overline{P}^l \\ \overline{P}^l - \overline{P}^{l-1} & \overline{P}^{l-1} \ge SD \end{cases},$$

for generators with $UT > 1$. When $UT = 1$, (46) must be replaced with:

$$p^l(t) \le (\overline{P}^l - \overline{P}^{l-1})u(t) - C^v(l)v(t) \qquad l \in \mathcal{L} \tag{47a}$$

$$p^l(t) \le (\overline{P}^l - \overline{P}^{l-1})u(t) - C^w(l)w(t+1) \qquad l \in \mathcal{L}. \tag{47b}$$

Notice that both (46) and (47) enforce that the piecewise production variables never exceed SU (SD) when the generator is turning on (off). These additional terms are not necessary for validity, they only serve to tighten the model. Finally, when $UT = 1$ and $SU \ne SD$, one can improve (47), in a similar fashion to (23a) and (23b):

$$p^l(t) \le (\overline{P}^l - \overline{P}^{l-1})u(t) - C^v(l)v(t) - [C^v(l) - C^w(l)]^+ w(t+1) \qquad l \in \mathcal{L} \tag{48a}$$

$$p^l(t) \le (\overline{P}^l - \overline{P}^{l-1})u(t) - C^w(l)w(t+1) - [C^w(l) - C^v(l)]^+ v(t) \qquad l \in \mathcal{L}. \tag{48b}$$

We show in the online supplement (Knueven et al. 2018d, Section A) that when $UT \ge 2$, (46) can be used to build an ideal formulation for a generator with piecewise production costs, minimum up/down times and start-up/shut-down ramping limits. In a similar fashion, we also show when $UT = 1$, (48) can be used to build an ideal formulation for a generator with start-up/shut-down ramping limits and piecewise production costs. We additionally show the start-up cost formulation from Knueven et al. (2018c) can be added to this without loosing strength. The inclusion of such strengthening for the piecewise linear costs may improve extended LMP calculation when the generator is turning on or turning off (Chen and Wang 2017, Hua and Baldick 2017). The generalization to considering the ramping trajectories when starting-up or shutting-down, as in (39), in the limits on $p^l(t)$ is straightforward but tedious.

Another common way to model convex piecewise production costs is to consider the typical epigraph formulation using $p'(t)$ and $c^p(t)$. Atakan et al. (2018) formulate this as:

$$c^p(t) \geq C^l(p'(t) - (\overline{P}^{l-1} - \underline{P})) + \overline{C}^{l-1} \qquad \forall l \in \mathcal{L}. \tag{49}$$

Just as before, we can devise a locally ideal version for these production costs. This is sometimes called the *perspective formulation* (Frangioni and Gentile 2006, Frangioni et al. 2009). The form used here for convex piecewise linear production costs is given by (Hua and Baldick 2017):

$$c^p(t) \geq C^l p'(t) + (\overline{C}^{l-1} - C^l(\overline{P}^{l-1} - \underline{P}))u(t) \qquad \forall l \in \mathcal{L}. \tag{50}$$

As above, notice how the status variable u serves to adjust upward the production costs when $u(t)$ is fractional, which tightens the formulation. If the p variables are used this constraint simplifies a bit:

$$c^p(t) \geq C^l p(t) + (\overline{C}^{l-1} - C^l\overline{P}^{l-1})u(t) \qquad \forall l \in \mathcal{L}. \tag{51}$$

We detail SOS2-type formulations for piecewise production costs in the online supplement (Knueven et al. 2018d, Section B.6).

3.7. Time-Dependent Start-Up Costs

When a thermal unit is switched off, heat begins to dissipate. If the unit is started-up while there is still heat remaining, it will cost less to bring it back up to operating temperature, and hence on-line. Silbernagl et al. (2016) introduced a formulation for start-up costs that explicitly models temperature, exploiting this fact and providing a physical basis for their model. However, as this formulation is a restriction of that from Carrion and Arroyo (2006), we will not consider it here.

Most UC models use an approximated and discretized version of these time-dependent start-up costs by considering different start-up types for each generator, often just hot, cold, and sometimes warm. The formulations given below, however, apply to an arbitrary number of start-up types.

One simple start-up cost formulation, introduced by Nowak and Römisch (2000) and used by Carrion and Arroyo (2006), relies on the auxiliary variable $c^{SU}(t)$ for each time period t and the status variable u:

$$c^{SU}(t) \geq C^s \left(u(t) - \sum_{i=1}^{\underline{T}^s} u(t-i) \right) \qquad \forall s \in \mathcal{S} \tag{52}$$

$$c^{SU}(t) \geq 0 \qquad \forall t \in \mathcal{T}. \tag{53}$$

Brandenberg et al. (2017) showed the convex hull for non-decreasing start-up costs has an exponential number of facets in the u, c^{SU} space, but provided a linear-time separation algorithm.

3.7.1. Formulations with Additional Variables A common approach for start-up cost modeling is to add new variables to keep track of the generator's off-time and new constraints to tie these to the typical 3-binary variables.

One possibility is to use indicator variables for each start-up type $s \in \mathcal{S}_g$, for each $g \in \mathcal{G}$ and each $t \in \mathcal{T}$. This was originally proposed in Muckstadt and Wilson (1968), and has been more recently considered in Simoglou et al. (2010). We will use the more general formulation from Morales-España et al. (2013a):

$$\delta^s(t) \leq \sum_{i=\underline{T}^s}^{\underline{T}^{s+1}-1} w(t-i) \qquad s \in \mathcal{S} \setminus \{S\} \tag{54}$$

$$v(t) = \sum_{s=1}^{S} \delta^s(t) \tag{55}$$

$$c^{SU}(t) = \sum_{s=1}^{S} C^s \delta^s(t), \tag{56}$$

where $c^{SU}(t)$ can be eliminated from the formulation by substituting it for $\sum_{s=1}^{S} C^s \delta^s(t)$. We always make this substitution for cost variables in the objective function which are defined by equalities. These same indicator variables can be used to track power output during different start-up and shut-down types, more accurately reflecting the generator's output as it comes on-line or goes off-line (Morales-España et al. 2013b). We also note that we can project out $\delta^S(t)$, the variable representing the coldest start-up type, by replacing (55) and (56) with

$$v(t) \geq \sum_{s=1}^{S-1} \delta^s(t) \quad \text{and} \tag{57}$$

$$c^{SU}(t) = c^S v(t) - \sum_{s=1}^{S-1} (C^S - C^s)\delta^s(t), \tag{58}$$

respectively. Convex hull results for time-dependent start-up costs using these variables can be found in Queyranne and Wolsey (2017) and Silbernagl (2016). In both cases, a quadratic number of constraints is required. Additionally, both convex hull descriptions are somewhat cumbersome in their formulations.

Another formulation using indicator variables is presented in Knueven et al. (2018c). This formulation uses the down-time arcs $x(t,t')$, but only for $[t,t') \in \mathcal{X}$ such that $t' - TC < t \le t' - DT$:

$$\sum_{t'=t-TC+1}^{t-DT} x(t',t) \le v(t) \tag{59}$$

$$\sum_{t'=t+DT}^{t+TC-1} x(t,t') \le w(t) \tag{60}$$

$$c^{SU}(t) = C^S v(t) - \sum_{s=1}^{S-1} (C^S - C^s) \left(\sum_{t'=t-\underline{T}^{s+1}+1}^{t-\underline{T}^s} x(t',t) \right). \tag{61}$$

If $\underline{T}^s < \underline{T}^{s+1} - 1$ for some $s \in \mathcal{S} \setminus \{S\}$, then some of these arcs may be redundant, in the sense that there is more than one of them per start-up type for a given time period. However, Knueven et al. (2018c) showed this formulation to be as strong for increasing (non-decreasing) start-up costs as a perfect formulation. Like the other indicator variable formulations, this formulation can be used to model power output for different start-up and shut-down types. We detail additional start-up cost formulations in the online supplement (Knueven et al. 2018d, Section B.7).

3.8. Shut-Down Costs

Shut-down costs are often 0, but in the case when there are shut-down costs, they can be modeled in a few ways. For example, if the w variables are included, then the shut-down cost is simply:

$$c^{SD}(t) = C^w w(t). \tag{62}$$

The additional cases are described in the online supplement (Knueven et al. 2018d, Section B.8).

3.9. System Constraints

For the purposes of this paper, we will hold the system constraints constant, up to substitution of the variables used to describe the generators.

3.9.1. Renewables We will consider only aggregate renewables available at each bus n at time t, $p_{W,n}(t)$, and further assume they have zero marginal cost. To the extent that renewables bid into the market, they could be modeled as thermal generators as described above. Allowing for some portion of renewables production to be must-take, this is modeled simply as

$$\underline{W}_n(t) \leq p_{W,n}(t) \leq \overline{W}_n(t) \qquad \forall t \in \mathcal{T}, \forall n \in \mathcal{N}, \tag{63}$$

where either parameter $\underline{W}_n(t)$ or $\overline{W}_n(t)$ may be 0 depending on policy and availability.

3.9.2. Network Constraints We will use the linear B, θ "DC" approximation for the transmission network, when one is provided. Consider the following network constraints:

$$f_k(t) = B_k\left(\theta_{k(n)}(t) - \theta_{k(m)}(t)\right) \qquad \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \tag{64a}$$

$$-F_k \leq f_k(t) \leq F_k \qquad \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \tag{64b}$$

$$-\pi \leq \theta_n(t) \leq \pi \qquad \forall n \in \mathcal{N}, \forall t \in \mathcal{T} \tag{64c}$$

$$\theta_1(t) = 0 \qquad \forall t \in \mathcal{T}. \tag{64d}$$

Equation (64a) is Kirchhoff's law, and equation (64b) enforces capacity constraints for the branches. We additionally bound the voltage angles to be no more than π in magnitude (64c) and fix the first bus as the reference bus (64d). The flow balance constraints are

$$\sum_{g \in \mathcal{G}_n} p_g(t) + p_{W,n}(t) + \sum_{k \in \delta^+(n)} f_k(t) - \sum_{k \in \delta^-(n)} f_k(t) + s_n(t) = D_n(t) \qquad \forall n \in \mathcal{N}, \forall t \in \mathcal{T}, \tag{65}$$

where the constraint is satisfied if $s_n(t) = 0$. System operators commonly relax certain constraints – for example, if a demand cannot be met without an extraordinarily high cost, it may make sense for economic reasons to relax the flow balance constraint and put a high penalty on $s_n(t) \neq 0$. This is modeled linearly using the typical auxiliary variables $s_n^+(t)$

and $s_n^-(t)$ to represent (respectively) the positive and negative part of the slack at bus n at time t, along with the constraints:

$$s_n(t) = s_n^+(t) - s_n^-(t) \qquad \forall n \in \mathcal{N}, \forall t \in \mathcal{T} \tag{66a}$$

$$s_n^+(t),\ s_n^-(t) \geq 0 \qquad \forall n \in \mathcal{N}, \forall t \in \mathcal{T}. \tag{66b}$$

The penalty cost c_{LP} can then be put on both the $s_n^+(t)$ and $s_n^-(t)$ variables in the objective function. If appropriate, the variables $p_g(t)$ are replaced by the equivalent expression $p'_g(t) + \underline{P}u_g(t)$, depending on the variables used to represent generator output.

Finally, if there is only one bus (i.e., network constraints are relaxed), (64) is dropped and the sums involving the $f_k(t)$ variables in equation (65) are empty, resulting in a simple system-wide demand constraint.

3.9.3. Reserve Requirement In this paper we simply consider a system-wide spinning reserve requirement $R(t)$. Other ancillary services tend to vary by market.

The reserve requirement in Carrion and Arroyo (2006) and Ostrowski et al. (2012) is modeled as additional power available above demand

$$\sum_{g \in \mathcal{G}} \overline{p}_g(t) + s_R(t) \geq \sum_{n \in \mathcal{N}} D_n(t) + R(t), \qquad \forall t \in \mathcal{T} \tag{67}$$

whereas in Morales-España et al. (2013a) the reserve requirement is modeled separately from total demand:

$$\sum_{g \in \mathcal{G}} r_g(t) + s_R(t) \geq R(t) \qquad \forall t \in \mathcal{T}. \tag{68}$$

Which approach is used depends on how reserve production is modeled for the generators. If the $\overline{p}_g(t)$ or $\overline{p}'_g(t)$ variables are used, then equation (67) is the natural choice for the reserve requirement, substituting $\overline{p}'_g(t) + \underline{P}_g u_g(t)$ for $\overline{p}_g(t)$ if the $\overline{p}'_g(t)$ variables are used. Conversely, if the $r_g(t)$ variables are used to model reserve production, then equation (68) is more sensible. In either case, we note the following relationship between these variables holds: $r_g(t) = \overline{p}_g(t) - p_g(t) = \overline{p}'_g(t) - p'_g(t)$. In both cases, we also relax the global reserve requirement and add the penalty term $c_R s_R(t)$ to the objective for each $t \in \mathcal{T}$.

3.10. Objective Function

Finally, the objective is minimizing system operation cost:

$$\min \sum_{t \in \mathcal{T}} \left(\sum_{g \in \mathcal{G}} \left(c_g^p(t) + C_g^R u_g(t) + c_g^{SU}(t) + c_g^{SD}(t) \right) + \sum_{n \in \mathcal{N}} C_{LP} \left(s_n^+(t) + s_n^-(t) \right) + C_{RP} s_R(t) \right). \tag{69}$$

When $c_g^p(t)$, $c_g^{SU}(t)$, or $c_g^{SD}(t)$ are expressed using equality constraints, we make the appropriate substitution directly into (69).

4. State-of-the-Art UC Formulations

Our analysis considers a range of UC formulations from the literature. Where possible, we mirror authors' original formulations. However, some papers in the literature do not deal with every aspect of the UC problem discussed in this paper. For example, many papers do not consider a network (Carrion and Arroyo 2006, Ostrowski et al. 2012, Morales-España et al. 2013a, Yang et al. 2017, Atakan et al. 2018), and even influential papers (e.g., Ostrowski et al. (2012) and Morales-España et al. (2013a)) do not consider piecewise production costs. We attempt to fill in these missing pieces by making a judgment about what would have been state-of-the-art at the time the source paper was published, and explicitly mention these differences as they arise. In this way, we hope to track the improvements in UC formulations over the past twelve years. When we state variable names when describing these formulations, they should be assumed to be T-dimensional vectors, e.g., $u_g \in \mathbb{Z}^T$, $p_g \in \mathbb{R}^T$. Similarly, the indexing set for those variables included in a model is implied.

For brevity, in the main text of this paper we only consider recent UC formulations that demonstrate competitive performance computationally. However, we additionally considered six other formulations from the literature, labeled **CA**, **OAV-O**, **OAV-UD**, **OAV**, **OAV-T**, and **ALS**. The first is from Carrion and Arroyo (2006), the last from Atakan et al. (2018), and the rest are variants tested in Ostrowski et al. (2012). In general, these formulations are not computationally competitive, as detailed with full computational results in the online supplement (Knueven et al. 2018d, Section C). This leaves two competitive UC formulations, which we will denote **MLR** and **KOW**, which represent the current state-of-the-art.

Morales-España et al. (2013a) proposed many changes to the base UC formulation as compared to Carrion and Arroyo (2006), including the use of $p'_g(t)$ and $r_g(t)$ variables for

Table 1 Specification of the formulation proposed in Morales-España et al. (2013a)

Morales-España et al. (MLR)	
objective:	(69)
variables:	$u_g, v_g, w_g, \delta_g^s, p'_g, r_g, c_g^p, p_{W,n}, f_k, \theta_n, s_n^+, s_n^-, s_n, s_R$
up-time/down-time:	(2), (3), (4), (5)
generation limits:	(20), (21)
ramp limits:	(26), (27)
piecewise production:	(50)
start-up cost:	(54), (55), (56)
shut-down cost:	(62)
system constraints:	(64), (65), (66), (68)

Table 2 Specification of the formulation proposed in Knueven et al. (2018b)

Knueven et al. (KOW)	
objective:	(69)
variables:	$u_g, v_g, w_g, x_g, p'_g, r_g, p_g^l, p_{W,n}, f_k, \theta_n, s_n^+, s_n^-, s_n, s_R$
up-time/down-time:	(2), (3), (4), (5)
generation limits:	(20), (21)
ramp limits:	(26), (27)
piecewise production:	(43), (44), (46) if $UT_g > 1$, else (47)
start-up cost:	(59), (60), (61)
shut-down cost:	(62)
system constraints:	(64), (65), (66), (68)

power and reserves, start-up type indicator variables $\delta_g^s(t)$, and tighter bounds on generator production. We add the perspective formulation for piecewise cost curves (50) and the (B,θ) network formulation, which was referenced in Morales-España et al. (2013a) but not considered therein for simplicity. Table 1 gives the complete formulation, which we refer to as **MLR**.

Knueven et al. (2018b) reported a UC formulation similar to that of Morales-España et al. (2013a), with start-up costs formulated as in (Knueven et al. 2018c) in addition to a tighter description of piecewise production costs. As with **MLR**, we add the (B, θ) network formulation. The resulting formulation has the property that identical generators that are not ramping-constrained (i.e., $RU_g, RD_g \geq \overline{P}_g - \underline{P}_g$) can be exactly aggregated. Such aggregation can result in significant speed-ups for UC instances with identical generators as symmetry issues are mitigated. Table 2 gives the complete formulation, which we refer to as **KOW**.

5. Novel Formulations

We now introduce two novel UC formulations that draw on various ideas presented above, yielding novel combinations of formulation components that have not been explicitly con-

Table 3 Specification of the "tight" formulation

	Tight (T)
objective:	(69)
variables:	$u_g, v_g, w_g, x_g, p'_g, \overline{p}'_g, p^l_g, p_{W,n}, f_k, \theta_n, s^+_n, s^-_n, s_n, s_R$
up-time/down-time:	(2), (3), (4), (5)
generation limits:	(17), (23), (38), (40) if $T^{RU}_g > UT_g - 2$, (41)
ramp limits:	(35), (36)
piecewise production:	(43), (44), (46) if $UT_g > 1$, else (48)
start-up cost:	(59), (60), (61)
shut-down cost:	(62)
system constraints:	(64), (65), (66), (67)

sidered previously in the literature. The first formulation builds what should be a tight formulation, sacrificing some compactness yet exhibiting good computational performance. We refer to this formulation, presented in Table 3, as **T**. The differentiating features of the **T** formulation are the enhanced generation limits discussed in Section 3.5 and the tighter formulation for piecewise production cost discussed in Section 3.6. The **T** formulation also uses the two-period ramping limits from the ramp-up and ramp-down polytopes introduced by Damcı-Kurt et al. (2016) and the tight start-up cost formulation introduced by Knueven et al. (2018c). Finally, as with formulation **KOW**, formulation **T** has the property that identical generators with redundant ramping constraints can be aggregated exactly.

The second formulation we consider is also based on some of the tighter UC formulations reported in the literature, but emphasizes compactness at the potential loss of tightness. We refer to this formulation, presented in Table 4, as **Co**. This formulation is most similar to **MLR** (Morales-España et al. 2013a), with two key differences: (1) we use the $\overline{p}'_g(t)$ variables for the reserve production because they empirically yield improved performance in practice relative to the $r_g(t)$ variables, and (2) we use the two-period ramping limits described in Damcı-Kurt et al. (2016) in place of the ramping limits used in **MLR**. As with the **MLR** formulation, we use the perspective formulation to model piecewise production costs. Unlike the **KOW** and **T** formulations, **MLR** and **Co** in general only allow for exact aggregation of generators with minimum up-time/down-time constraints and simple ($\underline{P}$, $\overline{P}$) power bounds.

6. Computational Experiments

We now analyze the performance of various UC formulations, considering a variety of realistic test instances. All UC models were expressed using the Python-based Pyomo algebraic modeling library (`www.pyomo.org`) (Hart et al. 2011, 2017). In order to facilitate

Table 4 **Specification of the "compact" formulation**

Compact (Co)	
objective:	(69)
variables:	$u_g, v_g, w_g, \delta^s_g, p'_g, \overline{p}'_g, c^p_g, p_{W,n}, f_k, \theta_n, s^+_n, s^-_n, s_n, s_R$
up-time/down-time:	(2), (3), (4), (5)
generation limits:	(17), (20), (21)
ramp limits:	(35), (36)
piecewise production:	(50)
start-up cost:	(54), (55), (56)
shut-down cost:	(62)
system constraints:	(64), (65), (66), (67)

the comparison of a wide range of UC formulations, we developed a modular modeling framework that allows users to easily combine disparate formulation components from the literature to form both existing and novel UC models. This framework is available as part of the EGRET software package (`https://github.com/grid-parity-exchange/Egret`). While it is beyond the present scope to evaluate all possible UC formulations that can be expressed in the framework, our aim in this software release is to enable researchers and practitioners to quickly evaluate potential novel UC formulations by providing reference implementations of most of the formulation ideas found in the existing literature. Additionally, we solicit and will actively integrate new formulations for UC model components, as well as novel combinations that prove to be effective in practice.

6.1. Instances

We next describe the set of UC instances we consider in our computational experiments. We use realistic UC instances, based on real-world data, in order to avoid pathological issues that many of the synthetic instances reported in the UC literature possess. For example, the instances from Carrion and Arroyo (2006), Ostrowski et al. (2012), and Morales-España et al. (2013a) have excessive symmetry, which ends up dominating the solve time when not properly addressed, e.g., by the approaches reported by Ostrowski et al. (2015) and Knueven et al. (2018b). In all cases we consider a 48-hour scheduling horizon, mirroring the more difficult day-ahead UC (as opposed to reliability or intra-day UC) found in power systems operations contexts.

We consider three test sets, each based on realistic data. The first set, "RTS-GMLC," consists of twelve instances (corresponding to distinct days) associated with the newly introduced RTS-GMLC system (`https://github.com/GridMod/RTS-GMLC`) (Barrows et al. 2019). We consider variants both with and without a transmission network, for a total of 24 instances. The RTS-GMLC system is relatively small, with 73 thermal units. Reserves are set at 3% of system load. The second set, "CAISO," is based on publicly available data from the California Independent System Operator (CAISO) in the US. For the CAISO set, we consider five demand/renewable profiles and four system reserve levels, yielding a total of 20 instances. All CAISO instances consist of 410 thermal generators. Because it is sensitive critical infrastructure, we do not consider the transmission network. The third and final test set, "FERC," is based on publicly available generator data obtained from the US Federal Energy Regulatory Commission (FERC) (Krall et al. 2012), with demand, reserve quantities, and wind profiles taken from the PJM system operator in the US. The generator fleet from Krall et al. (2012) is itself based on market data from PJM. Twelve demand/renewable profiles are considered, and the renewable data is scaled up to 30% penetration to create twelve additional instances. The FERC instances are the largest we consider, with approximately 900 thermal generators. In total, we solve 68 UC instances for each formulation considered in the computational experiments reported below. The load, reserve, and generator data for these test instances is available as part of the IEEE PES Power Grid Lib - Unit Commitment benchmark library (`https://github.com/power-grid-lib/pglib-uc`). Further details on the test sets are available in the online supplement (Knueven et al. 2018d, Section D).

6.2. Novel UC Formulations

To demonstrate the utility of our modular framework for composing UC formulations, we sampled at random 1,000 formulations and solved each of the resulting formulations on each of our twenty-four RTS-GMLC instances. The formulations were sampled without replacement by selecting at random each of the eight components of a generator formulation (status variables, power variables, reserve variables, generation limits, ramping limits, production costs, up-time/down-time constraints, and start-up costs) based on the formulations discussed herein and the online supplement (Knueven et al. 2018d). Formulations with incompatible components were discarded. Each run used the Gurobi 8.0.1 (Gurobi Optimization, Inc. 2018) solver, limited to a single thread, with a 60 second time limit and a 0.0 optimality gap threshold. From these 1,000 formulations, we selected two UC formulations that scored in both the top 1% of shifted geometric mean optimality gap and geometric mean time. To compute the shifted geometric mean optimality gap, we added 1 to the optimality gap and computed the geometric mean of the resulting values. The two

Table 5 Specification of the first randomly sampled UC formulation

Random1 (R1)	
objective:	(69)
variables:	$\tilde{u}_g, v_g, w_g, \delta^s_g, p'_g, \overline{p}'_g, c^p_g, p_{W,n}, f_k, \theta_n, s^+_n, s^-_n, s_n, s_R$
up-time/down-time:	(3), (9), (10), (11)
generation limits:	(17), (20), (21)
ramp limits:	(35), (36)
piecewise production:	(50)
start-up cost:	(54), (57), (58)
shut-down cost:	(62)
system constraints:	(64), (65), (66), (67)

Table 6 Specification of the second randomly sampled UC formulation

Random2 (R2)	
objective:	(69)
variables:	$u_g, v_g, w_g, \delta^s_g, p'_g, \overline{p}'_g, c^p_g, p_{W,n}, f_k, \theta_n, s^+_n, s^-_n, s_n, s_R$
up-time/down-time:	(2), (3), (4), (5)
generation limits:	(17), (23), (38), (40) if $T^{RU}_g > UT_g - 2$, (41)
ramp-up limits:	(24) if $RU_g \leq SD_g - \underline{P}_g$ or $UT_g < 2$, else (28)
ramp-down limits:	(25) if $RD_g \leq SU_g - \underline{P}_g$ or $UT_g < 2$, else (30) if $UT_g < 3$ or $DT_g < 2$, else (31)
2-ramp-up limits:	(29) if $RU_g > SD_g - \underline{P}_g$, $UT_g \geq 2$, and $DT_g \geq 2$
2-ramp-down limits:	(32) if $RD_g > SU_g - \underline{P}_g$, $UT_g \geq 3$, and $DT_g \geq 2$
piecewise production:	(50)
start-up cost:	(54), (55), (56)
shut-down cost:	(62)
system constraints:	(64), (65), (66), (67)

selected formulations are shown in Tables 5 and 6. Although we did not explicitly count the number of formulations our framework can express, we were able to enumerate over 100,000 unique formulations for UC. Since this is an underestimate of the total number of formulations our framework enables, we have explored no more than 1% of the candidate space with our 1,000 sample selection.

Inspection indicates that the first random formulation (denoted **R1**) is a modification of that introduced in Atakan et al. (2018) (see Table 6 in Knueven et al. (2018d)), but replaces the generation limits with those from Morales-España et al. (2013a), the modified start-up costs also from Morales-España et al. (2013a) (as presented in Section 3.7.1), and the perspective production cost formulation. Conversely, the second random formulation (denoted **R2**) is a mixture of diverse UC formulation ideas from the literature. Inspection indicates that **R2** essentially draws aspects from both the **T** and **Co** formulations, sharing variables with formulation **Co**, and adding the Morales-España et al. (2013a) start-up cost formulation and the perspective production cost formulation. However, **R2** further brings in the combination of generation limits used in formulation **T** and all of the ramping limits from Ostrowski et al. (2012), making it one of the denser UC formulations in terms of the constraints on the power variables.

6.3. Results

All computational experiments were conducted on a Dell PowerEdge T620 with two Intel Xeon E5-2670 processors, 256GB of RAM, and running the Ubuntu 16.04.4 Linux operating system. Gurobi 8.0.1 was the MIP solver used in all tests, with the default settings preserved except for the time limit. We attempt to solve all instances to a 0.01% relative optimality gap. No other intensive jobs were running on the system at the time the experiments were carried out.

We report the performance of each of the six UC formulations examined in summary tables, which report the following quantities: wall clock time, optimality gap, number of time outs (instances for which the time limit was reached), number of times a solution was found in the fastest overall time, and number of times a solution was found in the second fastest overall time. To rank the UC formulations, "times best" refers to the fastest solve time or if every formulation times out, the lowest optimality gap. "Times second" similarly refers to the second fastest solve or if every or all but one formulation times out, the second lowest optimality gap. In columns labeled "Time (s)" we report the geometric mean solve time in seconds, substituting the time limit for instances which timed out. In columns labeled "Opt gap (%)" we report a shifted geometric mean optimality gap, where for each instance solved we replace the terminating optimality gap reported by the solver with the default tolerance of 0.01%. Because the optimality gap can be (close to) 0, we first added 1 to the optimality gap, computed the geometric mean, and subtracted 1 from the result; the latter is the reported mean. We adjusted the terminating optimality gap as the solver often terminated with a gap significantly less than 0.01%, and we did not wish to bias our reporting with this behavior. Thus, if every instance timed out, "Time (s)" would report the time limit, and conversely if every instance solved "Opt gap (%)" would report 0.01%. Full summary tables and detailed results are provided in the online supplement (Knueven et al. 2018d, Sections E and G).

6.3.1. RTS-GMLC Summary computational results for the RTS-GMLC instances are reported in Table 7. Due to their relatively small size, we set the time limit for each solve of

Table 7 Summary computational results for RTS-GMLC instances.

Formulation	Time (s)	Opt gap (%)	Time outs	Times best	Times 2nd
MLR	86.19	0.0165%	5	0	0
KOW	94.81	0.0165%	4	0	1
T	50.65	0.0121%	2	4	4
Co	43.11	0.0121%	**1**	1	4
R1	**32.37**	**0.0114%**	**1**	**14**	6
R2	36.94	0.0114%	**1**	4	**8**

Table 8 Summary computational results for CAISO instances.

Formulation	Time (s)	Opt gap (%)	Time outs	Times best	Times 2nd
MLR	117.1	0.0119%	2	0	1
KOW	79.02	0.0102%	2	4	**7**
T	**56.90**	**0.0100%**	**0**	**15**	3
Co	100.6	**0.0100%**	**0**	0	5
R1	147.4	0.0102%	1	0	1
R2	104.1	0.0100%	1	1	3

these instances to 300 seconds. We first note that two state-of-the-art formulations from the UC literature – **MLR** and **KOW** – perform very similarly, likely because they share many formulation components. Both the **MLR** and **KOW** formulations are outperformed by the **T** and **Co** formulations. Between formulations **T** and **Co**, **Co** is on average faster, but **T** yields the fastest solve time more often. Even though **T** has an additional time-out over **Co**, their shifted geometric mean optimality gaps are identical. Finally, we observe that the best of the randomly sampled UC formulations – **R1** and **R2** – outperform even the **T** and **Co** formulations. Together, these two formulations claim the majority of first and second place spots available across the formulations, and further possess the shortest geometric mean solve times and lowest shifted geometric mean optimality gap. We emphasize that these two random UC formulations were based on tests on the RTS-GMLC instances, so this result is not entirely surprising. However, the result does highlight that novel UC formulations can yield performance above that of state-of-the-art formulations reported in the literature.

6.3.2. CAISO Summary computational results for the CAISO instances are reported in Table 8. A time limit of 300 seconds for each solve of these instances was imposed, identical to that used for the RTS-GMLC experiments. The results indicate that formulation **MLR** is competitive on these instances, but is outperformed by **KOW** and the other

Table 9 Summary computational results for FERC instances.

Formulation	Time (s)	Opt gap (%)	Time outs	Times best	Times 2nd
MLR	340.5	0.0555%	3	4	1
KOW	390.2	0.0117%	2	3	5
T	**268.6**	**0.0104%**	**1**	**8**	4
Co	309.5	0.0596%	3	4	5
R1	308.9	0.0480%	3	3	**6**
R2	373.1	0.0665%	4	0	1

four formulations introduced in this paper. Formulations **R1** and **R2** perform somewhat similarly on these instances, with **R2** leading in terms of time-to-solution. Formulation **R2**'s single time-out has a optimality gap of just over 0.01%, which is why its reported shifted geometric mean optimality gap is 0.0100%. However, in contrast to the results on RTS-GMLC, the clear winner on the CAISO instances is formulation **T**, which yields the smallest geometric mean runtime and is fastest on three quarters of the instances. Further, as documented in the online supplement, the largest solve time given formulation **T** was 129.9 seconds, compared to a maximum time of 261.9 seconds for **Co** and 300+ seconds for the other four formulations.

6.3.3. FERC Summary computational results for the FERC instances are reported in Table 9. Because the FERC instances have over twice the number of thermal generators as the CAISO instances, we increase the solve time limit to 600 seconds. The results indicate that UC formulations using some variant of the start-up cost formulation from Morales-España et al. (2013a) (**MLR**, **Co**, **R1**, and **R2**) all perform similarly. In contrast, the start-up cost formulation from Knueven et al. (2018c) appears to be essential to achieving high-quality solutions across the FERC instance set, i.e., as can be seen for results obtained by the UC formulations **KOW** and **T**. Variant **T** slightly outperforms **KOW** in terms of geometric mean time to solution. Further, even on the one FERC instance for which formulation **T** hits the solve time limit, the terminating optimality gap is only 0.018%.

6.3.4. Summary Analysis Across the three test instance sets considered, the newly introduced "Tight" formulation **T** performs the best – establishing what we believe to be a new state-of-the-art in UC performance. Formulation **T** has the fewest time-outs overall, and is consistently the fastest on the larger and more difficult CAISO and FERC instances. In the worst case for **T**, Gurobi returns with a terminating gap of 0.05%, which is the

Table 10 Selected results for changing various components of the tight formulation T. For each row, we swap equations for the component in formulation T (e.g. up-time/down-time) with that defined by the equations given.

	RTS-GMLC		CAISO		FERC	
	Time (s)	Opt gap (%)	Time (s)	Opt gap (%)	Time (s)	Opt gap (%)
T	50.65	0.0121%	56.90	0.0100%	268.6	0.0103%
up-time/down-time						
RT 2bin: (4)(7)	48.29	0.0120%	51.30	0.0100%	235.1	0.0100%
generation limits						
MLR: (20)(21)	51.29	0.0122%	52.44	0.0100%	254.8	0.0100%
GMR: (20)(23)	51.37	0.0122%	52.42	0.0100%	254.1	0.0100%
ramping limits						
MLR: (26)(27)	49.28	0.0119%	48.53	0.0100%	242.6	0.0100%
piecewise production						
CA: (43)(44)(45)	48.93	0.0119%	57.87	0.0100%	230.2	0.0100%

best worst-case performance across all UC formulations examined. The performance of **T** is weakest on the RTS-GMLC instances, and in particular on the RTS-GMLC instances with a network. Upon closer examination, we believe this discrepancy is likely due to the lower number of nodes explored in the branch-and-cut process relative to the more compact formulations, i.e., **Co**, **R1**, and **R2**. Our analysis of solver logs suggests that Gurobi has difficulty generating cuts at the root node of the branch-and-cut tree when the (B, θ) linearized network representation is considered. Typically, system operators do not use this network representation when solving UC in practice, and instead rely on a Power Transfer Distribution Factor (PTDF) or related network formulations (Van den Bergh et al. 2014, Chen and Wang 2017). Because the vast majority of transmission lines will not have binding thermal limits in an optimal UC solution, those likely not to be at their limits can be dropped from the formulation (Chen et al. 2016) or treated as lazy constraints. Further, one can generate cover inequalities for the PTDF transmission constraints, which may tighten the relaxation (Wu 2016).

6.4. Detailed Analysis of the "Tight" Formulation

We undertook a detailed computational study to determine what aspects of formulation **T** are essential to its superior performance and conversely which aspects could possibly be improved upon. For brevity, we report the full results in the online supplement (Knueven et al. 2018d, Section F), but provide selected results in Table 10. Overall, the results clearly indicate that there is some room for improvement over the base formulation **T**. Examining Table 10, formulating the up-time and down-time constraints using only two binary variables (in this case $u_g(t)$ and $v_g(t)$) resulted in a surprising performance improvement on the larger CAISO and FERC instances, with no degradation in performance on the RTS-GMLC instances.

Further examining Table 10, we see computational performance is moderately improved when considering the simpler MLR or GMR generation limits from Morales-España et al. (2013a) and Gentile et al. (2017), respectively. Interestingly, the simple ramping limits from Morales-España et al. (2013a) improved the performance of **T**; this is likely because with the variable upper bounds used in **T**, the ramping constraints become mostly redundant. Finally, there is some evidence that the production costs formulation from Carrion and Arroyo (2006) exhibited some improvement in the FERC test instances over the tight but more complicated formulation introduced in this paper. However, because of the risk of over-fitting for these test cases, we will not explore combinations of these options, though they may prove beneficial for the practitioner.

7. Conclusion

This paper has undertaken a thorough examination of the state-of-the-art in formulations for the UC problem, with detailed computational analysis. The computational results presented herein suggests that with a modern formulation, realistic UC problems with up to a thousand generators and 48 time periods are easily solvable with modest hardware and a modern commercial MIP solver. Therefore, the current research challenges in deterministic UC are (1) the interaction between UC modeling components and the transmission system (Van den Bergh et al. 2014, Wu 2016), (2) virtual transactions, which may weaken our ability to tighten system constraints (Chen et al. 2016), (3) realistic modeling of the transmission system, including the need for reactive power support (Castillo et al. 2016), and (4) better modeling of the ancillary service products, which tend to vary by market. Additional model changes will have to be considered with the continued increase in uncertain energy supply due to renewables, which could entail stochastic variants of UC, robust variants of UC, or additional reserve products, though all of these alternatives would have the deterministic formulations presented here at their core.

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Online Supplement for

On Mixed Integer Programming Formulations for the Unit Commitment Problem

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A Ideal Formulation for Generator with Piecewise Production Costs and Start-up/Shutdown Ramping Limits

Theorem 1. *The system* (2)*,* (4)*,* (5)*,* (43)*,* (44)*,* (46) *with variable bounds is an ideal formulation for a generator with minimum up/down times, piecewise production costs, and start-up/shutdown ramping limits if $UT \geq 2$. If $UT = 1$, then the system* (2)*,* (4)*,* (5)*,* (43)*,* (44)*,* (48) *with variable bounds is an ideal formulation for a generator with piecewise production costs and start-up/shutdown ramping limits.*

Proof. When $UT = 2$, a straightforward extension of [12, Theorem 5] with the $p^l(t)$ variables playing the same role as the p_t variables in the proof of [12, Theorem 5] shows this is true for the system (2), (4), (5), (46) with variable bounds. An application of [12, Lemma 4] shows the addition of the equality constraints (43) and (44) do not affect integrality. Notice that generation limits on $p'(t)$ are redundant because of (43) with (46). Finally, the above is also true when $UT = 1$ with inequalities (48) in place of (46). □

Remark 1. *When the start-up cost formulation* (59),(60),(61) *from [19] is added to the formulation from Theorem 1 and start-up costs are non-decreasing, the resulting polytope has integer optimal vertices.*

Remark 1 is a direct result of Theorem 1 and Knueven et al. [17, Remark 1]. It implies that a 1-UC for a generator with minimum up/down times, piecewise production costs, start-up/shutdown ramping limits, and non-decreasing time-dependent start-up costs can be solved as a linear program.

B Other Formulations

This section details formulations choices in addition to those in the main text.

B.1 Additional formulations for minimum up-time/down-time

Given Garver's [11] state variables, one possibility for writing the up-time/down-time is given by Dillon et al. [7] and more recently by Arroyo and Conejo [1]:

$$\sum_{i=t}^{t+UT-1} u(i) \geq UT v(t) \qquad t \in \{U+1, \dots, T-UT+1\} \tag{70a}$$

$$\sum_{i=t}^{T} (u(i) - v(t)) \geq 0 \qquad t \in \{T-UT+2, \dots, T\} \tag{70b}$$

$$\sum_{i=t}^{t+DT-1} (1-u(i)) \geq DT w(t) \qquad t \in \{D+1, \dots, T-DT+1\} \tag{71a}$$

$$\sum_{i=t}^{T} (1-u(i) - w(t)) \geq 0 \qquad t \in \{T-DT+2, \dots, T\}. \tag{71b}$$

Another common formulation is to just consider one binary variable to represent the state of generator g at time t, $u(t)$. Based on the logical constraint (2), the v and w variables are completely determined by the u variables, and so in some sense are redundant. However, results both theoretical and computational suggest this is not entirely true.

Takriti et al. [31] formulate the minimum up-time/down-time constraints as:

$$u(t) - u(t-1) \leq u(\tau) \qquad \forall \tau \in \{t+1, \dots, \min\{t+UT-1, T\}\}, t \in \mathcal{T} \tag{72}$$

$$u(t-1) - u(t) \leq 1 - u(\tau) \qquad \forall \tau \in \{t+1, \dots, \min\{t+DT-1, T\}\}, t \in \mathcal{T}. \tag{73}$$

These constraints are simple, but could be up to quadratic in T if the minimum up-times or down-times are long.

Carrion and Arroyo [4] formulate these same constraints as

$$\sum_{i=t}^{t+UT-1} u(i) \geq UT(u(t) - u(t-1)) \qquad t \in \{U+1, \dots, T-UT+1\} \tag{74a}$$

$$\sum_{i=t}^{T} (u(i) - (u(t) - u(t-1))) \geq 0 \qquad t \in \{T-UT+2, \dots, T\} \tag{74b}$$

$$\sum_{i=t}^{t+DT-1} (1 - u(i)) \geq DT(u(t-1) - u(t)) \qquad t \in \{D+1, \dots, T-DT+1\} \tag{75a}$$

$$\sum_{i=t}^{T} (1 - u(i) - (u(t-1) - u(t))) \geq 0 \qquad t \in \{T-DT+2, \dots, T\}, \tag{75b}$$

which are in the same form as (70) and (71), but with the v and w variables projected out.

However, unlike with the three- and two-binary formulations for minimum up-time/down-time, there is no compact convex hull description for these constraints in the u variables alone. The convex hull of the minimum up-time/down-time constraints has an exponential number of facets in the space of the u variables [21]. Hence the two- or three-binary formulations are often preferred, as the minimum up-time/down-time constraints have a compact (linear) convex hull description which can be directly incorporated into UC MIP formulations.

B.2 Additional Generation and Ramping Limits from Carrion and Arroyo [4]

To enforce the shutdown ramp limits, Carrion and Arroyo [4] defined the constraint

$$\overline{p}_g(t) \leq \overline{P}_g u_g(t+1) + SD_g(u_g(t) - u_g(t+1)) \qquad t \in \mathcal{T} \setminus \{T\}, \tag{76}$$

which is the same as constraint (19) with the shutdown variable w removed. Similarly, Carrion and Arroyo [4] formulated the ramping limits as

$$\overline{p}(t) - p(t-1) \leq RU u(t-1) + SU\big(u(t) - u(t-1)\big) + \overline{P}\big(1 - u(t)\big) \tag{77}$$

$$p(t-1) - p(t) \leq RD u(t) + SD\big(u(t-1) - u(t)\big) + \overline{P}\big(1 - u(t-1)\big), \tag{78}$$

which eliminate the variables v and w from (24) and (25), respectively.

B.3 Additional Ramping Inequalities from Damcı-Kurt et al. [6]

Like Ostrowski et al. [23], Damcı-Kurt et al. [6] also gave polynomial classes of ramp-up and ramp-down inequalities which hold under certain conditions. In some ways these are complementary to those given by Ostrowski et al. [23], as they hold under somewhat complementary conditions, when $RD = RU$ and $SU = SD$. Both ramping inequalities are extensions of (33) and (34). When $SU \geq \underline{P} + RU$ ($RU \leq SU - \underline{P}$) we have

$$p(t+j) - p(t) \leq (\underline{P} + jRU)u(t+j) \\ + \sum_{i=1}^{j} \min\{(SU - \underline{P} - iRU), (\overline{P} - \underline{P} - jRU)\}v(t+i) - \underline{P}u(t), \tag{79a}$$

which holds for $\forall j \in \left\{1, \dots, \min\left\{T - t, \frac{SU-\underline{P}}{RU}\right\}\right\}, t \in \mathcal{T}$. Note this is in contrast to inequalities (28) and (29), which in addition to requirements on UT and DT also require $RU > SD - \underline{P}$. Damcı-Kurt et al. [6] also showed (79a) is a facet of the ramp-up polytope if and only if $\underline{P} + jRU < \overline{P}$. There is an analogous class of ramp-down inequalities, which are valid when $SD \geq \underline{P} + RD$ ($RD \leq SD - \underline{P}$)

$$p(t) - p(t+j) \leq (\underline{P} + jRD)u(t) - \underline{P}u(t+j) \\ + \sum_{i=1}^{j} \min\{(SD - \underline{P} - (j-i+1)RD), (\overline{P} - \underline{P} - jRD)\}w(t+i), \tag{79b}$$

$\forall j \in \left\{1, \dots, \min\left\{T - t, \frac{SD-\underline{P}}{RD}\right\}\right\}, t \in \mathcal{T}$. Like its ramp-up counterpart (79a), (79b) is a facet of the ramp-down polytope if and only if $\underline{P} + jRD < \overline{P}$. For both inequalities (79a) and (79b), it is not often the case that $\left\lfloor \frac{SU-\underline{P}}{RU} \right\rfloor \geq 2$ or $\left\lfloor \frac{SD-\underline{P}}{RD} \right\rfloor \geq 2$, and when these quantities are 1 these two inequalities are just (33) and (34) respectively. Hence the inequalities (79a) and (79b) may not be very useful in practice.

B.4 The Generator Polytope

Knueven et al. [16] presented the first perfect formulations for a generator with ramping constraints as well as time-dependent start-up costs, work that was independently discovered by Frangioni and Gentile [9, 10] and Guan et al. [13]. While the formulations are very similar, all having on the order of $|\mathcal{T}|^3$ and constraints, they differ in how they "link" the underlying economic dispatch polytopes. Knueven et al. [16] shows that only $\mathcal{O}(|\mathcal{T}|)$ many linking constraints are needed to describe the instances with no time-dependent start-up costs, and Guan et al. [13] gives the same result for instances with time-dependent start-up costs. We present here a formulation from Knueven et al. [16] which directly incorporates reserve capability. Note that any of these formulations can be extended to include most ancillary service products while still maintaining integrality, so long as the dispatch is a polytope when the on/off schedule is fixed. That is, there is a perfect formulation for Π_g, in the sense that the vertices of said formulation are integer in the binary variables. First, we to need define an additional set and variable. Let $[t, t') \in \mathcal{Y}_g$ be the feasible intervals of operation for generator g with respect to its minimum uptime, that is, $[t, t') \in \mathcal{T} \times \mathcal{T}$ such that $t' \geq t + UT_g$, including times (as necessary) before and after the planning period $\mathcal{T}$, and $y_g(t,t')$ be the binary indicator arc for start-up at time t, shutdown at time t', committed for $i \in [t,t')$, $[t,t') \in \mathcal{Y}_g$, for generator $g \in \mathcal{G}$. Then put broadly, said formulation is

$$A^{[i,j)}p^{[i,j)} + \overline{A}^{[i,j)}\overline{p}^{[i,j)} \leq b^{[i,j)}y(i,j) \qquad \forall [i,j) \in \mathcal{Y} \tag{80a}$$

$$\sum_{[i,j)\in\mathcal{Y}} p^{[t,t')}(t) = p(t) \qquad \forall t \in \mathcal{T} \tag{80b}$$

$$\sum_{[i,j)\in\mathcal{Y}} \overline{p}^{[t,t')}(t) = \overline{p}(t) \qquad \forall t \in \mathcal{T} \tag{80c}$$

$$\sum_{\{[k,l)\in\mathcal{X}|l=t\}} x(k,l) = \sum_{\{[i,j)\in\mathcal{Y}|i=t\}} y(i,j) \qquad \forall t \in \mathcal{T} \tag{80d}$$

$$\sum_{\{[i,j)\in\mathcal{Y}|j=t\}} y(i,j) = \sum_{\{[k,l)\in\mathcal{X}|k=t\}} x(k,l) \qquad \forall t \in \mathcal{T}, \tag{80e}$$

where the $p^{[i,j)}(t)$ ($\overline{p}^{[i,j)}(t)$) represent the power (power available) at time t given the generator started-up at time i and shutdown at time j, and the polytope $\{p^{[i,j)}, \overline{p}^{[i,j)} \mid A^{[i,j)}p^{[i,j)} + \overline{A}^{[i,j)}\overline{p}^{[i,j)} \leq b^{[i,j)}\}$ represents the physical (minimum power, ramp up/down limits, maximum power) when the generator is turned on at time i and turned off at time j. So long as the power and ancillary services provided can be represented as a (bounded) polytope when the commitment status is fixed, (80) is a perfect formulation for said generator [16]. Constraints (80b) and (80c) serve to calculate the total power produced (or available) for the generator. Constraints (80d) and (80e) ensure that an off-arc is followed by an on-arc and visa versa. Notice that time-dependent start-up costs and fixed running costs can be accounted for by placing the appropriate coefficients on the $x(i,j)$ and $y(i,j)$ variables respectively. Piecewise production costs can be handled by considering $f^{[i,j)}(p^{[i,j)})$, where $f^{[i,j)}$ is a piecewise convex function. (Note such production costs should be incorporated into constraints (80a).)

While this extended formulation could be directly considered in a full-scale UC problem, its size, which is $\mathcal{O}(T^3)$, poses computational challenges. Namely, the extra copies of the continuous variables are not well-handled by linear programming solvers, in addition to increasing the size of the formulation. Knueven et al. [16] used it as a basis for a cut-generation routine for this reason.

However, there may be some cases where incorporating (80) directly in the UC formulation is worth while. As shown in Knueven et al. [18], if there are several copies of the same generator at the same location, sometimes (80) is advantageous and can be directly incorporated into the formulation, so long as the multiple generators are aggregated. Knueven et al. [18] demonstrated this aggregation can be done while maintaining optimality and feasibility. Such a formulation may be useful especially in capacity expansion problems, where one may have the option of building many generators of the same type. Further, since the commitment decisions are often relaxed as part of a two-stage framework, having a convex hull representation provides the best approximation for dispatch. For the same reason, (80) may also be useful as part of the pricing problem, as it ensures convex hull prices are obtained [14]. Though in this context it may still be best to solve the pricing linear program using some sort of decomposition technique. Finally, we note that a version of (80) can be used in a two-stage stochastic setting by replicating (80a)–(80c) for each scenario, giving the convex hull for generator scheduling under uncertainty.

Guan et al. [13] introduced three extended formulations for ramping. The first which is similar to (80) except it replaces the shortest path polytope (80d)–(80e) with an integer polytope directly taken from the dual of a modified version of the dynamic program for 1-UC suggested by Frangioni and Gentile [8]. Guan et al. [13] gave two other related extended formulations, one for a traditional deterministic setting and the other for a stochastic setting. We discuss the deterministic formulation below; the stochastic formulation is similar.

As in Guan et al. [13], consider the case when $S = SU = SD$ and $R = RU = RD$, and there are no reserves, so we can assume $r_t = 0$ or $p_t = \overline{p}_t$, for all $t \in \mathcal{T}$. Further define $\alpha_1 = \max\{t \in \mathcal{T} \mid \underline{P} + tR \leq \overline{P}\}$ and $\alpha_2 = \max\{t \in \mathcal{T} \mid S + tR \leq \overline{P}\}$ and let

$$\mathcal{Q} = \{0, (\underline{P} + tR)_{t=0}^{\alpha_1}, (S + tR)_{t=0}^{\alpha_2}, (\overline{P} - tR)_{t=0}^{\alpha_1}\}. \tag{81}$$

In this case, $\mathcal{Q}$ are the p-components of the vertices of the generator polytope [13]. Of course, if we add the additional reserve variables or drop the assumptions on the ramp-rates, we may have more vertices, and so need to enlarge the set $\mathcal{Q}$. Guan et al. [13] made such an extension to piecewise production costs. Given this, one can define a state-space for a given time t which is dependent on UT, DT, and $|\mathcal{Q}|$ as follows. $\mathscr{S}_0(t)$ are all the states when the generator is off-line at time t, which is defined by the 4-tuple ($p(t)$, $u(t)$, $v(t)$, $d(t)$), where $p(t) = u(t) = v(t) = 0$ as the generator is not operating, and $d(t)$ is how many time periods the generator has been off at time t, up to DT. $\mathscr{S}_1(t)$ are all the states when the generator is on-line at time t, again defined by a 4-tuple ($p(t)$, $u(t)$, $v(t)$, $d(t)$), when $p(t) \in \mathcal{Q}$, and $d(t)$ is how many time periods the generator has been on-line up to UT. Let $\mathscr{S}(t) = \mathscr{S}_0(t) \cup \mathscr{S}_1(t)$. Then we can put in the feasible transition arcs between $\mathscr{S}(t)$ and $\mathscr{S}(t+1)$ which respect the ramping, start-up/shutdown limits, and minimum up-time/down-time constraints, and similarly between $\mathscr{S}(t)$ and $\mathscr{S}(t-1)$. Guan et al. [13] showed that this flow formulation is integer in the variables for the state-transition arcs, and hence integer in the p, u, and v variables as well (see [13] for the full description). Additionally, the number of nodes in a state $\mathscr{S}(t)$ is $\mathcal{O}(DT + 3 \cdot UT\lceil(\overline{P} - \underline{P})/R\rceil)$, so this formulation may be useful for generators whose UT, DT, and/or $\lceil(\overline{P} - \underline{P})/R\rceil$ are small. Notice that the formulation (80) has the property that the set $\mathcal{Y}$ (and hence the number of variables and constraints) grows larger as UT gets smaller, so in some sense these two extended formulations are complementary. However, the formulation from [13] is not as extensible as that of (80), in that a full vertex description in the continuous variables p (and $\overline{p}$, if included) is needed to specify the formulation. This may be especially problematic when the assumption $R = RU = RD$ does not hold.

B.5 Additional Variable Upper Bounds

Ostrowski et al. [23] stated inequality (37) as

$$p(t) \le \overline{P}u(t+K(t)) + \sum_{i=1}^{K(t)} (SD + (i-1)RD)\, w(t+i) - \sum_{i=1}^{K(t)} \overline{P}v(t+i), \tag{82}$$

where $K(t) := \max\{\tau \in \{1,\dots,UT\} \mid SD + (\tau-1)RD < \overline{P} \text{ and } \tau + t < T\}$. Note that the $K(t) = K^{SD}(t)+1$ for given t, where like for (37), $K^{SD}(t) = \min\{T^{RD}, UT-1, T-t-1\}$. To see that this is equivalent to (37), we will show that (2) can be applied to (82) to obtain (37). Note that by (2), $v(t+i) = u(t+i) - u(t+i-1) + w(t+i)$. Applying this to the final sum in (82), we have

$$p(t) \le \overline{P}u(t+K(t)) + \sum_{i=1}^{K(t)} \left(SD + (i-1)RD - \overline{P}\right) w(t+i) - \sum_{i=1}^{K(t)} \overline{P}(u(t+i) - u(t+i-1)). \tag{83}$$

Canceling the inside terms of the last sum leaves in its place $\overline{P}(u(t+K(t)) - u(t))$, which when canceled with the first term yields

$$p(t) \le \overline{P}u(t) + \sum_{i=1}^{K(t)} \left(SD + (i-1)RD - \overline{P}\right) w(t+i), \tag{84}$$

which gives exactly (37) after pulling a negative out of and reindexing the sum because $K(t) = K^{SD}(t)+1$.

Damcı-Kurt et al. [6] gave two exponential classes of variable upper bound inequalities with polynomial time separation algorithms. However, such classes of inequalities require a separation procedure, which is usually implemented as a callback to the solver. Pan and Guan [24] also presented a linear and two exponential classes of variable bound inequalities for the generator polytope with minimum up/down times and ramp-up/ramp-down constraints. Again we will not consider the exponential classes here, but the linear class is a form of the inequalities alluded to above. First, we note that [24] assumed no difference between ramp-up and ramp-down, so let $R = RU = RD$ and $S = SU = SD$. Then we have

$$p(t) \le Su(t) + (\overline{P} - S)(u(t+1) - v(t+1)) - \sum_{i=1}^{k-1} (\overline{P} - S - (i-1)R)v(t-i+1), \tag{85}$$

which is valid for $k \in \{1,\dots,\min\{UT, T^R+2\}\}$ and $t \in \{k,\dots,T-1\}$, where $T^R = \left\lfloor \frac{\overline{P}-S}{R} \right\rfloor$. Pan and Guan [24] proved this is facet of the generator polytope when $k = \min\{UT, T^R+2\}$ and (1) $UT \le 3$ or (2) $UT \ge 4$ and $t = T-1$. Using the transformation given by (2) we can relate (85) to the those above. As

$u(t+1) - v(t+1) = u(t) - w(t+1)$, we can re-arrange (85) as (re-indexing the sum as well):

$$p(t) \leq \overline{P}u(t) + (\overline{P} - S)w(t+1) - \sum_{i=0}^{\min\{UT-2, T^R\}} (\overline{P} - S - iR)v(t-i). \tag{86}$$

Further, now it is clear which S plays the role of SU and SD, and similarly for R, so we arrive at the following valid inequality

$$\overline{p}(t) \leq \overline{P}u(t) + (\overline{P} - SD)w(t+1) - \sum_{i=0}^{\min\{UT-2, T^{RU}\}} (\overline{P} - SU - iRU)v(t-i), \tag{87}$$

which is just (38).

Finally we outline the extension of (39) to when $UT = T^{RU} + T^{RD} + 1$. In this case, we can derive a couple of inequalities in the spirit of (23):

$$\begin{aligned} p'(t) \leq (\overline{P} - \underline{P})u(t) - \sum_{i=0}^{T^{RU}-1} (\overline{P} - (SU + iRU))v(t-i) - \sum_{i=0}^{T^{RD}} (\overline{P} - (SD + iRD))w(t+1+i) \\ - \left[(SD + T^{RD}RD) - (SU + T^{RU}RU)\right]^{+} v(t - T^{RU}) \end{aligned} \tag{88a}$$

$$\begin{aligned} p'(t) \leq (\overline{P} - \underline{P})u(t) - \sum_{i=0}^{T^{RU}} (\overline{P} - (SU + iRU))v(t-i) - \sum_{i=0}^{T^{RD}-1} (\overline{P} - (SD + iRD))w(t+1+i) \\ - \left[(SU + T^{RU}RU) - (SD + T^{RD}RD)\right]^{+} w(t + T^{RD} + 1). \end{aligned} \tag{88b}$$

When $UT \leq T^{RD} + T^{RD}$, the number of potential valid and strong inequalities for upper-bound continues to double.

B.6 Additional Piecewise Production Cost Formulations

Another possibility for modeling convex piecewise linear production costs is a relaxed SOS2 model. Let $\lambda^l \in [0,1]^T$, for $l \in \mathcal{L}$, represent the fraction of generation coming from piecewise segment l, and $\lambda^0 \in [0,1]^T$ represent the faction of generation coming from the unit operating at $\underline{P}$. Then, another formulation for convex piecewise linear production is:

$$\sum_{l \in \mathcal{L} \cup \{0\}} (\overline{P}^l - \underline{P})\lambda^l(t) = p'(t) \tag{89}$$

$$\sum_{l \in \mathcal{L} \cup \{0\}} (\overline{C}^l - \overline{C}^0)\lambda^l(t) = c^p(t) \tag{90}$$

$$\sum_{l \in \mathcal{L} \cup \{0\}} \lambda^l(t) = 1. \tag{91}$$

The cost of operating the generator at minimum power is factored in to c^R. Sridhar et al. [30] showed that if (91) is replaced with

$$\sum_{l \in \mathcal{L} \cup \{0\}} \lambda^l(t) = u(t), \tag{92}$$

then the resulting formulation is locally ideal, in the same sense as (42)–(44).

The formulation proposed by Chen and Wang [5] is also locally ideal, and comes from noticing that the coefficient on $\lambda^0(t)$ is 0 in both (89) and (90). Hence $\lambda^0(t)$ can be dropped from both equations without issue, and we can then project out $\lambda^0(t)$, leaving:

$$\sum_{l \in \mathcal{L}} (\overline{P}^l - \underline{P}) \lambda^l(t) = p'(t) \tag{93}$$

$$\sum_{l \in \mathcal{L}} (\overline{C}^l - \overline{C}^0) \lambda^l(t) = c^p(t) \tag{94}$$

$$\sum_{l \in \mathcal{L}} \lambda^l(t) \leq u(t). \tag{95}$$

Chen and Wang [5] observed that projecting out $\lambda^0(t)$ had a dramatic effect on the root relaxation time for MISO's day-ahead UC.

B.7 Additional Start-up Cost Formulations

One class of start-up cost formulations use points on the epigraph of $c^{SU}(t)$ to calculate start-up costs, without needing additional variables. An example of such a formulation, introduced by Knueven et al. [19], uses the start-up and shutdown indicator variables v and w:

$$c^{SU}(t) \geq C^s v(t) - \sum_{k=1}^{s-1} \left((C^s - C^k) \sum_{i=\underline{T}^k}^{\underline{T}^{k+1}-1} w(t-i) \right) \qquad s \in \mathcal{S} \tag{96}$$

$$c^{SU}(t) \geq 0. \tag{97}$$

It was shown in Silbernagl [28] that the formulation (54)–(56) has the same linear programming relaxation value as the formulation given by (96),(97).

Silbernagl et al. [29] showed constraint (52) could be strengthened by increasing the coefficients in the sum:

$$c^{SU}(t) \geq C^s \left(u(t) - \sum_{i=1}^{DT} u(t-i) \right) - \sum_{k=1}^{s-1} \left((C^s - C^k) \sum_{i=\underline{T}^k+1}^{\underline{T}^{k+1}} u(t-i) \right) \qquad \forall s \in \mathcal{S}. \tag{98}$$

It is not difficult to show the inequalities (96) dominate (98) which dominate (52) [19].

Yang et al. [33] and Atakan et al. [2] both proposed a different auxiliary variable, $\overline{c}^{SU}(t)$, which represents the start-up cost over a hot-start. Yang et al. [33] then wrote the start-up costs as

$$\overline{c}^{SU}(t) \geq (C^s - C^1)\left(v(t) - \sum_{i=1}^{\underline{T}^s} u(t-i)\right) \qquad \forall s \in \mathcal{S} \setminus \{1\} \tag{99}$$

$$\overline{c}^{SU}(t) \geq 0 \tag{100}$$

$$c^{SU}(t) = c^1 v(t) + \overline{c}^{SU}(t), \tag{101}$$

where (101) is substituted into the objective function. Note that Yang et al. [33] only considered two start-up types, though the above formulation is its natural extension to arbitrary start-up types. Substituting (101) into (99) and (100) we see that this formulation is dominated by (96),(97). Atakan et al. [2] proposes a similar formulation, and explicitly models arbitrary start-up types, replacing (99) with:

$$\overline{c}^{SU}(t) \geq (C^s - C^1)\left(v(t) - \sum_{i=DT}^{\underline{T}^s} v(t-i) - \tilde{u}(t-\underline{T}^s)\right) \qquad \forall s \in \mathcal{S} \setminus \{1\}. \tag{102}$$

This same constraint can be written with $u(t)$ as:

$$\overline{c}^{SU}(t) \geq (C^s - C^1)\left(v(t) - \sum_{i=DT}^{\underline{T}^s-1} v(t-i) - u(t-\underline{T}^s)\right) \qquad \forall s \in \mathcal{S} \setminus \{1\}. \tag{103}$$

Hence we see that (103) dominates (99) and requires no more non-zeros. Notice that we can also replace (99) with a modified version of (96)

$$\overline{c}^{SU}(t) \geq (C^s - C^1)v(t) - \sum_{k=1}^{s-1}\left((C^s - C^k)\sum_{i=\underline{T}^k}^{\underline{T}^{k+1}-1} w(t-i)\right) \qquad \forall s \in \mathcal{S} \setminus \{1\} \tag{104}$$

and using (2) we can "push" $u(t-\underline{T}^s)$ through the sum in (103) to arrive at:

$$\overline{c}^{SU}(t) \geq (C^s - C^1)\left(v(t) - u(t-DT) + \sum_{i=DT}^{\underline{T}^s-1} w(t-i)\right) \qquad \forall s \in \mathcal{S} \setminus \{1\}. \tag{105}$$

It is clear that (104) dominates (105), while requiring fewer non-zeros.

Table 1: Specification of the formulation proposed in Carrion and Arroyo [4] (**CA**)

	Carrion-Arroyo (CA)
objective:	(69)
variables:	$u_g, p_g, \overline{p}_g, p^l_g, c^{SU}_g, c^{SD}_g, p_{W,n}, f_k, \theta_n, s^+_n, s^-_n, s_n, s_R$
up-time/down-time:	(3), (74), (75)
generation limits:	(18), (76)
ramping limits:	(77), (78)
piecewise production:	(43), (44), (45)
start-up cost:	(97), (52)
shutdown cost:	(107), (108)
system constraints:	(64), (65), (66), (67)

B.8 Additional Shutdown Cost Formulations

If the v variables are in the formulation but not the w variables, then the shutdown cost can be modeled using the implied shutdown status variable:

$$c^{SD}(t) = C^w(v(t) - u(t) - u(t-1)). \tag{106}$$

In this case $c^{SD}(t)$ can be substituted out of the objective function. If neither the w nor v variables are in the model, then $c^{SD}(t)$ must remain a variable in the model, and we add the constraints:

$$c^{SD}(t) \geq C^w(u(t-1) - u(t)) \tag{107}$$

$$c^{SD}(t) \geq 0. \tag{108}$$

C Other Formulations from the Literature

The first paper to consider a mostly complete UC formulation is Carrion and Arroyo [4]. The formulation given by that paper is shown in Table 1 and will be referred to as **CA**. Note that given the variables involved in this formulation, we may apply some of the transformations discussed above to arrive at the formulation given. For example, the formulation in Table 1 includes constraint (43), which has the term $p'_g(t)$. In this case, the transformation given by (12) is applied to arrive at a constraint valid for this model (and which is the same as that in Carrion and Arroyo [4]). Note the only difference between the formulation given by Table 1 and that labeled "MILP-UC" in Carrion and Arroyo [4] is the addition of the network constraints and the slacks on nodal balance and reserves.

Carrion and Arroyo [4] also tested a 3-binary formulation, however the formulation considered is not precisely specified. A possible variant of this formulation is considered in Ostrowski et al. [23], and is called the

Table 2: Specification of the "original" formulation from Ostrowski et al. [23] (**OAV-O**)

Ostrowski et al. "original" (OAV-O)	
objective:	(69)
variables:	$u_g, v_g, w_g, p_g, \overline{p}_g, c_g^p, c_g^{SU}, p_{W,n}, f_k, \theta_n, s_n^+, s_n^-, s_n, s_R$
up-time/down-time:	(2), (3), (70), (71)
generation limits:	(18), (19)
ramping limits:	(24), (25)
piecewise production:	(51)
start-up cost:	(97), (52)
shutdown cost:	(62)
system constraints:	(64), (65), (66), (67)

Table 3: Specification of the "Up/Downtime" formulation from Ostrowski et al. [23] (**OAV-UD**)

Ostrowski et al. "Up/Downtime" (OAV-UD)	
objective:	(69)
variables:	$u_g, v_g, w_g, p_g, \overline{p}_g, c_g^p, c_g^{SU}, p_{W,n}, f_k, \theta_n, s_n^+, s_n^-, s_n, s_R$
up-time/down-time:	(2), (3), (4), (5)
generation limits:	(18), (19)
ramping limits:	(24), (25)
piecewise production:	(51)
start-up cost:	(97), (52)
shutdown cost:	(62)
system constraints:	(64), (65), (66), (67)

Table 4: Specification of a formulation from Ostrowski et al. [23] (**OAV**)

Ostrowski et al. (OAV)	
objective:	(69)
variables:	$u_g, v_g, w_g, p_g, \overline{p}_g, c^p_g, c^{SU}_g, p_{W,n}, f_k, \theta_n, s^+_n, s^-_n, s_n, s_R$
up-time/down-time:	(2), (3), (4), (5)
generation limits:	(18), (19)
ramp-up limits:	(24) if $RU_g \leq SD_g - \underline{P}_g$ or $UT_g < 2$, else (28)
ramp-down limits:	(25) if $RD_g \leq SU_g - \underline{P}_g$ or $UT_g < 2$, else (30) if $UT_g < 3$ or $DT_g < 2$, else (31)
piecewise production:	(51)
start-up cost:	(97), (52)
shutdown cost:	(62)
system constraints:	(64), (65), (66), (67)

Table 5: Specification of a tighter formulation from Ostrowski et al. [23] (**OAV-T**)

Ostrowski et al. tighter (OAV-T)	
objective:	(69)
variables:	$u_g, v_g, w_g, p_g, \overline{p}_g, c^p_g, c^{SU}_g, p_{W,n}, f_k, \theta_n, s^+_n, s^-_n, s_n, s_R$
up-time/down-time:	(2), (3), (4), (5)
generation limits:	(18), (19), (37)
ramp-up limits:	(24) if $RU_g \leq SD_g - \underline{P}_g$ or $UT_g < 2$, else (28)
ramp-down limits:	(25) if $RD_g \leq SU_g - \underline{P}_g$ or $UT_g < 2$, else (30) if $UT_g < 3$ or $DT_g < 2$, else (31)
2-ramp-up limits:	(29) if $RU_g > SD_g - \underline{P}_g$, $UT_g \geq 2$, and $DT_g \geq 2$
2-ramp-down limits:	(32) if $RD_g > SU_g - \underline{P}_g$, $UT_g \geq 3$, and $DT_g \geq 2$
piecewise production:	(51)
start-up cost:	(97), (52)
shutdown cost:	(62)
system constraints:	(64), (65), (66), (67)

"original" formulation therein. Most of the generator description except for start-up costs, shutdown costs, and production costs are based on the formulation from Arroyo and Conejo [1]. As piecewise production costs are not specified in [23], we use the perspective formulation referenced therein, which is (51). Similarly [23] did not consider a network. The formulation considered is given in Table 2 and will be referred to as **OAV-O**. Another 3-binary formulation replaces (70) and (71) in Ostrowski et al. "original" with (72) and (73), respectively. This is called "Up/Downtime" in Ostrowski et al. [23], and we refer to it as **OAV-UD**. It is given in Table 3. Note that in all the formulations derived from Ostrowski et al. [23], we add the perspective piecewise production (51), the systems constraints, as well as the upper-bound constraint (19), the last (which limits the reserves when the generator is turning-off) being necessary for this formulation to agree with Carrion and Arroyo [4].

Table 6: Specification of the formulation proposed in Atakan et al. [2] (**ALS**)

	Atakan et al. (ALS)
objective:	(69)
variables:	$\tilde{u}_g, v_g, w_g, p'_g, \overline{p}_g, c^p_g, \overline{c}^{SU}_g, p_{W,n}, f_k, \theta_n, s^+_n, s^-_n, s_n, s_R$
up-time/down-time:	(3), (9), (10), (11)
generation limits:	(16), (19)
ramp limits:	(35), (36)
piecewise production:	(49)
start-up cost:	(100), (101), (102)
shutdown cost:	(62)
system constraints:	(64), (65), (66), (67)

Because Ostrowski et al. [23] proposed adding the additional constraints (37), (28)–(32) as user cuts, a feature unique to CPLEX [15], we will consider two variants of the formulation proposed in Ostrowski et al. [23]. The first of which replaces the consecutive-period ramping constraints with their tighter counterparts (28), (30), (31), when possible, and the other also adds the upper bound constraint (37) and the two-period ramping constraints (29) and (32). The two formulations are in Tables 4 and 5 and will be referred to as **OAV** and **OAV-T**, respectively. Note we add the network constraints, the perspective piecewise production (51), and (19) to these models.

Recently, Atakan et al. [2] presented a reformulation for UC using the state-transition variables discussed in Section 3.1.2. Other aspects of this formulation are the combination of the $p'_g(t)$ variables with the $\overline{p}_g(t)$ variables and the inclusion of the ramping constraints from Damcı-Kurt et al. [6]. Relying heavily on the transformations (8), (12), and (13), in Table 6 we give the formulation from Atakan et al. [2] with the addition of network constraints. We will refer to this formulation as **ALS** and it is given in Table 6.

D Detailed Summary of Instances

D.1 RTS-GMLC

The RTS-GMLC system [3] is an update to the RTS-96, and is publicly available on GitHub at `https://github.com/GridMod/RTS-GMLC`. As is, none of the generators have ramping constraints over an hourly time horizon. In order to create instances which are sensitive to various formulation changes, we considered a 3% reserve margin, let $SU_g = SD_g = \underline{P}_g$ for all $g \in \mathcal{G}$, and reduced the ramp rates by a factor of three. The later change is not overly restrictive, in that units marked as coal can ramp up/down in two to three hourly time periods, whereas the units marked as gas can ramp up/down in a single hourly time period.

The RTS-GMLC system has 73 thermal generators, 81 renewable generators including wind, hydro, utility-scale photo-voltaic, and rooftop photo-voltaic generators. Each thermal generator has four piecewise production segments and between one and three start-up types. The rooftop photo-voltaic and hydro generation is modeled as must-take, but the wind and utility-scale photo-voltaic units are fully curtailable. The system also includes a network with 73 buses and 120 transmission lines. Hourly day-ahead data are provided for both load and renewable generation for the year 2020. For this test set we selected twelve days from the year: Jan 27, Feb 9, Mar 5, Apr 3, May 5, Jun 9, Jul 6, Aug 12, Sep 20, Oct 27, Nov 25, and Dec 23. Jan 27, Feb 9, Nov 25, and Dec 23 were selected because they contain significant wind-ramping events, Apr 3 and Jun 9 were selected because there is very little renewable generation online, and the remaining days were selected arbitrarily to have a test-set containing one day per month to capture seasonal variations. The initial generator status for all days is given from the provided PLEXOS solution. For each day we consider an instance with and without the network constraints, for a total of 24 instances.

D.2 CAISO

The "CAISO" instances have 410 schedulable thermal generators and another 200 must-run thermal generators. These instances are based on publicly-available market and regulatory data from the California Independent System Operator (CAISO). No network data is included. As the generator offer curve provided is quadratic, we approximate each curve using $L_g = 2$ for each $g \in \mathcal{G}$. Each generator only has two start-up categories. Five 48-hour demand scenarios are considered, one for each season based on historical data, and another, "Scenario400," representing a hypothetical scenario where wind is on average 40% of energy demand. Wind generation is treated as fully curtailable. The wind profile for this instance was collected from historical data and then scaled appropriately. Four different reserve margins were considered for each demand profile, 0%, 1%, 3%, and 5%. In total then we have 20 CAISO UC instances.

This system consists of mostly small and flexible generating units: only 20 of the 410 schedulable units have irredundant ramping constraints (that is, $RU_g > (\overline{P}_g - \underline{P}_g)$ and $RD_g > (\overline{P}_g - \underline{P}_g)$), and account for 75% of the schedulable capacity. Additionally, 370 of the schedulable units have $UT_g = DT_g = 1$, accounting for 66% of the schedulable capacity. These are the same instances considered in [19, 18].

D.3 FERC

The "FERC" instances are based on two sets of generators publicly available from the Federal Regulatory Energy Commission (FERC) in the United States [20]. Twelve 48-hour instances were constructed from publicly available load, reserve, and wind data from 2015 provided by PJM [25, 26]. One day was selected from each month so as to capture the seasonal variation, with the "Summer" generators being used for months April – September and the "Winter" generators for the remaining months. PJM was selected for the load profile because the FERC generator set is itself based on data from PJM. To create additional more interesting test instances, we scaled the provided wind profile by a factor of 15 to increase the average wind

Table 7: Summary computational results for RTS-GMLC instances.

Formulation	Time (s)	Opt gap (%)	Time outs	Times best	Times 2nd
CA	300.0	13.806%	24	0	0
OAV-O	172.8	0.3335%	13	0	0
OAV-UD	163.2	0.2541%	13	0	0
OAV	169.7	0.2352%	12	0	0
OAV-T	178.0	0.2498%	12	0	0
MLR	86.19	0.0165%	5	0	0
ALS	58.29	0.0122%	2	1	1
KOW	94.81	0.0165%	4	0	1
T	50.65	0.0121%	2	4	4
Co	43.11	0.0121%	**1**	1	4
R1	**32.37**	**0.0114%**	**1**	**14**	6
R2	36.94	0.0114%	**1**	4	**8**

energy supply to 30% of demand. In both cases, wind generation is treated as fully curtailable. Hence there are a total of 24 FERC instances, twelve "low-wind" instances which use the data as provided and twelve "high-wind" instances which scale the wind by a factor of 15. These are the same instances considered in [16, 19].

The Summer set of generators has 978 units and the Winter set has 934 units. Missing data is as in [16]. Piecewise production curves are based market data such that $1 \leq L_g \leq 10$, and each generator has at most two start-up types. Given the number of generators and the potential number of piecewise production points, this test set is very large as measured by the number of nonzeros in the constraint matrix, regardless of the formulation chosen. As compared to CAISO, this system has many fewer flexible generators; generators with irredundant ramping constraints comprise only 50% of units and 35% of capacity for both the Winter and Summer set of generators.

E Summary Computational Results, Including Other Formulations

E.1 RTS-GMLC

Summary computational results for the RTS-GMLC instances are reported in Table 7. As in the main text, the time limit was set at 300 seconds. Full results are in Tables 17 and 20. We note formulation **CA** is completely uncompetitive on these instances, and fails to find and certify quality solution (with a optimality gap less than 1%) for every instance within the time allotted. The **OAV** formulations all perform similarly, and are again uncompetitive, but at least manage to find and certify a quality solution in most instances.

Table 8: Summary computational results for CAISO instances.

Formulation	Time (s)	Opt gap (%)	Time outs	Times best	Times 2nd
CA	300.0	1.0987%	20	0	0
OAV-O	300.0	6.7647%	20	0	0
OAV-UD	300.0	6.2269%	20	0	0
OAV	300.0	3.6319%	20	0	0
OAV-T	300.0	6.1679%	20	0	0
MLR	117.1	0.0119%	2	0	1
ALS	260.4	0.0577%	12	0	0
KOW	79.02	0.0102%	2	4	**7**
T	**56.90**	**0.0100%**	**0**	**15**	3
Co	100.6	**0.0100%**	**0**	0	5
R1	147.4	0.0102%	1	0	1
R2	104.1	0.0100%	1	1	3

Formulation **ALS**, while clearly superior to **MLR** and **KOW** on this test set, is outperformed by **Co**, **R1**, and **R2**.

E.2 CAISO

Summary computational results for the CAISO instances are reported in Table 8. As in the main text, a time limit of 300 seconds was also set for these instances. Full results are in Tables 18 and 21. On these instances we see that the **CA** and **OAV** variants are uncompetitive. The CAISO generation set has mostly small, flexible generators with $UT = DT = 1$, so the poor representation of the start-up costs used for **ALS** is problematic, though it is still able to find and certify a quality solution in all cases by the time limit.

E.3 FERC

We report summary computational results for the FERC instances in Table 9. The time limit was set at 600 seconds, as in the main text. Full results are in Tables 19 and 22.

Here again we see that the **CA** and **OAV** variants are uncompetitive. Interestingly, however, we do see a difference in solution quality and verification at 600 seconds when comparing **OAV-O** to the other **OAV** variants, demonstrating the effectiveness of the up-time and down-time constraints from Rajan and Takriti [27]. The slightly weaker start-up cost formulation from Atakan et al. [2] is again the likely reason **ALS** performs slightly worse than the variants **MLR**, **Co**, **R1**, and **R2**.

Table 9: Summary computational results for FERC instances.

Formulation	Time (s)	Opt gap (%)	Time outs	Times best	Times 2nd
CA	600.0	43.333%	24	0	0
OAV-O	599.7	11.716%	23	0	0
OAV-UD	588.2	1.4575%	20	0	0
OAV	588.4	1.3104%	21	0	0
OAV-T	582.4	1.4389%	22	0	0
MLR	340.5	0.0555%	3	4	1
ALS	394.7	0.0933%	7	2	2
KOW	390.2	0.0117%	2	3	5
T	**268.6**	**0.0104%**	**1**	**8**	4
Co	309.5	0.0596%	3	4	5
R1	308.9	0.0480%	3	3	**6**
R2	373.1	0.0665%	4	0	1

Table 10: Summary of computational results, changing the power variable used.

	RTS-GMLC		CAISO		FERC	
	Time (s)	Opt gap (%)	Time (s)	Opt gap (%)	Time (s)	Opt gap (%)
T	50.65	0.0121%	56.90	0.0100%	268.6	0.0103%
CA: p_g [4]	55.59	0.0131%	63.86	0.0100%	415.6	0.0223%

F Analysis of the "Tight" Formulation

Because formulation **T** is the best performer across the test bed, we examined it further to determine which components are the most critical to its success. Approximately 30 variants of the Tight formulation were considered by swapping one component (power variables, reserve variables, up-time/down-time constraints, generation limits, ramp limits, piecewise production, start-up costs) for another considered in this paper. The motivation here is two-fold. First, this process may discover a better formulation, and second, the practitioner has a sense of what changes they should prioritize if considering a formulation change.

We first consider changing the power variable from $p'_g(t)$ to the simpler $p_g(t)$. A summary of the results are presented in Table 10, and the full results are in Tables 23–25 in Appendix G. As we can see, holding all else constant, using the variable $p_g(t)$ to represent total output leads to a degradation in overall performance.

Next we consider changing the variable used for reserve allocation. Formulation **T** uses $\overline{p}'_g(t)$, so we considered both $\overline{p}_g(t)$ from Carrion and Arroyo [4] and $r_g(t)$ from Morales-España et al. [22]. The full results are available in appendix Tables 26–28. Overall again it seems like the $\overline{p}'_g(t)$ variables are the best choice, but the results are more mixed than for the power variable selection.

For this experiment we considered two other ways of formulating the minimum up-time and down-time.

Table 11: Summary of computational results, changing the reserve variable used.

	RTS-GMLC		CAISO		FERC	
	Time (s)	Opt gap (%)	Time (s)	Opt gap (%)	Time (s)	Opt gap (%)
T	50.65	0.0121%	56.90	0.0100%	268.6	0.0103%
MLR: r_g [22]	76.91	0.0128%	80.01	0.0100%	230.4	0.0106%
CA: $\overline{p}_g$ [4]	52.41	0.0121%	50.77	0.0100%	397.2	0.0101%

Table 12: Summary of computational results, changing the up-time and down-time constraints used.

	RTS-GMLC		CAISO		FERC	
	Time (s)	Opt gap (%)	Time (s)	Opt gap (%)	Time (s)	Opt gap (%)
T	50.65	0.0121%	56.90	0.0100%	268.6	0.0103%
RT 2bin: (4)(7) [27]	48.29	0.0120%	51.30	0.0100%	235.1	0.0100%
DEKT: (70)(71) [7]	59.30	0.0124%	46.31	0.0100%	410.0	0.0163%

The first is simply the up-time/down-time polytope given by Rajan and Takriti [27] with just the unit on variable $u_g(t)$ and the start-up variable $v_g(t)$, given by (4) and (7). The other is the older formulation originally given by Dillon et al. [7] and more recently by Arroyo and Conejo [1], which was discussed in Section B.1. Summary results are reported in Table 12; full results are given in Tables 29–31. It is obvious that the DEKT formulation leads to an overall degradation in performance, whereas the 2bin variant of the up-time/down-time polytope seems to improve performance over the variant which included the turn-off variables. The "RT 2bin" formulation for the up-time/down-time constraints solved every instance of CAISO and FERC to within 0.01% of optimality within the time limit, while timing out on an additional RTS-GMLC instance over **T**. Still, its shifted geometric mean optimality gap is slightly less on RTS-GMLC while being faster on average.

In Table 13 we show the computational summary for changes to the generation limits for formulation **T**; the complete results are in Tables 32–34. As we can see, the MLR, GMR, and GMR+PG variants are all competitive with the generation limits used in **T**, which are essentially GMR+PG+(41). Conversely, the PG, AC, OAV, and CA variants result in degraded performance on at least two fo the three instance sets.

Table 14 summarizes the computational results for formulation **T** under changes to the ramping limits. Overall it is difficult to distinguish between the various formulations. However, the sparser MLR formulation from [22] has a definite advantage, whereas the denser ramping limits from the **OAV-T** formulation only serve to reduce performance. The performance of the MLR ramping inequalities is likely because the strong variable upper bound inequalities used in **T** make the ramping constraints mostly redundant. The complete results are in the appendix, Tables 35–37.

Table 13: Summary of computational results, changing the generation limits used.

	RTS-GMLC		CAISO		FERC	
	Time (s)	Opt gap (%)	Time (s)	Opt gap (%)	Time (s)	Opt gap (%)
T	50.65	0.0121%	56.90	0.0100%	268.6	0.0103%
MLR: (20)(21) [22]	51.29	0.0122%	52.44	0.0100%	254.8	0.0100%
GMR: (20)(23) [12]	51.37	0.0122%	52.42	0.0100%	254.1	0.0100%
PG: (38) [24]	57.22	0.0128%	88.97	0.0100%	299.2	0.0100%
GMR+PG: (23)(38)	50.10	0.0116%	50.70	0.0100%	255.0	0.0103%
AC: (19) [1]	72.94	0.0145%	93.44	0.0100%	362.1	0.0101%
OAV: (19)(37) [23]	75.44	0.0125%	96.72	0.0100%	229.7	0.0101%
CA: (18)(76) [4]	73.78	0.0179%	92.97	0.0100%	307.7	0.0100%

Table 14: Summary of computational results, changing the ramping limits used.

	RTS-GMLC		CAISO		FERC	
	Time (s)	Opt gap (%)	Time (s)	Opt gap (%)	Time (s)	Opt gap (%)
T	50.65	0.0121%	56.90	0.0100%	268.6	0.0103%
DKRA: (35)(36)(79)[6]	50.45	0.0119%	52.59	0.0100%	281.9	0.0100%
MLR: (26)(27) [22]	49.28	0.0119%	48.53	0.0100%	242.6	0.0100%
AC: (24)(25) [1]	51.67	0.0124%	53.59	0.0100%	313.7	0.0107%
OAV ramp limits [23]	51.27	0.0119%	59.60	0.0100%	268.3	0.0101%
OAV-T ramp limits [23]	53.76	0.0121%	59.94	0.0100%	324.0	0.0120%

Table 15: Summary of computational results, changing the production cost formulation used.

	RTS-GMLC		CAISO		FERC	
	Time (s)	Opt gap (%)	Time (s)	Opt gap (%)	Time (s)	Opt gap (%)
T	50.65	0.0121%	56.90	0.0100%	268.6	0.0103%
CA: (43)(44)(45) [4]	48.93	0.0119%	57.87	0.0100%	230.2	0.0100%
Wu: (42)(43)(44) [32]	49.64	0.0136%	55.55	0.0100%	323.9	0.0101%
KOW prod. costs [18]	50.28	0.0119%	52.63	0.0100%	282.2	0.0100%
CW: (93)(94)(95) [5]	50.00	0.0114%	47.68	0.0100%	320.7	0.0101%
SOS2: (89)(90)(91)	59.90	0.0146%	50.67	0.0100%	277.5	0.0100%
SLL: (89)(90)(92) [30]	46.83	0.0115%	48.25	0.0100%	304.1	0.0102%
Epi: (49)	57.33	0.0143%	68.24	0.0100%	277.3	0.0134%
HB: (50) [14]	57.27	0.0119%	71.53	0.0100%	364.5	0.0130%

Table 16: Summary of computational results, changing the start-up cost formulation used.

	RTS-GMLC		CAISO		FERC	
	Time (s)	Opt gap (%)	Time (s)	Opt gap (%)	Time (s)	Opt gap (%)
T	50.65	0.0121%	56.90	0.0100%	268.6	0.0103%
MLR: (54)(55)(56) [22]	49.23	0.0117%	66.37	0.0100%	291.9	0.0571%
KOW 3bin: (96)(97) [19]	49.25	0.0121%	104.3	0.0106%	326.9	0.0788%
A-K: (100)(101)(104) [2, 19]	49.65	0.0120%	55.86	0.0100%	276.4	0.0609%
MLR proj: (54)(57)(58)	44.15	0.0121%	65.47	0.0100%	289.3	0.0520%

The literature has many possibilities for formulating piecewise production costs; Table 15 summarized the computational results under changes to the production cost formulation for **T**. The full results are in the appendix, Tables 38–40. It is clear from Table 15 that the SOS2 and Epi variants result in performance degradation, and HB does not perform as well as we might expect. Gurobi seems to prefer formulations with additional variables, as even the non-ideal CA variant out-performs HB across the board. Interestingly, CA does surprisingly well, only timing out on the difficult RTS-GMLC instance "2020-04-03 NC." This may be in part because CA is sparse: the limits on the $p_g^l(t)$ variables are just variable bounds and hence are not additional rows in the simplex tableau.

Lastly, we consider the effect changing the start-up cost formulation has on the performance of formulation **T**. The summary results are in Table 16, and the full results are in the appendix, Tables 41–43. On the RTS-GMLC and CAISO instances, none of the other formulations are a significant improvement over **T**, and on the FERC instances the start-up costs formulation from Knueven et al. [19] allows **T** to achieve a high-quality solution in 10 minutes on all instances.

G Full Computational Results

In this appendix we report the full computational results for the formulations and instances tested.

Table 17: Computational results for RTS-GMLC instances. Wall clock times are reported in seconds. When the 300 second time limit is reached, the terminating optimality gap is reported in parentheses. The best value for each instance is bold-faced. Each instance is named by the date from which the beginning of the 48-hour load profile was selected and whether the system with considered with (NC) or without (CP) a network.

Instance	**MLR** (2013)	**ALS** (2018)	**KOW** (2018)	**T**	**Co**	**R1**	**R2**
2020-01-27 CP	(0.056%)	219.7	(0.073%)	**88.8**	101.5	94.8	99.7
2020-02-09 CP	49.8	**20.4**	47.6	23.0	28.0	21.3	23.3
2020-03-05 CP	91.5	34.7	92.2	**25.3**	26.6	25.9	40.2
2020-04-03 CP	(0.047%)	(0.020%)	(0.032%)	42.6	51.2	51.2	**39.1**
2020-05-05 CP	37.3	34.9	56.7	**19.5**	25.0	25.8	20.9
2020-06-09 CP	14.6	12.1	18.6	5.8	6.3	**4.7**	6.4
2020-07-06 CP	13.3	10.8	13.2	6.8	6.8	**5.3**	9.7
2020-08-12 CP	29.0	12.1	14.1	12.2	**6.0**	6.8	7.4
2020-09-20 CP	42.4	11.6	39.5	**7.4**	12.7	8.3	8.5
2020-10-27 CP	79.6	39.2	59.7	24.3	21.4	**21.1**	29.0
2020-11-25 CP	115.0	47.0	176.2	33.4	34.3	**26.7**	35.7
2020-12-23 CP	35.1	20.7	53.9	19.4	21.7	**13.4**	15.5
2020-01-27 NC	103.2	89.1	188.4	157.6	53.7	56.3	**38.6**
2020-02-09 NC	193.8	192.1	206.5	212.6	60.9	**55.8**	56.6
2020-03-05 NC	(0.017%)	300.0	217.8	(0.021%)	292.4	**113.5**	230.1
2020-04-03 NC	(0.048%)	(0.054%)	(0.051%)	(0.050%)	(0.061%)	**(0.043%)**	(0.044%)
2020-05-05 NC	197.1	205.2	(0.039%)	212.8	249.2	**67.6**	108.6
2020-06-09 NC	37.1	26.6	38.4	49.8	18.9	19.2	**16.0**
2020-07-06 NC	40.0	25.5	55.7	39.3	22.9	**16.1**	18.9
2020-08-12 NC	41.4	42.3	49.4	67.3	56.5	37.8	**26.2**
2020-09-20 NC	81.1	51.9	135.8	69.3	40.8	**39.4**	39.8
2020-10-27 NC	(0.037%)	222.7	295.2	153.7	191.9	**135.4**	168.9
2020-11-25 NC	206.7	93.5	209.9	164.9	100.3	**64.0**	108.8
2020-12-23 NC	219.3	211.6	236.9	226.1	244.3	**82.1**	89.8

Table 18: Computational results for CAISO instances. Wall clock times are reported in seconds. When the 300 second time limit is reached, the terminating optimality gap is reported in parentheses. The best value for each instance is bold-faced. Each instance is named by the date or hypothetical scenario from which the beginning of the 48-hour load profile was selected, followed by the reserve level, considered as a percentage of load.

Instance	**MLR** (2013)	**ALS** (2018)	**KOW** (2018)	**T**	**Co**	**R1**	**R2**
2014-09-01 0%	72.5	189.0	39.1	51.7	37.2	71.2	**37.2**
2014-12-01 0%	60.7	(0.014%)	**33.7**	45.1	66.7	122.8	64.0
2015-03-01 0%	36.4	207.5	**27.4**	32.1	90.9	118.3	38.7
2015-06-01 0%	23.7	190.4	**12.1**	19.1	43.3	87.8	47.3
Scenario400 0%	98.9	(0.139%)	50.4	**31.0**	92.3	162.6	146.4
2014-09-01 1%	81.6	163.8	56.5	**25.4**	35.8	88.8	41.6
2014-12-01 1%	116.7	(0.012%)	85.0	**84.5**	103.2	138.8	92.2
2015-03-01 1%	95.1	(0.012%)	39.6	**36.4**	87.7	123.3	131.9
2015-06-01 1%	93.9	176.2	69.9	**38.3**	54.5	98.2	77.7
Scenario400 1%	163.4	(0.283%)	99.6	**66.5**	131.6	207.5	140.3
2014-09-01 3%	297.1	(0.010%)	198.7	**83.5**	128.3	142.2	127.1
2014-12-01 3%	165.2	(0.021%)	**73.4**	106.8	152.8	176.3	104.9
2015-03-01 3%	83.2	241.7	88.0	**67.4**	109.9	114.5	88.5
2015-06-01 3%	166.8	284.3	159.2	**57.8**	124.9	107.3	160.9
Scenario400 3%	(0.017%)	(0.324%)	(0.010%)	**103.8**	261.9	273.1	(0.010%)
2014-09-01 5%	191.8	(0.025%)	132.3	**92.7**	138.6	198.4	111.8
2014-12-01 5%	268.1	(0.046%)	147.4	**129.9**	156.9	235.4	187.0
2015-03-01 5%	97.6	(0.018%)	75.7	**54.6**	145.1	176.6	140.3
2015-06-01 5%	145.8	260.3	113.8	**56.7**	147.1	246.6	136.7
Scenario400 5%	(0.041%)	(0.169%)	(0.014%)	**106.9**	165.6	(0.015%)	279.7

Table 19: Computational results for FERC instances. Wall clock times are reported in seconds. When the 600 second time limit is reached, the terminating optimality gap is reported in parentheses. The best value for each instance is bold-faced. Each instance is named by the date from which the beginning of the 48-hour load profile was selected and whether it is low-wind (LW) (wind supply is 2% of energy demand for the year) or high-wind (HW) (wind supply is scaled to be 30% of energy demand for the year).

Instance	**MLR** (2013)	**ALS** (2018)	**KOW** (2018)	**T**	**Co**	**R1**	**R2**
2015-01-01 LW	265.3	348.1	310.5	107.6	**105.0**	185.3	161.4
2015-02-01 LW	543.2	(0.022%)	266.3	**133.5**	508.0	570.2	438.2
2015-03-01 LW	423.0	**221.9**	(0.049%)	244.0	237.6	282.3	337.4
2015-04-01 LW	414.1	240.0	311.2	411.6	**184.7**	279.4	398.2
2015-05-01 LW	160.4	**128.8**	310.4	221.6	190.5	136.4	182.6
2015-06-01 LW	**120.6**	266.4	311.2	282.6	128.4	159.2	276.9
2015-07-01 LW	555.9	338.1	521.2	429.6	**237.4**	469.8	509.8
2015-08-01 LW	551.2	365.8	**273.6**	281.5	401.5	358.7	527.1
2015-09-01 LW	541.9	542.6	**394.8**	541.8	501.6	403.8	439.8
2015-10-01 LW	165.4	287.0	320.1	201.9	**141.1**	148.5	217.6
2015-11-02 LW	166.2	419.2	317.5	**166.0**	282.2	217.8	368.9
2015-12-01 LW	217.3	251.5	383.8	**114.5**	118.5	154.9	152.9
2015-01-01 HW	(0.793%)	(1.379%)	415.4	**210.4**	(1.151%)	(0.895%)	(1.263%)
2015-02-01 HW	(0.256%)	(0.430%)	435.6	**136.0**	(0.055%)	(0.027%)	(0.076%)
2015-03-01 HW	411.5	(0.016%)	559.7	379.2	420.5	**277.5**	573.9
2015-04-01 HW	508.5	369.8	595.3	**244.6**	296.6	252.3	290.3
2015-05-01 HW	**122.3**	277.7	246.7	244.4	258.0	235.5	191.5
2015-06-01 HW	**254.6**	385.7	263.6	351.2	435.8	319.3	409.1
2015-07-01 HW	**223.9**	(0.016%)	(0.010%)	548.1	(0.020%)	493.3	549.2
2015-08-01 HW	391.2	591.5	448.1	**361.7**	436.9	386.3	459.5
2015-09-01 HW	(0.075%)	(0.191%)	**439.5**	(0.018%)	554.5	(0.024%)	(0.044%)
2015-10-01 HW	486.1	499.8	(0.012%)	351.6	331.0	**294.3**	336.8
2015-11-02 HW	586.9	(0.025%)	343.2	**321.1**	492.8	512.1	(0.021%)
2015-12-01 HW	346.7	483.6	497.5	287.3	385.9	**283.0**	526.6

Table 20: Computational results for RTS-GMLC instances, older formulations. Wall clock times are reported in seconds. When the 300 second time limit is reached, the terminating optimality gap is reported in parentheses. Each instance is named by the date from which the beginning of the 48-hour load profile was selected and whether the system with considered with (NC) or without (CP) a network.

Instance	**CA** (2006)	**OAV-O** (2012)	**OAV-UD** (2012)	**OAV** (2012)	**OAV-T** (2012)
2020-01-27 CP	(29.439%)	(1.617%)	(1.363%)	(1.295%)	(1.161%)
2020-02-09 CP	(7.566%)	103.5	63.5	62.5	75.3
2020-03-05 CP	(11.437%)	204.0	163.8	154.0	169.5
2020-04-03 CP	(11.256%)	(0.044%)	(0.017%)	(0.031%)	(0.037%)
2020-05-05 CP	(9.583%)	144.0	69.1	77.0	101.3
2020-06-09 CP	(2.676%)	42.7	38.0	37.8	47.6
2020-07-06 CP	(2.536%)	59.9	62.6	64.3	89.4
2020-08-12 CP	(3.548%)	59.1	43.7	44.3	56.8
2020-09-20 CP	(9.302%)	60.2	53.0	56.4	66.8
2020-10-27 CP	(18.483%)	(0.180%)	(0.019%)	(0.155%)	(0.099%)
2020-11-25 CP	(23.601%)	(1.220%)	(0.654%)	(0.829%)	(0.790%)
2020-12-23 CP	(8.972%)	88.3	80.8	114.0	169.2
2020-01-27 NC	(35.790%)	(0.531%)	(0.448%)	(0.547%)	(0.489%)
2020-02-09 NC	(18.107%)	(0.828%)	(0.545%)	(0.372%)	(0.480%)
2020-03-05 NC	(13.130%)	(0.807%)	(0.571%)	(0.635%)	(0.579%)
2020-04-03 NC	(16.058%)	(0.487%)	(0.355%)	(0.084%)	(0.262%)
2020-05-05 NC	(16.950%)	(0.312%)	(0.260%)	(0.165%)	(0.252%)
2020-06-09 NC	(5.735%)	100.0	108.4	88.5	117.6
2020-07-06 NC	(3.136%)	117.4	132.0	220.7	126.3
2020-08-12 NC	(7.401%)	110.1	174.3	224.6	204.9
2020-09-20 NC	(13.968%)	(0.016%)	(0.011%)	273.3	179.6
2020-10-27 NC	(26.777%)	(0.786%)	(0.713%)	(0.619%)	(0.703%)
2020-11-25 NC	(35.007%)	(0.674%)	(0.553%)	(0.540%)	(0.665%)
2020-12-23 NC	(10.101%)	(0.417%)	(0.493%)	(0.266%)	(0.369%)

Table 21: Computational results for CAISO instances, older formulations. Wall clock times are reported in seconds. When the 300 second time limit is reached, the terminating optimality gap is reported in parentheses. Each instance is named by the date or hypothetical scenario from which the beginning of the 48-hour load profile was selected, followed by the reserve level, considered as a percentage of load.

Instance	**CA** (2006)	**OAV-O** (2012)	**OAV-UD** (2012)	**OAV** (2012)	**OAV-T** (2012)
2014-09-01 0%	(0.266%)	(0.249%)	(0.299%)	(0.260%)	(0.286%)
2014-12-01 0%	(0.540%)	(0.301%)	(0.273%)	(0.244%)	(0.257%)
2015-03-01 0%	(1.073%)	(0.414%)	(0.261%)	(0.476%)	(0.406%)
2015-06-01 0%	(0.248%)	(0.199%)	(0.217%)	(0.210%)	(0.211%)
Scenario400 0%	(2.684%)	(63.420%)	(52.212%)	(1.852%)	(50.837%)
2014-09-01 1%	(0.358%)	(0.608%)	(0.390%)	(0.697%)	(2.334%)
2014-12-01 1%	(0.489%)	(0.344%)	(0.297%)	(0.346%)	(0.359%)
2015-03-01 1%	(0.960%)	(0.334%)	(0.363%)	(0.465%)	(0.597%)
2015-06-01 1%	(0.318%)	(0.305%)	(0.352%)	(0.266%)	(0.275%)
Scenario400 1%	(3.182%)	(44.939%)	(23.811%)	(16.637%)	(50.386%)
2014-09-01 3%	(0.278%)	(0.188%)	(4.311%)	(5.619%)	(4.056%)
2014-12-01 3%	(0.560%)	(0.480%)	(0.404%)	(0.217%)	(0.571%)
2015-03-01 3%	(1.649%)	(0.356%)	(0.307%)	(12.475%)	(0.458%)
2015-06-01 3%	(0.458%)	(2.655%)	(1.931%)	(7.124%)	(9.985%)
Scenario400 3%	(3.158%)	(17.791%)	(38.164%)	(7.291%)	(9.758%)
2014-09-01 5%	(0.353%)	(0.194%)	(4.865%)	(0.286%)	(1.633%)
2014-12-01 5%	(0.750%)	(13.422%)	(1.922%)	(3.532%)	(0.411%)
2015-03-01 5%	(1.342%)	(0.811%)	(0.626%)	(0.726%)	(0.677%)
2015-06-01 5%	(0.365%)	(0.327%)	(0.319%)	(0.356%)	(0.342%)
Scenario400 5%	(3.048%)	(8.326%)	(8.579%)	(16.152%)	(6.425%)

Table 22: Computational results for FERC instances, older formulations. Wall clock times are reported in seconds. When the 600 second time limit is reached, the terminating optimality gap is reported in parentheses. Each instance is named by the date from which the beginning of the 48-hour load profile was selected and whether it is low-wind (LW) (wind supply is 2% of energy demand for the year) or high-wind (HW) (wind supply is scaled to be 30% of energy demand for the year).

Instance	**CA** (2006)	**OAV-O** (2012)	**OAV-UD** (2012)	**OAV** (2012)	**OAV-T** (2012)
2015-01-01 LW	(2.710%)	(0.552%)	(0.093%)	(0.083%)	(0.137%)
2015-02-01 LW	(2.593%)	(1.268%)	(0.471%)	(0.779%)	(0.878%)
2015-03-01 LW	(1.747%)	(0.264%)	(0.016%)	(0.071%)	(0.115%)
2015-04-01 LW	(23.563%)	(2.157%)	(0.013%)	(0.013%)	(0.040%)
2015-05-01 LW	(2.151%)	592.1	566.5	468.7	355.4
2015-06-01 LW	(4.241%)	(0.949%)	529.2	540.6	(0.096%)
2015-07-01 LW	(38.678%)	(1.635%)	(0.029%)	(0.042%)	(0.194%)
2015-08-01 LW	(4.372%)	(1.458%)	(0.692%)	(0.750%)	(1.003%)
2015-09-01 LW	(11.635%)	(3.155%)	(0.559%)	(0.644%)	(0.689%)
2015-10-01 LW	(34.530%)	(3.988%)	(0.085%)	(0.282%)	(0.913%)
2015-11-02 LW	(57.216%)	(4.354%)	(2.145%)	(0.820%)	(1.267%)
2015-12-01 LW	(28.359%)	(1.672%)	(0.569%)	(0.247%)	(0.899%)
2015-01-01 HW	(89.979%)	(14.934%)	(5.845%)	(5.765%)	(5.596%)
2015-02-01 HW	(77.601%)	(6.419%)	(3.069%)	(3.723%)	(3.871%)
2015-03-01 HW	(15.793%)	(6.696%)	(1.935%)	(2.150%)	(2.383%)
2015-04-01 HW	(92.072%)	(66.194%)	(6.103%)	(6.252%)	(6.239%)
2015-05-01 HW	(58.044%)	(0.123%)	510.5	533.1	495.2
2015-06-01 HW	(90.847%)	(11.658%)	(0.775%)	(0.619%)	(1.623%)
2015-07-01 HW	(73.523%)	(7.871%)	(1.236%)	(1.788%)	(1.059%)
2015-08-01 HW	(89.531%)	(26.264%)	(3.219%)	(1.932%)	(3.238%)
2015-09-01 HW	(66.341%)	(43.264%)	(4.737%)	(3.331%)	(1.774%)
2015-10-01 HW	(99.977%)	(36.145%)	524.8	(0.038%)	(0.082%)
2015-11-02 HW	(97.372%)	(40.326%)	(3.014%)	(2.306%)	(2.004%)
2015-12-01 HW	(86.932%)	(27.849%)	(0.737%)	(0.147%)	(0.742%)

Table 23: Computational results for RTS-GMLC instances. Wall clock times are reported in seconds. When the 300 second time limit is reached, the terminating optimality gap is reported in parentheses. Each instance is named by the date from which the beginning of the 48-hour load profile was selected and whether the system with considered with (NC) or without (CP) a network. We replace the variable $p'_g(t)$ in formulation **T** with that listed in each heading.

Instance	**T**	CA [4] p_g
2020-01-27 CP	88.8	94.4
2020-02-09 CP	23.0	23.7
2020-03-05 CP	25.3	26.9
2020-04-03 CP	42.6	71.4
2020-05-05 CP	19.5	34.2
2020-06-09 CP	5.8	8.2
2020-07-06 CP	6.8	7.3
2020-08-12 CP	12.2	13.1
2020-09-20 CP	7.4	10.6
2020-10-27 CP	24.3	24.9
2020-11-25 CP	33.4	35.4
2020-12-23 CP	19.4	21.2
2020-01-27 NC	157.6	100.2
2020-02-09 NC	212.6	244.7
2020-03-05 NC	(0.021%)	183.3
2020-04-03 NC	(0.050%)	(0.044%)
2020-05-05 NC	212.8	(0.051%)
2020-06-09 NC	49.8	48.9
2020-07-06 NC	39.3	68.2
2020-08-12 NC	67.3	48.2
2020-09-20 NC	69.3	78.0
2020-10-27 NC	153.7	195.1
2020-11-25 NC	164.9	122.8
2020-12-23 NC	226.1	271.8

Table 24: Computational results for CAISO instances. Wall clock times are reported in seconds. When the 300 second time limit is reached, the terminating optimality gap is reported in parentheses. Each instance is named by the date or hypothetical scenario from which the beginning of the 48-hour load profile was selected, followed by the reserve level, considered as a percentage of load. We replace the variable $p'_g(t)$ in formulation **T** with that listed in each heading.

	T	CA [4]
Instance		p_g
2014-09-01 0%	51.7	59.9
2014-12-01 0%	45.1	33.6
2015-03-01 0%	32.1	34.5
2015-06-01 0%	19.1	22.4
Scenario400 0%	31.0	52.0
2014-09-01 1%	25.4	43.9
2014-12-01 1%	84.5	57.9
2015-03-01 1%	36.4	53.3
2015-06-01 1%	38.3	58.0
Scenario400 1%	66.5	68.4
2014-09-01 3%	83.5	82.4
2014-12-01 3%	106.8	98.6
2015-03-01 3%	67.4	85.3
2015-06-01 3%	57.8	96.5
Scenario400 3%	103.8	104.2
2014-09-01 5%	92.7	78.3
2014-12-01 5%	129.9	140.0
2015-03-01 5%	54.6	59.8
2015-06-01 5%	56.7	74.5
Scenario400 5%	106.9	85.9

Table 25: Computational results for FERC instances. Wall clock times are reported in seconds. When the 600 second time limit is reached, the terminating optimality gap is reported in parentheses. Each instance is named by the date from which the beginning of the 48-hour load profile was selected and whether it is low-wind (LW) (wind supply is 2% of energy demand for the year) or high-wind (HW) (wind supply is scaled to be 30% of energy demand for the year). We replace the variable $p'_g(t)$ in formulation **T** with that listed in each heading.

	T	CA [4]
Instance		p_g
2015-01-01 LW	107.6	304.2
2015-02-01 LW	133.5	246.6
2015-03-01 LW	244.0	414.5
2015-04-01 LW	411.6	472.2
2015-05-01 LW	221.6	360.8
2015-06-01 LW	282.6	473.3
2015-07-01 LW	429.6	329.5
2015-08-01 LW	281.5	480.2
2015-09-01 LW	541.8	581.1
2015-10-01 LW	201.9	312.0
2015-11-02 LW	166.0	366.9
2015-12-01 LW	114.5	210.6
2015-01-01 HW	210.4	420.1
2015-02-01 HW	136.0	441.7
2015-03-01 HW	379.2	457.2
2015-04-01 HW	244.6	375.0
2015-05-01 HW	244.4	383.6
2015-06-01 HW	351.2	503.1
2015-07-01 HW	548.1	(0.273%)
2015-08-01 HW	361.7	(0.015%)
2015-09-01 HW	(0.018%)	(0.039%)
2015-10-01 HW	351.6	571.8
2015-11-02 HW	321.1	498.1
2015-12-01 HW	287.3	335.1

Table 26: Computational results for RTS-GMLC instances. Wall clock times are reported in seconds. When the 300 second time limit is reached, the terminating optimality gap is reported in parentheses. Each instance is named by the date from which the beginning of the 48-hour load profile was selected and whether the system with considered with (NC) or without (CP) a network. We replace the variable $\overline{p}'_g(t)$ in formulation **T** with that listed in each heading.

Instance	**T**	MLR [22] r_g	CA [4] $\overline{p}_g$
2020-01-27 CP	88.8	(0.021%)	90.9
2020-02-09 CP	23.0	53.6	18.0
2020-03-05 CP	25.3	48.8	21.7
2020-04-03 CP	42.6	(0.029%)	48.5
2020-05-05 CP	19.5	42.0	15.9
2020-06-09 CP	5.8	11.9	6.8
2020-07-06 CP	6.8	16.0	6.8
2020-08-12 CP	12.2	12.9	9.0
2020-09-20 CP	7.4	26.9	14.2
2020-10-27 CP	24.3	50.8	23.2
2020-11-25 CP	33.4	79.7	36.3
2020-12-23 CP	19.4	56.7	20.5
2020-01-27 NC	157.6	97.3	152.8
2020-02-09 NC	212.6	212.8	193.2
2020-03-05 NC	(0.021%)	162.5	(0.020%)
2020-04-03 NC	(0.050%)	(0.046%)	(0.050%)
2020-05-05 NC	212.8	227.0	248.6
2020-06-09 NC	49.8	46.2	66.1
2020-07-06 NC	39.3	52.1	44.1
2020-08-12 NC	67.3	56.2	56.0
2020-09-20 NC	69.3	65.6	111.4
2020-10-27 NC	153.7	192.4	178.4
2020-11-25 NC	164.9	147.4	146.2
2020-12-23 NC	226.1	213.4	210.0

Table 27: Computational results for CAISO instances. Wall clock times are reported in seconds. When the 300 second time limit is reached, the terminating optimality gap is reported in parentheses. Each instance is named by the date or hypothetical scenario from which the beginning of the 48-hour load profile was selected, followed by the reserve level, considered as a percentage of load. We replace the variable $\overline{p}'_g(t)$ in formulation **T** with that listed in each heading.

Instance	**T**	MLR [22] r_g	CA [4] $\overline{p}_g$
2014-09-01 0%	51.7	40.8	36.8
2014-12-01 0%	45.1	33.4	35.3
2015-03-01 0%	32.1	29.3	30.7
2015-06-01 0%	19.1	13.8	17.6
Scenario400 0%	31.0	30.3	34.8
2014-09-01 1%	25.4	61.4	28.5
2014-12-01 1%	84.5	82.0	60.9
2015-03-01 1%	36.4	38.4	45.2
2015-06-01 1%	38.3	68.5	40.8
Scenario400 1%	66.5	191.8	62.2
2014-09-01 3%	83.5	242.9	67.9
2014-12-01 3%	106.8	71.9	98.1
2015-03-01 3%	67.4	99.5	77.4
2015-06-01 3%	57.8	120.4	35.8
Scenario400 3%	103.8	126.8	86.0
2014-09-01 5%	92.7	114.1	70.8
2014-12-01 5%	129.9	158.1	58.8
2015-03-01 5%	54.6	123.1	56.7
2015-06-01 5%	56.7	173.6	104.9
Scenario400 5%	106.9	266.0	67.8

Table 28: Computational results for FERC instances. Wall clock times are reported in seconds. When the 600 second time limit is reached, the terminating optimality gap is reported in parentheses. Each instance is named by the date from which the beginning of the 48-hour load profile was selected and whether it is low-wind (LW) (wind supply is 2% of energy demand for the year) or high-wind (HW) (wind supply is scaled to be 30% of energy demand for the year). We replace the variable $\overline{p}'_g(t)$ in formulation **T** with that listed in each heading.

Instance	**T**	MLR [22] r_g	CA [4] $\overline{p}_g$
2015-01-01 LW	107.6	175.7	310.0
2015-02-01 LW	133.5	108.1	297.1
2015-03-01 LW	244.0	254.0	420.4
2015-04-01 LW	411.6	364.3	577.4
2015-05-01 LW	221.6	110.1	304.0
2015-06-01 LW	282.6	200.9	243.1
2015-07-01 LW	429.6	(0.013%)	548.3
2015-08-01 LW	281.5	191.8	456.0
2015-09-01 LW	541.8	465.6	543.1
2015-10-01 LW	201.9	131.5	296.0
2015-11-02 LW	166.0	120.6	356.8
2015-12-01 LW	114.5	112.8	281.5
2015-01-01 HW	210.4	223.8	340.2
2015-02-01 HW	136.0	290.3	286.3
2015-03-01 HW	379.2	193.6	394.9
2015-04-01 HW	244.6	270.0	447.2
2015-05-01 HW	244.4	103.8	294.0
2015-06-01 HW	351.2	181.7	496.3
2015-07-01 HW	548.1	(0.021%)	534.1
2015-08-01 HW	361.7	443.1	478.8
2015-09-01 HW	(0.018%)	294.5	(0.012%)
2015-10-01 HW	351.6	362.8	355.5
2015-11-02 HW	321.1	335.4	478.6
2015-12-01 HW	287.3	219.6	544.7

Table 29: Computational results for RTS-GMLC instances. Wall clock times are reported in seconds. When the 300 second time limit is reached, the terminating optimality gap is reported in parentheses. Each instance is named by the date from which the beginning of the 48-hour load profile was selected and whether the system with considered with (NC) or without (CP) a network. We replace the up-time/down-time constraints from **T** with those listed.

Instance	**T**	RT 2bin [27] (4)(7)	DEKT [7] (70)(71)
2020-01-27 CP	88.8	90.9	105.8
2020-02-09 CP	23.0	18.7	23.9
2020-03-05 CP	25.3	24.0	26.3
2020-04-03 CP	42.6	49.1	53.9
2020-05-05 CP	19.5	24.3	26.8
2020-06-09 CP	5.8	6.2	6.8
2020-07-06 CP	6.8	9.4	10.6
2020-08-12 CP	12.2	7.6	12.4
2020-09-20 CP	7.4	9.6	11.3
2020-10-27 CP	24.3	22.7	27.9
2020-11-25 CP	33.4	31.7	53.0
2020-12-23 CP	19.4	18.1	16.6
2020-01-27 NC	157.6	135.7	180.8
2020-02-09 NC	212.6	(0.011%)	283.2
2020-03-05 NC	(0.021%)	(0.018%)	(0.020%)
2020-04-03 NC	(0.050%)	(0.050%)	(0.052%)
2020-05-05 NC	212.8	120.9	169.6
2020-06-09 NC	49.8	52.9	55.9
2020-07-06 NC	39.3	38.3	43.6
2020-08-12 NC	67.3	43.8	43.1
2020-09-20 NC	69.3	97.4	124.7
2020-10-27 NC	153.7	192.4	214.2
2020-11-25 NC	164.9	127.6	(0.015%)
2020-12-23 NC	226.1	102.9	227.2

Table 30: Computational results for CAISO instances. Wall clock times are reported in seconds. When the 300 second time limit is reached, the terminating optimality gap is reported in parentheses. Each instance is named by the date or hypothetical scenario from which the beginning of the 48-hour load profile was selected, followed by the reserve level, considered as a percentage of load. We replace the up-time/down-time constraints from **T** with those listed.

Instance	**T**	RT 2bin [27] (4)(7)	DEKT [7] (70)(71)
2014-09-01 0%	51.7	42.0	55.6
2014-12-01 0%	45.1	23.3	39.5
2015-03-01 0%	32.1	23.6	31.3
2015-06-01 0%	19.1	12.0	18.2
Scenario400 0%	31.0	38.5	32.0
2014-09-01 1%	25.4	28.0	25.9
2014-12-01 1%	84.5	56.9	34.5
2015-03-01 1%	36.4	16.9	33.2
2015-06-01 1%	38.3	41.7	32.5
Scenario400 1%	66.5	86.7	40.7
2014-09-01 3%	83.5	83.4	51.5
2014-12-01 3%	106.8	57.5	68.6
2015-03-01 3%	67.4	61.5	66.7
2015-06-01 3%	57.8	95.4	38.5
Scenario400 3%	103.8	108.1	91.7
2014-09-01 5%	92.7	71.0	59.0
2014-12-01 5%	129.9	100.9	98.7
2015-03-01 5%	54.6	71.9	33.3
2015-06-01 5%	56.7	95.0	75.0
Scenario400 5%	106.9	102.8	107.8

Table 31: Computational results for FERC instances. Wall clock times are reported in seconds. When the 600 second time limit is reached, the terminating optimality gap is reported in parentheses. Each instance is named by the date from which the beginning of the 48-hour load profile was selected and whether it is low-wind (LW) (wind supply is 2% of energy demand for the year) or high-wind (HW) (wind supply is scaled to be 30% of energy demand for the year). We replace the up-time/down-time constraints from **T** with those listed.

Instance	**T**	RT 2bin [27] (4)(7)	DEKT [7] (70)(71)
2015-01-01 LW	107.6	102.9	197.5
2015-02-01 LW	133.5	88.0	268.1
2015-03-01 LW	244.0	211.7	318.1
2015-04-01 LW	411.6	369.7	271.8
2015-05-01 LW	221.6	117.1	192.0
2015-06-01 LW	282.6	187.6	345.1
2015-07-01 LW	429.6	389.7	480.3
2015-08-01 LW	281.5	275.0	374.8
2015-09-01 LW	541.8	422.1	(0.012%)
2015-10-01 LW	201.9	125.4	350.9
2015-11-02 LW	166.0	140.1	213.3
2015-12-01 LW	114.5	135.5	427.3
2015-01-01 HW	210.4	254.2	(0.030%)
2015-02-01 HW	136.0	175.8	380.7
2015-03-01 HW	379.2	277.3	471.3
2015-04-01 HW	244.6	275.8	580.4
2015-05-01 HW	244.4	184.4	433.0
2015-06-01 HW	351.2	246.8	395.3
2015-07-01 HW	548.1	410.4	(0.026%)
2015-08-01 HW	361.7	421.6	(0.034%)
2015-09-01 HW	(0.018%)	474.6	(0.026%)
2015-10-01 HW	351.6	401.5	(0.019%)
2015-11-02 HW	321.1	330.3	583.3
2015-12-01 HW	287.3	286.0	(0.075%)

Table 32: Computational results for RTS-GMLC instances. Wall clock times are reported in seconds. When the 300 second time limit is reached, the terminating optimality gap is reported in parentheses. Each instance is named by the date from which the beginning of the 48-hour load profile was selected and whether the system with considered with (NC) or without (CP) a network. We replace the generation limits except (17) from **T** with those listed.

Instance	**T**	MLR [22] (20)(21)	GMR [12] (20)(23)	PG [24] (38)	GMR+PG (23)(38)	AC [1] (19)	OAV [23] (19)(37)	CA [4] (18)(76)
2020-01-27 CP	88.8	146.7	147.3	108.0	146.8	177.3	147.3	149.5
2020-02-09 CP	23.0	23.2	23.5	24.6	23.8	28.3	34.1	32.5
2020-03-05 CP	25.3	27.6	27.8	33.1	32.3	31.9	39.3	30.0
2020-04-03 CP	42.6	70.7	72.8	149.4	40.2	(0.025%)	(0.016%)	175.5
2020-05-05 CP	19.5	23.7	23.9	31.0	20.0	26.0	27.2	43.2
2020-06-09 CP	5.8	6.4	6.3	6.6	6.2	8.8	9.4	9.2
2020-07-06 CP	6.8	10.6	10.7	10.8	7.0	11.1	15.1	7.1
2020-08-12 CP	12.2	10.6	10.7	13.9	9.3	14.6	21.0	26.0
2020-09-20 CP	7.4	7.7	7.9	12.0	9.0	9.8	15.8	15.6
2020-10-27 CP	24.3	20.5	20.6	21.7	26.6	33.8	47.7	42.7
2020-11-25 CP	33.4	30.8	30.5	35.0	28.9	48.3	52.5	51.7
2020-12-23 CP	19.4	20.4	20.4	35.7	22.1	29.6	24.4	28.2
2020-01-27 NC	157.6	89.1	88.4	103.6	139.9	172.1	158.5	193.7
2020-02-09 NC	212.6	109.7	108.7	98.5	264.4	255.7	224.5	243.5
2020-03-05 NC	(0.021%)	(0.018%)	(0.018%)	(0.012%)	129.3	(0.017%)	(0.016%)	258.5
2020-04-03 NC	(0.050%)	(0.054%)	(0.054%)	(0.075%)	(0.049%)	(0.054%)	(0.059%)	(0.052%)
2020-05-05 NC	212.8	262.4	265.5	296.2	263.6	(0.052%)	279.9	293.3
2020-06-09 NC	49.8	54.7	54.2	52.9	46.1	92.9	68.2	85.1
2020-07-06 NC	39.3	44.2	43.5	93.6	44.3	64.9	87.0	71.4
2020-08-12 NC	67.3	53.3	53.3	48.8	50.8	57.8	50.2	54.8
2020-09-20 NC	69.3	117.7	116.9	74.3	114.7	121.0	110.8	119.8
2020-10-27 NC	153.7	152.8	152.5	175.7	149.8	234.9	212.3	204.2
2020-11-25 NC	164.9	149.4	150.0	107.7	175.5	191.2	143.8	140.8
2020-12-23 NC	226.1	122.4	121.8	125.3	94.2	270.1	273.2	(0.157%)

Table 33: Computational results for CAISO instances. Wall clock times are reported in seconds. When the 300 second time limit is reached, the terminating optimality gap is reported in parentheses. Each instance is named by the date or hypothetical scenario from which the beginning of the 48-hour load profile was selected, followed by the reserve level, considered as a percentage of load. We replace the generation limits except (17) from **T** with those listed.

Instance	**T**	MLR [22] (20)(21)	GMR [12] (20)(23)	PG [24] (38)	GMR+PG (23)(38)	AC [1] (19)	OAV [23] (19)(37)	CA [4] (18)(76)
2014-09-01 0%	51.7	31.6	31.8	46.6	46.3	50.0	76.2	50.5
2014-12-01 0%	45.1	39.1	39.3	35.8	31.7	37.4	43.7	38.5
2015-03-01 0%	32.1	40.3	41.0	53.8	36.2	44.7	45.4	49.5
2015-06-01 0%	19.1	20.2	20.3	19.6	21.4	18.4	18.8	18.0
Scenario400 0%	31.0	40.6	39.8	46.6	45.4	57.9	61.8	61.3
2014-09-01 1%	25.4	26.7	26.2	45.2	21.9	48.4	44.2	73.2
2014-12-01 1%	84.5	76.2	75.9	140.8	81.3	141.8	153.1	98.0
2015-03-01 1%	36.4	33.9	33.7	66.3	35.6	57.8	67.5	54.8
2015-06-01 1%	38.3	38.5	38.6	74.6	40.1	132.7	76.4	142.8
Scenario400 1%	66.5	46.1	46.4	84.4	35.8	132.9	149.4	121.5
2014-09-01 3%	83.5	54.6	54.9	168.1	62.7	158.6	165.5	156.2
2014-12-01 3%	106.8	119.9	117.7	139.4	72.7	154.5	151.6	145.1
2015-03-01 3%	67.4	64.6	64.4	105.0	79.0	128.6	140.1	121.1
2015-06-01 3%	57.8	62.8	62.9	206.6	38.3	91.0	58.5	98.0
Scenario400 3%	103.8	109.7	110.3	175.7	102.6	167.0	227.1	(0.011%)
2014-09-01 5%	92.7	58.9	58.6	164.4	71.0	106.2	126.1	138.3
2014-12-01 5%	129.9	106.0	106.7	130.6	64.1	208.7	149.4	106.6
2015-03-01 5%	54.6	38.8	38.8	116.8	58.6	108.0	156.1	105.6
2015-06-01 5%	56.7	54.4	54.9	117.3	55.5	162.4	181.9	150.1
Scenario400 5%	106.9	128.8	128.1	168.9	128.1	221.9	224.6	170.5

Table 34: Computational results for FERC instances. Wall clock times are reported in seconds. When the 600 second time limit is reached, the terminating optimality gap is reported in parentheses. Each instance is named by the date from which the beginning of the 48-hour load profile was selected and whether it is low-wind (LW) (wind supply is 2% of energy demand for the year) or high-wind (HW) (wind supply is scaled to be 30% of energy demand for the year). We replace the generation limits except (17) from **T** with those listed.

Instance	**T**	MLR [22] (20)(21)	GMR [12] (20)(23)	PG [24] (38)	GMR+PG (23)(38)	AC [1] (19)	OAV [23] (19)(37)	CA [4] (18)(76)
2015-01-01 LW	107.6	106.3	106.5	167.6	107.9	183.5	123.3	163.0
2015-02-01 LW	133.5	93.3	93.1	199.9	86.8	302.1	241.1	238.7
2015-03-01 LW	244.0	196.2	193.7	243.4	194.1	327.2	328.2	536.5
2015-04-01 LW	411.6	383.2	382.3	423.5	366.4	600.0	253.3	387.0
2015-05-01 LW	221.6	177.4	177.4	201.7	213.1	253.1	103.3	212.3
2015-06-01 LW	282.6	304.1	303.6	443.8	287.4	340.0	171.1	406.9
2015-07-01 LW	429.6	398.4	400.6	528.5	508.5	594.0	463.9	407.6
2015-08-01 LW	281.5	386.5	381.9	357.3	364.7	386.5	133.0	415.2
2015-09-01 LW	541.8	437.7	436.9	354.3	407.3	(0.013%)	(0.013%)	544.9
2015-10-01 LW	201.9	126.1	123.8	226.1	181.1	302.3	176.0	138.2
2015-11-02 LW	166.0	148.2	146.2	243.8	124.7	219.7	163.2	106.6
2015-12-01 LW	114.5	115.1	114.9	198.5	131.4	334.0	256.3	229.0
2015-01-01 HW	210.4	273.7	272.8	222.3	254.6	401.1	211.7	280.6
2015-02-01 HW	136.0	171.3	171.7	248.8	152.7	353.6	188.2	268.9
2015-03-01 HW	379.2	248.6	246.2	351.1	247.9	327.2	540.4	290.3
2015-04-01 HW	244.6	289.7	288.5	216.1	239.0	321.7	192.4	510.7
2015-05-01 HW	244.4	349.8	349.4	297.4	288.4	374.3	184.1	420.4
2015-06-01 HW	351.2	369.7	369.8	308.7	326.3	410.1	146.5	282.7
2015-07-01 HW	548.1	424.1	429.7	544.9	(0.018%)	572.6	344.6	400.9
2015-08-01 HW	361.7	402.4	406.6	392.2	432.3	469.0	348.6	484.0
2015-09-01 HW	(0.018%)	462.0	463.2	435.0	(0.010%)	475.4	243.1	412.9
2015-10-01 HW	351.6	342.9	329.7	326.2	318.2	331.0	369.2	346.5
2015-11-02 HW	321.1	301.3	300.5	359.1	244.0	298.1	142.7	186.6
2015-12-01 HW	287.3	269.3	278.3	278.8	239.2	312.6	227.9	345.5

Table 35: Computational results for RTS-GMLC instances. Wall clock times are reported in seconds. When the 300 second time limit is reached, the terminating optimality gap is reported in parentheses. Each instance is named by the date from which the beginning of the 48-hour load profile was selected and whether the system with considered with (NC) or without (CP) a network. We replace the ramping limits from **T** with those listed.

Instance	**T**	DKRA [6] (35)(36)(79)	MLR [22] (26)(27)	AC [1] (24)(25)	**OAV** [23] ramp limits	**OAV-T** [23] ramp limits
2020-01-27 CP	88.8	99.8	90.8	86.9	96.9	87.0
2020-02-09 CP	23.0	35.3	28.7	26.8	26.0	32.6
2020-03-05 CP	25.3	20.7	28.0	32.2	27.6	33.6
2020-04-03 CP	42.6	53.8	38.8	66.8	44.7	68.2
2020-05-05 CP	19.5	22.8	23.5	24.4	23.6	22.7
2020-06-09 CP	5.8	6.4	6.6	6.3	6.0	7.3
2020-07-06 CP	6.8	9.6	6.2	7.0	10.3	10.4
2020-08-12 CP	12.2	10.6	9.3	10.7	10.7	10.3
2020-09-20 CP	7.4	11.2	8.2	8.3	8.1	9.6
2020-10-27 CP	24.3	19.2	29.5	26.5	31.2	26.5
2020-11-25 CP	33.4	32.9	33.2	26.8	38.9	42.7
2020-12-23 CP	19.4	20.3	18.7	24.6	18.9	16.5
2020-01-27 NC	157.6	137.4	167.3	143.5	79.1	139.6
2020-02-09 NC	212.6	259.2	220.1	217.3	242.1	(0.015%)
2020-03-05 NC	(0.021%)	(0.015%)	(0.018%)	(0.019%)	(0.015%)	(0.020%)
2020-04-03 NC	(0.050%)	(0.049%)	(0.048%)	(0.058%)	(0.050%)	(0.045%)
2020-05-05 NC	212.8	138.9	133.4	143.8	225.2	242.5
2020-06-09 NC	49.8	53.4	45.9	51.3	52.9	50.4
2020-07-06 NC	39.3	42.2	59.5	33.1	47.8	47.6
2020-08-12 NC	67.3	61.6	50.3	53.3	46.4	46.9
2020-09-20 NC	69.3	67.9	63.1	109.2	110.0	65.4
2020-10-27 NC	153.7	178.7	186.0	165.4	151.5	175.1
2020-11-25 NC	164.9	129.8	132.1	121.2	145.0	119.5
2020-12-23 NC	226.1	91.0	116.4	196.0	108.5	116.7

Table 36: Computational results for CAISO instances. Wall clock times are reported in seconds. When the 300 second time limit is reached, the terminating optimality gap is reported in parentheses. Each instance is named by the date or hypothetical scenario from which the beginning of the 48-hour load profile was selected, followed by the reserve level, considered as a percentage of load. We replace the ramping limits from **T** with those listed.

Instance	**T**	DKRA [6] (35)(36)(79)	MLR [22] (26)(27)	AC [1] (24)(25)	**OAV** [23] ramp limits	**OAV-T** [23] ramp limits
2014-09-01 0%	51.7	57.0	42.3	41.1	62.0	61.6
2014-12-01 0%	45.1	32.0	30.9	35.2	24.2	25.1
2015-03-01 0%	32.1	36.2	34.8	39.4	34.8	36.7
2015-06-01 0%	19.1	20.2	17.1	21.9	22.7	22.1
Scenario400 0%	31.0	36.3	15.9	35.0	37.7	37.5
2014-09-01 1%	25.4	36.5	32.8	32.3	48.9	50.2
2014-12-01 1%	84.5	73.8	71.7	62.1	69.6	69.5
2015-03-01 1%	36.4	35.6	27.5	42.5	40.6	41.0
2015-06-01 1%	38.3	41.5	45.3	55.2	56.5	58.5
Scenario400 1%	66.5	63.4	68.2	39.1	44.5	44.2
2014-09-01 3%	83.5	55.5	77.2	69.6	76.4	76.2
2014-12-01 3%	106.8	89.5	71.0	82.2	110.9	111.1
2015-03-01 3%	67.4	63.5	68.0	76.0	71.3	70.8
2015-06-01 3%	57.8	35.3	71.4	54.3	36.7	36.8
Scenario400 3%	103.8	102.8	93.8	97.2	101.7	98.2
2014-09-01 5%	92.7	68.7	58.6	59.3	86.8	87.3
2014-12-01 5%	129.9	84.5	72.0	116.0	201.7	202.7
2015-03-01 5%	54.6	43.6	51.4	47.4	57.4	57.5
2015-06-01 5%	56.7	72.7	70.9	62.7	76.7	77.2
Scenario400 5%	106.9	101.7	55.4	101.5	118.6	120.5

Table 37: Computational results for FERC instances. Wall clock times are reported in seconds. When the 600 second time limit is reached, the terminating optimality gap is reported in parentheses. Each instance is named by the date from which the beginning of the 48-hour load profile was selected and whether it is low-wind (LW) (wind supply is 2% of energy demand for the year) or high-wind (HW) (wind supply is scaled to be 30% of energy demand for the year). We replace the ramping limits from **T** with those listed.

Instance	**T**	DKRA [6] (35)(36)(79)	MLR [22] (26)(27)	AC [1] (24)(25)	**OAV** [23] ramp limits	**OAV-T** [23] ramp limits
2015-01-01 LW	107.6	114.4	107.9	132.5	111.9	119.3
2015-02-01 LW	133.5	107.1	139.3	139.4	98.5	99.7
2015-03-01 LW	244.0	259.5	124.2	293.5	223.7	263.7
2015-04-01 LW	411.6	395.9	342.1	(0.017%)	400.0	359.8
2015-05-01 LW	221.6	224.9	166.3	263.9	229.1	277.9
2015-06-01 LW	282.6	341.5	302.3	358.1	328.6	449.0
2015-07-01 LW	429.6	595.3	203.5	484.4	514.1	(0.032%)
2015-08-01 LW	281.5	417.9	385.2	376.1	401.6	438.5
2015-09-01 LW	541.8	568.1	399.3	584.4	392.7	568.4
2015-10-01 LW	201.9	212.3	390.1	160.9	132.2	127.0
2015-11-02 LW	166.0	138.7	137.6	177.6	126.8	133.6
2015-12-01 LW	114.5	161.8	148.3	174.6	144.4	162.7
2015-01-01 HW	210.4	250.3	276.0	251.5	242.1	303.2
2015-02-01 HW	136.0	156.5	172.6	178.0	149.5	351.6
2015-03-01 HW	379.2	293.0	342.7	494.1	339.4	(0.013%)
2015-04-01 HW	244.6	282.4	250.9	296.4	258.1	308.0
2015-05-01 HW	244.4	255.0	117.2	278.8	227.8	246.3
2015-06-01 HW	351.2	464.6	373.0	431.6	386.1	437.5
2015-07-01 HW	548.1	522.3	282.8	579.0	489.1	(0.022%)
2015-08-01 HW	361.7	398.3	562.9	476.5	456.2	(0.011%)
2015-09-01 HW	(0.018%)	373.6	413.8	(0.019%)	(0.012%)	(0.021%)
2015-10-01 HW	351.6	349.3	346.1	368.3	326.7	454.1
2015-11-02 HW	321.1	348.8	192.7	409.5	340.7	396.2
2015-12-01 HW	287.3	268.6	272.9	256.9	319.0	372.4

Table 38: Computational results for RTS-GMLC instances. Wall clock times are reported in seconds. When the 300 second time limit is reached, the terminating optimality gap is reported in parentheses. Each instance is named by the date from which the beginning of the 48-hour load profile was selected and whether the system with considered with (NC) or without (CP) a network. We replace the production cost formulation from **T** with those listed.

Instance	**T**	CA [4] (43)(44)(45)	Wu [32] (42)(43)(44)	**KOW** [18] prod. costs	CW [5] (93)(94)(95)	SOS2 (89)(90)(91)	SLL [30] (89)(90)(92)	Epi (49)	HB [14] (50)
2020-01-27 CP	88.8	75.8	102.3	99.8	82.0	84.0	78.0	87.7	72.9
2020-02-09 CP	23.0	20.2	23.8	35.5	21.4	33.1	20.6	38.6	28.3
2020-03-05 CP	25.3	22.4	21.5	20.3	23.0	24.6	22.2	28.1	30.4
2020-04-03 CP	42.6	53.6	51.3	53.8	42.7	45.4	55.3	67.7	49.7
2020-05-05 CP	19.5	30.8	24.9	22.5	15.9	29.1	17.1	33.9	23.3
2020-06-09 CP	5.8	6.5	6.9	6.3	5.8	6.5	5.7	6.8	6.3
2020-07-06 CP	6.8	7.9	7.9	9.5	8.8	11.7	7.6	13.2	11.1
2020-08-12 CP	12.2	6.3	8.4	10.7	7.6	8.5	7.5	8.8	12.1
2020-09-20 CP	7.4	8.1	11.3	11.2	8.2	12.2	8.9	8.1	11.7
2020-10-27 CP	24.3	30.5	25.0	18.6	23.2	25.0	26.6	23.8	22.6
2020-11-25 CP	33.4	32.9	31.2	32.0	33.5	30.9	36.4	39.2	57.5
2020-12-23 CP	19.4	13.3	24.0	20.1	22.2	35.1	22.1	26.2	18.9
2020-01-27 NC	157.6	202.1	93.7	137.7	164.0	163.6	129.1	191.5	140.7
2020-02-09 NC	212.6	176.7	156.9	258.4	207.1	255.6	209.7	102.1	187.2
2020-03-05 NC	(0.021%)	210.5	125.8	(0.016%)	258.9	247.2	182.5	279.3	172.7
2020-04-03 NC	(0.050%)	(0.054%)	(0.067%)	(0.049%)	(0.043%)	(0.076%)	(0.047%)	(0.072%)	(0.055%)
2020-05-05 NC	212.8	125.2	(0.039%)	139.0	141.3	(0.055%)	254.6	(0.051%)	299.7
2020-06-09 NC	49.8	60.1	47.1	54.0	61.2	80.6	58.1	63.5	54.5
2020-07-06 NC	39.3	61.6	43.6	42.4	51.0	82.6	41.5	45.2	87.0
2020-08-12 NC	67.3	45.8	67.2	61.1	69.1	61.6	42.0	51.5	75.9
2020-09-20 NC	69.3	77.1	67.2	68.4	73.9	125.4	77.0	121.8	113.8
2020-10-27 NC	153.7	203.6	205.8	179.3	183.5	229.2	184.3	180.3	184.1
2020-11-25 NC	164.9	86.9	212.5	130.4	155.2	136.9	113.2	123.0	125.0
2020-12-23 NC	226.1	276.3	114.0	91.1	235.7	156.8	97.0	239.1	229.1

Table 39: Computational results for CAISO instances. Wall clock times are reported in seconds. When the 300 second time limit is reached, the terminating optimality gap is reported in parentheses. Each instance is named by the date or hypothetical scenario from which the beginning of the 48-hour load profile was selected, followed by the reserve level, considered as a percentage of load. We replace the production cost formulation from **T** with those listed.

Instance	**T**	CA [4] (43)(44)(45)	Wu [32] (42)(43)(44)	**KOW** [18] prod. costs	CW [5] (93)(94)(95)	SOS2 (89)(90)(91)	SLL [30] (89)(90)(92)	Epi (49)	HB [14] (50)
2014-09-01 0%	51.7	53.5	55.5	56.2	45.0	29.6	41.8	33.2	28.4
2014-12-01 0%	45.1	33.1	36.8	31.6	27.8	26.1	24.9	19.2	21.3
2015-03-01 0%	32.1	41.2	37.0	36.0	21.4	35.7	25.6	30.7	35.0
2015-06-01 0%	19.1	26.2	21.3	20.5	14.6	18.0	15.6	17.4	18.8
Scenario400 0%	31.0	34.7	29.7	35.6	27.2	36.3	28.0	62.8	43.9
2014-09-01 1%	25.4	27.1	24.5	36.9	22.5	25.2	26.2	38.3	45.2
2014-12-01 1%	84.5	77.9	45.5	72.3	54.8	34.5	42.6	110.1	112.5
2015-03-01 1%	36.4	55.9	34.6	36.2	25.4	45.7	24.6	48.6	32.9
2015-06-01 1%	38.3	59.5	47.2	42.0	37.1	64.3	37.8	59.7	68.8
Scenario400 1%	66.5	53.9	72.8	62.4	81.3	49.7	43.2	47.0	74.5
2014-09-01 3%	83.5	76.6	57.5	55.9	64.9	75.8	79.6	128.5	102.8
2014-12-01 3%	106.8	41.0	86.5	88.9	61.2	63.1	69.9	92.9	134.9
2015-03-01 3%	67.4	79.4	57.8	64.3	66.8	66.7	66.7	64.8	110.8
2015-06-01 3%	57.8	42.9	75.5	35.0	33.0	40.4	38.6	170.1	156.4
Scenario400 3%	103.8	99.9	102.8	105.0	105.7	117.1	149.5	132.8	114.1
2014-09-01 5%	92.7	63.5	80.8	68.9	62.0	76.7	49.1	139.0	81.5
2014-12-01 5%	129.9	134.5	93.7	84.6	85.6	77.2	118.4	131.7	159.6
2015-03-01 5%	54.6	77.2	64.8	44.1	56.7	54.1	42.9	71.1	62.3
2015-06-01 5%	56.7	100.6	86.6	72.9	84.4	87.7	128.2	127.4	137.0
Scenario400 5%	106.9	99.8	130.4	102.1	156.2	127.8	130.9	134.3	269.5

Table 40: Computational results for FERC instances. Wall clock times are reported in seconds. When the 600 second time limit is reached, the terminating optimality gap is reported in parentheses. Each instance is named by the date from which the beginning of the 48-hour load profile was selected and whether it is low-wind (LW) (wind supply is 2% of energy demand for the year) or high-wind (HW) (wind supply is scaled to be 30% of energy demand for the year). We replace the production cost formulation from **T** with those listed.

Instance	**T**	CA [4] (43)(44)(45)	Wu [32] (42)(43)(44)	**KOW** [18] prod. costs	CW [5] (93)(94)(95)	SOS2 (89)(90)(91)	SLL [30] (89)(90)(92)	Epi (49)	HB [14] (50)
2015-01-01 LW	107.6	118.8	164.6	114.4	268.4	183.8	226.5	94.2	319.3
2015-02-01 LW	133.5	109.5	194.4	110.7	212.5	143.0	160.9	75.0	149.7
2015-03-01 LW	244.0	241.6	266.4	255.9	296.0	241.6	340.5	331.7	432.4
2015-04-01 LW	411.6	281.7	391.5	395.7	470.7	382.5	287.7	494.0	527.6
2015-05-01 LW	221.6	152.9	265.3	233.7	297.2	201.1	253.2	308.7	370.3
2015-06-01 LW	282.6	247.0	538.6	341.8	305.4	321.8	293.9	491.8	380.8
2015-07-01 LW	429.6	481.0	503.1	594.8	461.6	563.3	364.3	(0.040%)	(0.025%)
2015-08-01 LW	281.5	243.6	393.4	416.0	363.5	306.0	334.7	387.9	487.2
2015-09-01 LW	541.8	328.1	(0.012%)	568.3	(0.013%)	463.1	551.1	(0.016%)	(0.013%)
2015-10-01 LW	201.9	222.1	182.3	206.8	244.6	232.6	238.0	152.6	251.8
2015-11-02 LW	166.0	139.5	170.1	138.5	216.2	176.0	210.8	93.9	248.4
2015-12-01 LW	114.5	135.7	225.8	161.7	272.6	177.0	189.4	90.6	256.0
2015-01-01 HW	210.4	196.0	285.2	249.5	237.9	194.8	210.0	162.2	279.5
2015-02-01 HW	136.0	172.5	207.6	156.8	267.0	227.8	233.9	180.6	307.5
2015-03-01 HW	379.2	249.7	500.0	291.9	298.7	293.0	433.2	367.1	(0.020%)
2015-04-01 HW	244.6	276.9	330.4	284.6	217.8	180.4	207.9	293.7	237.2
2015-05-01 HW	244.4	164.8	216.7	251.7	232.0	211.3	266.6	226.3	251.5
2015-06-01 HW	351.2	332.2	354.2	458.7	338.2	387.0	364.4	487.2	343.6
2015-07-01 HW	548.1	371.6	570.6	521.7	514.2	527.3	527.2	(0.039%)	(0.042%)
2015-08-01 HW	361.7	290.1	487.1	398.9	407.8	388.5	370.7	(0.015%)	543.8
2015-09-01 HW	(0.018%)	464.6	317.6	370.2	563.5	474.5	(0.014%)	(0.022%)	(0.022%)
2015-10-01 HW	351.6	293.2	364.4	349.2	338.4	391.2	325.5	309.6	417.4
2015-11-02 HW	321.1	208.6	481.0	353.5	351.7	259.4	388.0	158.2	259.6
2015-12-01 HW	287.3	230.4	386.1	272.1	293.3	251.6	362.8	365.5	313.5

Table 41: Computational results for RTS-GMLC instances. Wall clock times are reported in seconds. When the 300 second time limit is reached, the terminating optimality gap is reported in parentheses. Each instance is named by the date from which the beginning of the 48-hour load profile was selected and whether the system with considered with (NC) or without (CP) a network. We replace the start-up cost formulation from **T** with those listed.

Instance	**T**	MLR [22] (54)(55)(56)	KOW 3bin [19] (96)(97)	A-K [2, 19] (100)(101)(104)	MLR proj (54)(57)(58)
2020-01-27 CP	88.8	97.1	107.3	160.6	72.7
2020-02-09 CP	23.0	21.4	21.0	22.0	23.1
2020-03-05 CP	25.3	24.8	25.9	27.0	18.8
2020-04-03 CP	42.6	57.8	60.3	46.3	75.0
2020-05-05 CP	19.5	25.2	26.5	28.0	19.9
2020-06-09 CP	5.8	5.3	6.2	5.7	5.2
2020-07-06 CP	6.8	7.3	6.4	5.8	6.6
2020-08-12 CP	12.2	8.3	12.4	10.7	8.7
2020-09-20 CP	7.4	12.7	9.1	8.3	10.1
2020-10-27 CP	24.3	21.6	21.8	27.3	20.9
2020-11-25 CP	33.4	32.1	28.3	29.8	32.8
2020-12-23 CP	19.4	14.5	13.8	14.1	12.5
2020-01-27 NC	157.6	101.8	111.9	90.4	79.8
2020-02-09 NC	212.6	229.3	286.2	224.8	218.2
2020-03-05 NC	(0.021%)	(0.012%)	(0.014%)	(0.019%)	(0.021%)
2020-04-03 NC	(0.050%)	(0.049%)	(0.055%)	(0.050%)	(0.051%)
2020-05-05 NC	212.8	255.7	117.9	266.5	168.9
2020-06-09 NC	49.8	50.6	51.0	41.0	47.6
2020-07-06 NC	39.3	40.5	39.5	39.2	34.8
2020-08-12 NC	67.3	64.3	42.4	40.9	54.1
2020-09-20 NC	69.3	75.8	89.9	82.7	77.2
2020-10-27 NC	153.7	133.7	106.8	126.3	106.2
2020-11-25 NC	164.9	136.3	148.7	139.3	91.4
2020-12-23 NC	226.1	135.9	282.1	259.2	138.9

Table 42: Computational results for CAISO instances. Wall clock times are reported in seconds. When the 300 second time limit is reached, the terminating optimality gap is reported in parentheses. Each instance is named by the date or hypothetical scenario from which the beginning of the 48-hour load profile was selected, followed by the reserve level, considered as a percentage of load. We replace the start-up cost formulation from **T** with those listed.

Instance	**T**	MLR [22] (54)(55)(56)	KOW 3bin [19] (96)(97)	A-K [2, 19] (100)(101)(104)	MLR proj (54)(57)(58)
2014-09-01 0%	51.7	47.5	60.2	24.9	18.5
2014-12-01 0%	45.1	41.0	57.6	38.6	40.6
2015-03-01 0%	32.1	40.4	53.9	23.2	40.3
2015-06-01 0%	19.1	31.9	50.1	19.1	39.6
Scenario400 0%	31.0	67.0	159.9	65.9	79.6
2014-09-01 1%	25.4	38.7	64.8	33.6	40.1
2014-12-01 1%	84.5	87.9	122.9	72.9	90.7
2015-03-01 1%	36.4	49.8	58.9	49.6	65.3
2015-06-01 1%	38.3	56.7	87.3	29.7	39.2
Scenario400 1%	66.5	71.9	270.5	66.1	132.8
2014-09-01 3%	83.5	71.9	120.7	51.6	111.5
2014-12-01 3%	106.8	92.7	129.2	108.5	96.2
2015-03-01 3%	67.4	73.5	65.1	53.9	66.3
2015-06-01 3%	57.8	54.3	46.8	43.8	31.2
Scenario400 3%	103.8	120.1	(0.015%)	203.4	107.6
2014-09-01 5%	92.7	107.1	156.0	66.2	73.5
2014-12-01 5%	129.9	130.2	171.0	98.4	92.6
2015-03-01 5%	54.6	73.1	115.7	68.7	85.1
2015-06-01 5%	56.7	41.0	106.7	73.8	88.3
Scenario400 5%	106.9	177.3	(0.017%)	147.3	146.1

Table 43: Computational results for FERC instances. Wall clock times are reported in seconds. When the 600 second time limit is reached, the terminating optimality gap is reported in parentheses. Each instance is named by the date from which the beginning of the 48-hour load profile was selected and whether it is low-wind (LW) (wind supply is 2% of energy demand for the year) or high-wind (HW) (wind supply is scaled to be 30% of energy demand for the year). We replace the start-up cost formulation from **T** with those listed.

Instance	**T**	MLR [22] (54)(55)(56)	KOW 3bin [19] (96)(97)	A-K [2, 19] (100)(101)(104)	MLR proj (54)(57)(58)
2015-01-01 LW	107.6	221.1	254.5	185.6	203.3
2015-02-01 LW	133.5	562.2	(0.015%)	519.6	413.3
2015-03-01 LW	244.0	350.9	237.5	313.9	283.1
2015-04-01 LW	411.6	190.3	293.5	113.7	281.6
2015-05-01 LW	221.6	89.8	90.3	81.3	75.3
2015-06-01 LW	282.6	110.0	112.3	110.9	117.2
2015-07-01 LW	429.6	286.5	421.3	470.4	322.1
2015-08-01 LW	281.5	215.2	298.2	251.3	260.5
2015-09-01 LW	541.8	530.6	(0.010%)	276.7	452.0
2015-10-01 LW	201.9	188.5	155.4	198.6	217.6
2015-11-02 LW	166.0	257.8	441.1	354.7	283.0
2015-12-01 LW	114.5	212.1	163.1	177.3	210.2
2015-01-01 HW	210.4	(1.099%)	(1.335%)	(1.191%)	(0.984%)
2015-02-01 HW	136.0	(0.057%)	(0.218%)	(0.058%)	(0.048%)
2015-03-01 HW	379.2	389.7	533.8	447.0	492.0
2015-04-01 HW	244.6	342.7	447.2	268.7	240.4
2015-05-01 HW	244.4	163.1	154.5	178.9	141.0
2015-06-01 HW	351.2	163.2	201.1	177.3	206.3
2015-07-01 HW	548.1	337.1	431.5	283.7	370.3
2015-08-01 HW	361.7	382.9	427.7	338.4	356.6
2015-09-01 HW	(0.018%)	479.4	(0.051%)	466.2	428.7
2015-10-01 HW	351.6	395.1	352.7	320.6	391.0
2015-11-02 HW	321.1	578.6	(0.091%)	383.3	560.3
2015-12-01 HW	287.3	293.5	389.4	412.6	264.0

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