Contingency-Constrained Unit Commitment with Preventive and Corrective Transmission Switching

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Abstract

Preventive and corrective transmission switching have been previously shown to offer significant benefits to power system operation. The former primarily reduces operating costs, while the latter decreases potential levels of power imbalance caused by contingencies. Within the context of co-optimized electricity markets for energy and reserves, this paper addresses the incorporation of transmission switching in the contingency-constrained unit commitment problem. The proposed generation scheduling model differs from existing formulations due to the joint consideration of two major complicating factors. First, not only preventive but also corrective transmission switching actions are considered, thereby requiring binary post-contingency variables. In addition, the time-coupled operation of generating units is precisely characterized. The proposed model is cast as a challenging mixed-integer program that is solved by an exact nested column-and-constraint generation algorithm. Numerical results illustrate the economic, operational, and computational advantages of the proposed approach.

Nomenclature

The symbols used in this paper are defined in this section. Superscript “$m$” is used to represent new variables in the inner master problem. Superscripts “$(k)$” and “$(m)$” are used to denote the value of a variable at outer-loop iteration $k$ and inner-loop iteration $m$, respectively.
Constants

$\epsilon^i, \epsilon^o$  Inner- and outer-loop convergence parameters.

$F_l$  Rated capacity of transmission line $l$.

$F_{it}^{dn}, F_{it}^{up}$  Maximum down- and up-spinning reserve contributions of generator $i$ in period $t$.

$\underline{P}_{it}, \overline{P}_{it}$  Lower and upper production limits of generator $i$ in period $t$.

$A^c_{it}$  Parameter that is equal to 1 if generator $i$ is available in period $t$ under contingency state $c$, being 0 otherwise.

$A^c_{lt}$  Parameter that is equal to 1 if transmission line $l$ is available in period $t$ under contingency state $c$, being 0 otherwise.

$C_I$  Cost coefficient of power imbalance.

$C_{it}^{dn}, C_{it}^{up}$  Down- and up-spinning reserve costs offered by generator $i$ in period $t$.

$d_{bt}$  Power demand at bus $b$ in period $t$.

$K$  Number of unavailable system components.

$K^G$  Number of unavailable generators.

$K^L$  Number of unavailable transmission lines.

$LB, UB$  Lower and upper bounds for the total cost.

$M_l$  Big-M parameter related to transmission line $l$.

$RD_i, RU_i$  Ramp-down and ramp-up limits of generator $i$.

$SD_i, SU_i$  Shut-down and start-up ramp limits of generator $i$.

$x_l$  Reactance of line $l$.

Dual Variables

$\beta_{bt}$  Dual variable associated with the power balance equation at bus $b$ in period $t$ in the lower level of the oracle problem.

$\gamma_{it}, \chi_{it}$  Dual variables associated with the constraints imposing lower and upper bounds for $\tilde{p}_{it}$.

$\omega_{lt}, \zeta_{lt}$  Dual variables associated with the constraints relating line flows and phase angles for line $l$ in period $t$ in the lower level of the oracle problem.

$\pi_{lt}, \sigma_{lt}$  Dual variables associated with the constraints imposing lower and upper bounds for $\tilde{f}_{lt}$.

Functions

$f(\cdot)$  Vector of linear functions defining the set of contingency states.

$C_{it}^P(\cdot)$  Production cost function offered by generator $i$ in period $t$.

Sets

$B$  Set of bus indices $b$.

$C$  Set of contingency state indices $c$. The pre-contingency state is represented by $c = 0$.

$C_k$  Set of contingency state indices $c$ considered at outer-loop iteration $k$.
Feasibility set for the decision variables associated with generator $i$.

Set of generator indices $i$.

Set of indices $i$ of generators located at bus $b$.

Set of transmission line indices $l$.

Set of indices $l$ of switchable transmission lines.

Set of time period indices $t$.

Bus where generator $i$ is located.

Origin bus index of line $l$.

Destination bus index of line $l$.

**Decision Variables**

$\Phi$, $\Phi^{ap}$ Levels of system power imbalance resulting from the oracle problem and the inner master problem.

$\Phi_{bt}^-, \Phi_{bt}^+$ Variables used in the linearization of the absolute value of the power imbalance at bus $b$ in period $t$ under contingency state $c$.

$\Phi^w$ Worst-case system power imbalance.

$\theta_{bt}^c, \tilde{\theta}_{bt}$ Phase angles at bus $b$ in period $t$ under contingency state $c$ and in the lower level of the oracle problem.

$\tilde{\Phi}_{bt}^-, \tilde{\Phi}_{bt}^+$ Variables used in the linearization of the absolute value of the power imbalance at bus $b$ in period $t$ in the lower level of the oracle problem.

$a_{it}$ Binary variable that is equal to 1 if generator $i$ is available in period $t$, being 0 otherwise.

$a_{lt}$ Binary variable that is equal to 1 if transmission line $l$ is available in period $t$, being 0 otherwise.

$c_{it}^{ed}, c_{it}^{su}$ Shut-down and start-up costs of generator $i$ in period $t$.

$f_{lt}^c, \tilde{f}_{lt}$ Power flows of line $l$ in period $t$ under contingency state $c$ and in the lower level of the oracle problem.

$p_{it}^c, \tilde{p}_{it}$ Power outputs of generator $i$ in period $t$ under contingency state $c$ and in the lower level of the oracle problem.

$r_{it}^{dn}, r_{it}^{up}$ Down- and up-spinning reserve contributions of generator $i$ in period $t$.

$v_{it}$ Binary variable that is equal to 1 if generator $i$ is scheduled in period $t$, being 0 otherwise.

$z_{lt}^c, \tilde{z}_{lt}$ Binary variables that are equal to 1 if transmission line $l$ is connected in period $t$, being 0 otherwise, under contingency state $c$ and in the lower level of the oracle problem.
1 Introduction

Unit commitment (UC) is one of the main tools used by power system operators to manage energy resources one day ahead. The goal of this optimization problem is to schedule and dispatch generators to meet future demand with minimum operating costs [1-3]. In addition to supplying the demand, system operators have to consider the effect of contingencies and load fluctuations by co-optimizing energy with ancillary services, such as reserves and ramping [4].

One of the variants for UC which most accurately represent the real-world problem is contingency-constrained unit commitment (CCUC) [5-12]. While it has been shown that other formulations can lead to suboptimal or infeasible solutions once contingencies occur [5], CCUC ensures reserve deliverability, as the redispach of generators and power flows under each contingency state are explicitly considered. Furthermore, CCUC provides a suitable modeling framework for considering tight security criteria. An \( n - K \) criterion was first accounted for in the CCUC model proposed in [6], which was extended in [7] by including network constraints and transmission line contingencies. In [8], an adjustable robust optimization model was proposed, in which the worst-case contingency is found for any given pre-contingency schedule and the best corrective actions are determined to minimize the system power imbalance. Recent works also take into account renewable generation uncertainty, fast-acting units, and other practical aspects [9-12].

An operational feature that has also been studied is the modification of the transmission network’s topology through the so-called transmission switching (TS) or topology control, whereby lines can be switched on and off by the operator in each time period. Although the idea of switching off a functioning transmission line might seem odd, it has been shown to potentially reduce operating costs significantly [13-18]. This result stems from Kirchhoff’s voltage law, which adds an additional constraint to the problem for every loop in the system. By removing certain loops, it is possible to relieve system constraints and unlock cheap reserves.

In [13], Fisher et al. presented a generation dispatch model with TS that resulted in significant cost savings. The model described in [13] was extended in [14] with the addition of an \( n - 1 \) security criterion. Despite the consideration of contingencies, TS actions were only implemented in the pre-contingency state. Thus, we label them as preventive TS, in contrast to corrective TS, which comprises post-contingency TS actions. In [15], preventive TS was brought to the UC framework while also considering an \( n - 1 \) criterion. In [16], a master-subproblem structure was utilized to solve the CCUC problem with preventive TS. In [17], preventive and corrective TS were jointly considered for the first time. To that end, a single-period contingency-dependent model was devised wherein system operation was explicitly modeled for all possible contingencies. Finally, in [18], corrective TS was studied in a security-constrained UC framework. However, the corrective TS was conducted only after the UC was solved. This represents a major simplification, as TS does not affect the UC decisions and is, thus, not co-optimized with the scheduling of energy and reserves. Despite the relevance of existing works [15-18], new techniques are yet needed to address the co-optimization of energy, reserves, and TS actions in CCUC problems.
In this paper, we extend the works described in [15–18] by proposing a novel approach for CCUC with TS. Unlike [15,16], and [18], both preventive and corrective TS (PC-TS) actions are considered. In contrast to [18], the co-optimization of energy and reserve offers is accounted for. Moreover, the proposed model departs from [17] by precisely incorporating inter-temporal operational constraints. In addition, the contingency set is not limited to that associated with the $n - 1$ security criterion commonly adopted in the literature. From a methodological perspective, the proposed approach also differs significantly from the related literature. Due to its dimension, the resulting mixed-integer program is unsuitable for the state-of-the-art branch-and-cut algorithm used in [15] and [17]. Furthermore, the use of binary variables for corrective TS actions precludes the application of the method adopted in [16], which is based on Benders decomposition [19]. As an alternative to the methods used in [15–18], we utilize an exact decomposition technique based on the nested column-and-constraint generation algorithm (CCGA) proposed in [20], which exploits the countability of the set of possible corrective TS actions to devise an additional inner loop. Thus, finite convergence to optimality is guaranteed and a measure of the distance to the optimum is provided along the iterative process. The nested CCGA has been successfully applied to address other UC models wherein binary corrective actions were related to the operation of energy storage [21] and fast-acting units [22].

The main contributions of this paper are threefold:

1. To the best of our knowledge, a CCUC model is proposed, for the first time in the literature, to co-optimize energy and reserve offers, as well as PC-TS actions, while considering inter-temporal operational constraints and a general framework for security criteria.

2. An exact decomposition methodology is applied and shown to successfully address instances that are unable to be handled in practical time by straightforwardly solving the full problem.

3. A study of the benefits of PC-TS is presented for the IEEE 30-bus and 118-bus systems under several security criteria and a day-ahead setting. This study includes the quantification of potential reductions in offer costs and levels of power imbalance, as well as an analysis of the number of switchable lines required to effectively decrease the worst-case imbalance. It is worth emphasizing that previous approaches do not allow conducting such a study with current computing capabilities.

The remainder of this paper is organized as follows. In Section II, the CCUC problem with PC-TS is formulated. In Section III, the solution methodology is presented and explained. In Section IV, numerical results are provided and discussed. Finally, conclusions are drawn in Section V.
2 Problem Formulation

The proposed model is cast as a mixed-integer program driven by the minimization of the sum of the offer costs and the worst-case power imbalance cost:

$$\text{Minimize} \quad \sum_{t \in T} \sum_{i \in I} \left[ C_{pt}^{l} (p_{it}^0, v_{it}) + C_{up}^{l} \cdot r_{it}^{up} + C_{dn}^{l} \cdot r_{it}^{dn} + c_{it}^s + c_{it}^d \right] + C^T \Phi^w$$  \hspace{1cm} (1)

subject to:

$$\Phi^w \geq \sum_{t \in T} \sum_{b \in B} (\Phi_{bt}^- + \Phi_{bt}^+) \quad \forall c \in C$$  \hspace{1cm} (2)

$$\sum_{i \in I} p_{it}^0 + \sum_{t \in F_{l}(t)=b} f_{it}^l - \sum_{t \in F_{r}(t)=b} f_{it}^c = d_{bt} + \Phi_{bt}^- - \Phi_{bt}^+; \quad \forall b \in B, \forall t \in T, \forall c \in C$$  \hspace{1cm} (3)

$$- M_{il} (1 - A_{it}^c z_{it}^c) \leq f_{it}^l - \frac{1}{x_l} (\theta_{F_{r}(t)} - \theta_{F_{l}(t)}); \quad \forall l \in L, \forall t \in T, \forall c \in C$$  \hspace{1cm} (4)

$$A_{it}^c z_{it}^c \Phi^w \leq f_{it}^l \leq A_{it}^c z_{it}^l \Phi^w; \quad \forall l \in L, \forall t \in T, \forall c \in C$$  \hspace{1cm} (5)

$$z_{it}^c = 1; \quad \forall l \in L \setminus L^{TS}, \forall l \in T, \forall c \in C$$  \hspace{1cm} (6)

$$v_{it} A_{it}^c \Phi^w \leq p_{it}^0 \leq v_{it} A_{it}^c \Phi^w; \quad \forall i \in I, \forall t \in T, \forall c \in C$$  \hspace{1cm} (7)

$$A_{it}^c (p_{it}^0 - r_{it}^{dn}) \leq p_{it}^0 \leq A_{it}^c (p_{it}^0 + r_{it}^{up}); \quad \forall i \in I, \forall t \in T, \forall c \in C$$  \hspace{1cm} (8)

$$0 \leq r_{it}^{up} \leq \Phi^w; \quad \forall i \in I, \forall t \in T$$  \hspace{1cm} (9)

$$0 \leq r_{it}^{dn} \leq \Phi^w; \quad \forall i \in I, \forall t \in T$$  \hspace{1cm} (10)

$$p_{it}^0 - p_{it-1}^0 \leq RU_i (v_{it} - v_{it-1}) + \Phi^w; \quad \forall i \in I, \forall t \in T$$  \hspace{1cm} (11)

$$p_{it-1}^0 - p_{it}^0 \leq RD_i (v_{it} - v_{it-1}) + \Phi^w; \quad \forall i \in I, \forall t \in T$$  \hspace{1cm} (12)

$$\Phi_{bt}^- \geq 0, \Phi_{bt}^+ \geq 0; \quad \forall b \in B, \forall t \in T, \forall c \in C$$  \hspace{1cm} (13)

$$z_{it}^c \in \{0,1\}; \quad \forall l \in L^{TS}, \forall l \in T, \forall c \in C$$  \hspace{1cm} (14)

$$v_{it} \in \{0,1\}; \quad \forall i \in I, \forall t \in T.$$

The objective function minimized in (1) comprises the offered costs of pre-contingency power generation, up- and down-spinning reserve allocation, start-ups, and shut-downs, as well as the cost of the worst-case power imbalance. Constraints (2) ensure that \( \Phi^w \) represents the worst-case power imbalance by making it greater than or equal to the imbalance under every contingency state. Based on (23), a dc power flow is modeled by expressions (3), characterizing power balances, and (4)–(5), representing line flows while taking into account PC-TS and line availability. Line flow capacity limits are modeled in (6). As per (7), non-switchable lines are always switched on. In (8), power outputs are bounded. In (9), the relationship between power outputs and reserve contributions is characterized. Note that generator availability is considered in (8) and (9). In (10) and (11), bounds on up- and down-spinning reserve
contributions are respectively imposed. Expressions (12) and (13) model the ramping limitations, which are solely enforced in the pre-contingency state, as done in [9, 12, 15, 16, 21]. Start-up and shut-down offer costs as well as minimum up and down times are formulated in (14) in a compact way; more details can be found in [24]. Constraints (15) ensure the non-negativity of the variables used in the linearization of the absolute value of the power imbalance. Finally, PC-TS and generation scheduling are modeled by binary variables in (16) and (17), respectively. In this formulation, the contingency state $c = 0$ represents the pre-contingency state, wherein all generators and transmission lines are available.

In (1)–(17), contingency states associated with the prescribed security criterion are characterized through parameters $A_{c}$ and $A_{c}^{t}$. Thus, a compact formulation for the security criterion is as follows:

$$f\left(\{A_{c}^{t}\}_{t \in I}, \{A_{c}^{t}\}_{l \in L}\right) \geq 0; \quad \forall t \in T, \forall c \in C$$  \hspace{1cm} (18)

where $f(\cdot)$ is a set of linear constrained functions.

As an example, for an $n-K$ criterion, expressions (18) become:

$$\sum_{i \in I} A_{c}^{t} + \sum_{l \in L} A_{c}^{t} \geq |I| + |L| - K; \quad \forall t \in T, \forall c \in C.$$  \hspace{1cm} (19)

Similarly, an $n-K^{G}-K^{L}$ criterion, where $K^{G}$ and $K^{L}$ denote the number of out-of-service generators and transmission lines, respectively, gives rise to:

$$\sum_{i \in I} A_{c}^{G} \geq |I| - K^{G}; \quad \forall t \in T, \forall c \in C.$$  \hspace{1cm} (20)

$$\sum_{l \in L} A_{c}^{L} \geq |L| - K^{L}; \quad \forall t \in T, \forall c \in C.$$  \hspace{1cm} (21)

3 Solution Methodology

Straightforwardly solving problem (1)–(17) may be computationally intractable due to the need to explicitly model system operation under all contingency states associated with the pre-specified security criterion.

Alternatively, in the recent CCUC literature, the concept of umbrella contingencies [25, 26] has been widely utilized to devise decomposition techniques for problems structurally similar to (1)–(17). The idea that only a small portion of the set of credible contingencies $C$ needs to be considered in order to achieve the optimal solution has allowed the application of master-oracle structures such as Benders decomposition [19] and the CCGA [27] to CCUC. Hence, such decomposition methods have become the standard solution procedures for CCUC [6–9, 12]. In this context, the oracle problem, i.e., the subproblem, is a bilevel program responsible for finding the worst-case contingency state for a given schedule provided by the preceding master problem. Such a contingency state is then inserted into the master problem, which outputs a new schedule. The algorithm converges through the update of the upper and lower bounds given by the solutions obtained at each iteration.

Unfortunately, the presence of binary variables associated with corrective TS in the lower level of the oracle problem makes problem (1)–(17) unsuitable for the standard single-loop master-oracle structures.
used for CCUC \[19\[27\]. As a salient methodological feature, we propose the novel application of the nested CCGA \[20\], which is an exact decomposition technique that consists of two CCGA loops.

3.1 Outer Loop

The outer loop represents the master-oracle structure that is iterated until convergence to determine the solution of the original problem \((1)–(17)\). The outer loop converges once the bounds provided by the master problem and the oracle problem are within an outer-loop tolerance \(\epsilon^o\).

3.1.1 Master Problem

The master problem is a relaxation of the original problem \((1)–(17)\) where, at each outer-loop iteration \(k\), \(C\) is replaced with a subset of contingency states \(C_k\). Solving the master problem yields decisions \(p^0(k)_{it}, v^k_{it}, r^u(k)_{it}, r^d(k)_{it}\), which represent the optimal schedule for the set of states \(C_k\). Since the master problem constitutes a relaxation of the original problem, at each outer-loop iteration \(k\), its solution allows computing a lower bound for the optimal value of the objective function \((1)\):

\[
LB(k) = \sum_{t \in T} \sum_{i \in I} \left[ C^P_{it} (P^0(k)_{it}, v^k_{it}) + C^{up}_{it} r^{up}(k)_{it} + C^{dn}_{it} r^{dn}(k)_{it} + c^u(k)_{it} + c^d(k)_{it} \right] + C^f \Phi^{w(k)}. \tag{22}
\]

3.1.2 Oracle Problem

The goal of the oracle problem is to identify the worst-case contingency state for a given schedule obtained by the preceding master problem. To that end, availability parameters \(A^c_{it}\) and \(A^c_{lt}\) are replaced with binary variables \(a_{it}\) and \(a_{lt}\), respectively, and the worst-case setting is implemented by a bilevel programming framework \[7\[10\[12\]. As a result, system operation under contingency is implicitly modeled and, hence, indices \(c\) are dropped. In the bilevel oracle problem, the upper level is responsible for finding the contingency state maximizing the power imbalance, while the lower level obtains the optimal system reaction. The oracle problem for \((1)–(17)\) is presented below. For the sake of clarity, a tilde is used to denote the decision variables modeling system operation under contingency, whereas dual variables are shown in parentheses.

\[
\Phi^{k} = \max_{a_{it}, a_{lt}} \min_{\delta_{bt}, \Phi^+_{bt}, \Phi^-_{bt}} \sum_{t \in T} \sum_{b \in B} \left( \Phi^+_bt + \Phi^-bt \right) \tag{23}
\]

subject to:

\[
a_{it} \in \{0, 1\}; \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \tag{24}
\]

\[
a_{lt} \in \{0, 1\}; \quad \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{25}
\]

\[
f(\{a_{it}\}_{i \in \mathcal{I}}, \{a_{lt}\}_{l \in \mathcal{L}}) \geq 0; \quad \forall l \in \mathcal{T} \tag{26}
\]

\[
\sum_{i \in \mathcal{I}_b} \tilde{p}_{it} + \sum_{l \in \mathcal{L} | to(l) = b} \tilde{f}_{lt} - \sum_{l \in \mathcal{L} | fr(l) = b} \tilde{f}_{lt} = d_{bt} + \Phi^-_{bt} - \Phi^+_{bt} : (\beta_{bt}); \quad \forall b \in \mathcal{B}, \forall t \in \mathcal{T} \tag{27}
\]

\[
-M_t(1 - a_{lt} \tilde{z}_{lt}) \leq \tilde{f}_{lt} - \frac{1}{x_l} (\tilde{\theta}_{fr(l)l} - \tilde{\theta}_{to(l)l}) : (\omega_{lt}); \quad \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{28}
\]
\begin{align*}
\hat{f}_{lt} - \frac{1}{x_l} (\hat{\theta}_{fr(t)t} - \hat{\theta}_{tot(t)t}) &\leq M_l (1 - a_{lt}\tilde{z}_{lt}) : (\zeta_{lt}) \quad \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \\
- a_{lt}\tilde{z}_{lt} \bar{F}_l &\leq \hat{f}_{lt} \leq a_{lt}\tilde{z}_{lt} \bar{F}_l : (\pi_{lt}, \sigma_{lt}) \quad \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \\
\bar{z}_{lt} &\equiv 1; \quad \forall l \in \mathcal{L} \setminus \mathcal{L}^{TS}, \forall t \in \mathcal{T} \\
a_{lt} (p_{lt}^{0(k)} - r_{lt}^{dn(k)}) &\leq \bar{p}_{lt} \leq a_{lt} (p_{lt}^{0(k)} + r_{lt}^{up(k)}) : (\gamma_{lt}, \chi_{lt}) \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \\
\bar{\Phi}_{lt}^b &\geq 0, \bar{\Phi}_{lt}^b \geq 0; \quad \forall b \in \mathcal{B}, \forall t \in \mathcal{T} \\
\bar{z}_{lt} &\in \{0, 1\}; \quad \forall l \in \mathcal{L}^{TS}, \forall t \in \mathcal{T}.
\end{align*}

In (23), the max-min structure allows identifying the contingency state resulting in the maximum power imbalance while taking into consideration the best reaction, i.e., that minimizing the imbalance for every possible contingency state. Expressions (24) and (25) characterize the binary variables representing the availability of generators and transmission lines, respectively. In (26), the prescribed security criterion is enforced using the vector \( f(\cdot) \) explained in Section 2. Constraints (27) – (29) correspond to the dc power flow model. Bounds for post-contingency line flows are set in (30). In (31), non-switchable lines are forced to be switched on. Constraints (32) impose bounds on post-contingency power outputs based on the allocated reserves. Finally, (33) and (34) define the variables modeling power imbalance and corrective TS, respectively.

Note that, at each outer-loop iteration \( k \), \( \Phi^{(k)} \geq \Phi^{w(k)} \). Hence, the following upper bound for the optimal cost can be derived:

\[ UB^{(k)} = \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} \left[ C_{it}^{p} (p_{it}^{0(k)}, v_{it}^{(k)}) + C_{it}^{up} r_{it}^{up(k)} + C_{it}^{dn} r_{it}^{dn(k)} + c_{it}^{su} r_{it}^{su(k)} + c_{it}^{sd} r_{it}^{sd(k)} \right] + C^f \Phi^{(k)}. \]

### 3.2 Inner Loop

At each outer-loop iteration \( k \), an inner master problem and an inner subproblem are iterated until convergence to determine the solution of the bilevel oracle problem (23) – (34). The inner loop converges once the bounds provided by the inner master problem and the inner subproblem are within an inner-loop tolerance \( \epsilon^i \).

#### 3.2.1 Inner Subproblem

The inner subproblem comprises the lower-level optimization of the oracle problem for a given contingency state, i.e., the minimization in (23) subject to constraints (27) – (34) where \( a_{lt} \) and \( a_{lt} \) are replaced with the optimal values provided by the previous inner master problem. At each inner-loop iteration \( m \), the solution to the inner subproblem provides a lower bound for the optimal value of the objective function optimized in the oracle problem. The optimal line switching decisions resulting from the inner subproblem at inner-loop iteration \( m \), \( \tilde{z}_{lt}^{(m)} \), are fed to the following inner master problem.
3.2.2 Inner Master Problem

The inner master problem represents a single-level relaxation of (23)–(34). Following the methodology presented in [20], the inner master problem at outer-loop iteration \( k \) and inner-loop iteration \( j \) is formulated as follows:

\[
\begin{align*}
\text{Maximize} & \quad \Phi^o_p \\
\text{subject to:} & \quad \phi^o_p \leq \sum_{i \in \mathcal{I}} \left\{ \sum_{b \in \mathcal{B}} \beta^m_{bi} d_{bi} + \sum_{l \in \mathcal{L}} \left[ \omega^m_{li} M_l (a_{li} \cdot \tilde{z}^m_{li} - 1) + \zeta^m_{li} M_l (a_{li} \cdot \tilde{z}^m_{li} - 1) \\
& \quad - \pi^m_{li} a_{li} \tilde{z}^m_{li} + \sigma^m_{li} a_{li} \tilde{z}^m_{li} \right] + \sum_{j \in \mathcal{J}} \left[ \chi^m_{ij} a_{ij} (p_{ij}^0(k) - r_{ij}^d(k)) \\
& \quad - \gamma^m_{ij} a_{ij} (p_{ij}^0(k) + s_{ij}^a(k)) \right] \right\}; \quad m = 1, \ldots, j
\end{align*}
\]

Constraints (36)–(44)

Expression (37) contains nonlinearities in the form of products of decision variables. Using the algebraic results presented in [28], these bilinear terms can be linearized and, thus, the inner master problem can be cast as a single-level mixed-integer linear program. The optimal values of variables \( a_{ij} \) and \( a_{li} \) obtained from the resolution of problem (36)–(44) are used as parameters in the subsequent inner subproblem. Since the inner master problem is a relaxation of (23)–(34), its solution allows computing an upper bound for the optimal value of the objective function optimized in the oracle problem.

3.3 Algorithm Overview

Fig. 1 presents a flowchart of the proposed solution methodology. The fact that \( |\mathcal{C}_k| \ll |\mathcal{C}| \) addresses the problem of having to explicitly consider a prohibitive number of contingency states. The key idea is to add to \( \mathcal{C}_k \) the worst-case state identified by the oracle problem at each outer-loop iteration \( k \). The master problem converges to the full problem, as \( \mathcal{C}_k \) eventually becomes equal to \( \mathcal{C} \) should all possible contingency states be examined. It is worth emphasizing that the main advantage of this methodology is that attaining the optimal solution to the original problem (1)–(17) generally requires considering a small subset of contingency states. The nested iterative process is stopped when the difference between the...
upper and lower cost bounds is less than or equal to a pre-specified outer-loop tolerance $\epsilon^o$. For further details on the nested CCGA, we refer the interested reader to [20].

4 Numerical Results

In order to assess the benefits of PC-TS, numerical tests were conducted on a modified version of the IEEE 30-bus system and on the IEEE 118-bus system. In all cases, it was assumed that producers offer affine cost functions of the form $C_p^{it}(p_{it}^0, v_{it}) = C_{it}^f v_{it} + C_{it}^0 p_{it}^0$. For the sake of reproducibility, system data are provided in [29]. The cost of power imbalance $C^I$ was set to 10 times the cost of the most expensive generator. The outer-loop tolerance $\epsilon^o$ was set to 1%, while the inner-loop tolerance $\epsilon^i$ was set to 1 MW. For the 30-bus system, contingencies were considered for all 10 generators and 45 transmission lines. For the 118-bus system, contingencies were considered for the 17 generators with rated power capacity above 200 MW and for the 12 tie lines connecting the three areas in which the system can be split [30]. All tests were conducted in Julia language, utilizing the JuMP framework [31] and CPLEX 12.8, on an Intel Core i7-490K processor at 4.00 GHz and 32 GB of RAM.

4.1 Impact of PC-TS

Tables 1 and 2 summarize the impact of PC-TS for the IEEE 30-bus and 118-bus systems, respectively, over a single-period time span and under several security criteria. Both tables list the offer costs and the worst-case percent levels of power imbalance in terms of system load, as well as the total costs, which represent the values of the objective function (1), i.e., the offer cost plus the cost of power imbalance.
Table 1: Impact of PC-TS for the IEEE 30-Bus System

<table>
<thead>
<tr>
<th>Security Criterion</th>
<th>n - 1</th>
<th>n - 1 - 1</th>
<th>n - 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>No TS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Offer Cost ($)</td>
<td>52,077</td>
<td>50,318</td>
<td>50,253</td>
</tr>
<tr>
<td>Worst Imbalance (%)</td>
<td>3.5</td>
<td>4.2</td>
<td>7.8</td>
</tr>
<tr>
<td>Total Cost ($)</td>
<td>84,577</td>
<td>89,318</td>
<td>121,753</td>
</tr>
<tr>
<td>PC-TS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Offer Cost ($)</td>
<td>51,106</td>
<td>52,157</td>
<td>49,176</td>
</tr>
<tr>
<td>Worst Imbalance (%)</td>
<td>1.2</td>
<td>3.5</td>
<td>6.2</td>
</tr>
<tr>
<td>Total Cost ($)</td>
<td>62,484</td>
<td>84,657</td>
<td>106,260</td>
</tr>
</tbody>
</table>

Table 2: Impact of PC-TS for the IEEE 118-Bus System

<table>
<thead>
<tr>
<th>Security Criterion</th>
<th>n - 1</th>
<th>n - 1 - 1</th>
<th>n - 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>No TS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Offer Cost ($)</td>
<td>70,070</td>
<td>70,600</td>
<td>80,004</td>
</tr>
<tr>
<td>Worst Imbalance (%)</td>
<td>0.0</td>
<td>0.0</td>
<td>2.6</td>
</tr>
<tr>
<td>Total Cost ($)</td>
<td>70,070</td>
<td>70,600</td>
<td>177,399</td>
</tr>
<tr>
<td>PC-TS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Offer Cost ($)</td>
<td>69,638</td>
<td>70,181</td>
<td>76,140</td>
</tr>
<tr>
<td>Worst Imbalance (%)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Total Cost ($)</td>
<td>69,638</td>
<td>70,181</td>
<td>76,140</td>
</tr>
</tbody>
</table>

The results reported for the cases with no TS correspond to problem (1)–(17) wherein $L^{TS}$ is an empty set. On the other hand, for the cases with PC-TS, all lines are switchable, i.e., $L^{TS} = L$.

As can be observed, PC-TS yields reductions in the total cost ranging between 0.6% and 57.1%. In general, such reductions result from a decrease in both the offer costs and the worst-case levels of power imbalance. The 118-bus system under an $n - 2$ criterion deserves special mention, as PC-TS ensures power balance for all possible contingency states within the pre-specified security criterion, while disregarding TS resulted in a 2.6% worst-case imbalance.

4.2 Number of Switchable Lines

Considering all lines as switchable may lead to prohibitive amounts of binary variables for larger instances, thereby rendering the problem intractable. However, it can be shown that a small number of switchable lines is sufficient to obtain significant improvements over the case with no TS, even resulting in the same power imbalance reduction as when all lines are switchable. This finding may be beneficial for practical implementation purposes due to the considerable decrease in the number of binary variables of the resulting optimization problem.

Determining the smallest set of switchable lines yielding the best imbalance reduction may be computationally impractical, as it would require examining all combinations of switchable lines. As an alternative, we devise an iterative heuristic to rapidly obtain a reduced set that provides the best imbalance. At the first iteration, problem (1)–(17) is solved considering each transmission line as the only switchable line, thereby retrieving the single best switchable line in terms of power imbalance reduction.
At subsequent iterations, the process continues by testing all possible sets of switchable lines comprising those in the best set found at the previous iteration and each other single line. Thus, the best set of switchable lines is extended with one line per iteration until the best imbalance reduction is achieved. Hence, only $O(|L|)$ instances need to be solved, as opposed to the aforementioned combinatorial number. Furthermore, the minimum number of switchable lines necessary to obtain the best imbalance reduction is generally small. This result suggests that this iterative process can be conducted in practical time.

This heuristic was run for the instances yielding power imbalance with no TS (Tables 1 and 2). For such instances, Fig. 2 presents the worst-case percent power imbalance, in terms of system load, for each number of switchable transmission lines. As can be seen, a significant imbalance reduction can be obtained with the use of only a few switchable lines. For the 30-bus system, only one switchable line is necessary to achieve the best power imbalance reduction, while, for the 118-bus system, a subset of just five out of 186 lines is enough to guarantee power balance for all contingency states. More specifically, the resulting reduced sets of switchable lines are: line 1-3 for the 30-bus system under $n - 1$ and $n - 2$ criteria, line 4-6 for the 30-bus system under an $n - 1 - 1$ criterion, and lines 3-5, 7-12, 8-37, 11-12, and 17-30 for the 118-bus system under an $n - 2$ criterion.

### 4.3 Need for Decomposition

Next, with the goal of illustrating the computational viability of the proposed decomposition method, referred to as N-C CGA, we have solved the same six instances examined in Section 4.1 with off-the-shelf branch-and-cut software directly applied to problem (1)-(17). For quick reference, the latter approach is denoted by BC. For both approaches, the computing times required when all lines are deemed as switchable are compared with those required when the reduced sets of switchable lines reported in Section 4.2 are considered. The execution of BC was stopped when either a solution was found within a 1% optimality tolerance or a timeout limit of 60 min was reached. The results shown in Table 3 evidence...
Table 3: Computing Times for BC and N-CCGA (s)

<table>
<thead>
<tr>
<th>System</th>
<th>Approach</th>
<th>Security Criterion</th>
<th>( L^{TS} )</th>
<th>( n - 1 )</th>
<th>( n - 1 - 1 )</th>
<th>( n - 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>30-bus</td>
<td>BC</td>
<td>( L )</td>
<td>1,154.3</td>
<td>Timeout</td>
<td>Timeout</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Subset of ( L )</td>
<td></td>
<td>7.4</td>
<td>19.3</td>
<td>43.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>N-CCGA</td>
<td>( L )</td>
<td>32.1</td>
<td>8.9</td>
<td>13.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Subset of ( L )</td>
<td></td>
<td>1.9</td>
<td>7.1</td>
<td>8.5</td>
<td></td>
</tr>
<tr>
<td>118-bus</td>
<td>BC</td>
<td>( L )</td>
<td>Timeout</td>
<td>Timeout</td>
<td>Timeout</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Subset of ( L )</td>
<td></td>
<td>49.8</td>
<td>Timeout</td>
<td>Timeout</td>
<td></td>
</tr>
<tr>
<td></td>
<td>N-CCGA</td>
<td>( L )</td>
<td>6.2</td>
<td>3.2</td>
<td>565.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Subset of ( L )</td>
<td></td>
<td>1.7</td>
<td>1.8</td>
<td>16.2</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Results for the IEEE 118-Bus System over a 24-Hour Time Span

<table>
<thead>
<tr>
<th>Security Criterion</th>
<th>( n - 1 )</th>
<th>( n - 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Offer Cost ($)</td>
<td>1,359,845</td>
<td>1,580,116</td>
</tr>
<tr>
<td>Worst Imbalance (%)</td>
<td>0.0</td>
<td>1.62</td>
</tr>
<tr>
<td>Total Cost ($)</td>
<td>1,359,845</td>
<td>2,788,954</td>
</tr>
<tr>
<td>Computing Time (s)</td>
<td>536</td>
<td>1,193</td>
</tr>
<tr>
<td>PC-TS</td>
<td>Offer Cost ($)</td>
<td>1,355,532</td>
</tr>
<tr>
<td></td>
<td>Worst Imbalance (%)</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>Total Cost ($)</td>
<td>1,355,532</td>
</tr>
<tr>
<td></td>
<td>Computing Time (s)</td>
<td>930</td>
</tr>
</tbody>
</table>

*4.5% optimality gap.

that N-CCGA is significantly faster than BC and that the use of reduced sets of switchable lines is considerably advantageous over the case with \( L^{TS} = L \). More importantly, N-CCGA can solve, in a matter of seconds, instances for which BC failed to find a feasible solution within the allotted time.

4.4 Multi-Period Cases

Finally, we have tested the performance of the proposed N-CCGA on a 24-hour setting for the IEEE 118-bus system under \( n - 1 \) and \( n - 2 \) security criteria considering lines 3-5, 7-12, 8-37, 11-12, and 17-30 as switchable. Note that such multi-period instances of CCUC with PC-TS, even with reduced \( L^{TS} \), are intractable for BC. For illustration purposes, we compare the costs and power imbalance levels obtained without TS and with PC-TS. The results presented in Table 4 corroborate the capability of PC-TS to substantially reduce total costs, offer costs, and power imbalance levels.

As for computing times, it is worth highlighting that the \( n - 1 \) case with PC-TS required around 15 min on a regular computer, which is acceptable for day-ahead operation. The \( n - 2 \) case with PC-TS represents a much larger instance, and, admittedly, attaining a solution with a small optimality gap requires extensive computational effort. As an example, the solution reported in Table 4 featuring a
4.5% optimality gap, took almost 6 hours. This relatively large optimality gap notwithstanding, it is remarkable that the resulting schedule yields a significant 44.0% total cost reduction over the case with no TS, largely due to the power imbalance decrease from 1.62% to 0.16%. As a matter of fact, the proposed approach allows achieving substantial total cost reductions over the solution with no TS in practical time frames. This aspect is illustrated in Fig. 3 which depicts the evolution, with computing time, of the total cost incurred by the best solution found by N-CCGA over the optimal total cost with no TS. For this particular instance, a feasible solution yielding a 27.7% total cost reduction is attained after only 41 min, which is well within the several hours typically allotted in industry practice [32–34]. This promising result backs the interest of the proposed approach to system operators.

5 Conclusion

This paper has addressed the contingency-constrained unit commitment problem for the co-optimization of energy, reserves, as well as pre- and post-contingency transmission switching. For the first time in the literature, all the aforementioned features have been considered in a multi-period setting under general security criteria. Straightforwardly solving the proposed formulation proves to exceed current computing capabilities even for small instances. To address this issue, a decomposition method based on the nested column-and-constraint generation algorithm is applied. The solution methodology involves an outer loop wherein the original problem is decomposed into a master-oracle structure. The resulting bilevel oracle problem is responsible for obtaining the contingency state yielding the largest power imbalance for a given schedule. The presence of lower-level binary variables in the oracle problem is handled by an inner loop involving an inner master problem and an inner subproblem.

The numerical results presented in this work allow drawing four main conclusions:
1. Preventive and corrective transmission switching benefit system operation by consistently reducing the power imbalance in the worst contingency states, as well as energy and reserve costs.

2. It is possible to obtain the same power imbalance reduction with a small number of switchable lines.

3. The proposed decomposition methodology significantly outperforms the state-of-the-art branch-and-cut algorithm in terms of computing time.

4. Moderate computational effort is required to attain substantial cost savings over the no-TS solution for a medium-scale benchmark such as the IEEE 118-bus system over a 24-hour time span.

Ongoing research is focused on the incorporation of valid constraints to the master problem in order to improve the performance of the decomposition procedure. Further work will explore the computational savings that may be gained from the use of efficient solution algorithms as well as parallel and cloud computing. Another interesting avenue of research is the consideration of the uncertainty associated with renewable energy sources.

References


