Hub Location and Route Dimensioning: Strategic and Tactical Intermodal Transportation Hub Network Design

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Abstract

We propose a novel hub location model that jointly eliminates the traditional assumptions on the structure of the network and on the discount due to economies of scale in an effort to better reflect real-world logistics and transportation systems. Our model extends the hub literature in various facets: instead of connecting non-hub nodes directly to hub nodes, we consider routes with stopovers; instead of connecting pairs of hubs directly, we design routes that can visit several hub nodes; rather than dimensioning pairwise connections, we dimension routes of vehicles; and rather than working with a homogeneous fleet, we use intermodal transportation. Decisions pertinent to strategic and tactical hub layout and transportation network design are concurrently made through the proposed optimization scheme. An effective branch and cut algorithm is developed to solve realistic size problem instances in the Turkish network and to provide managerial insights.

1 Introduction

Campbell and O’Kelly (2012) provide an excellent synthesis of the 25 years of the vast hub location research following the seminal works O’Kelly (1986a,b, 1987). While the will to solve instances of realistic sizes has geared the literature towards models with strong assumptions, the real challenges have been in defining and solving realistic problems. Integrating the strategic level hub location decisions with the tactical and operational level transportation network designs has been identified as a crucial, yet rather challenging research direction. We take up this challenge with this paper.

On par with the academic interest, the practical motivation for the hub network design is on the rise. The explosive growth in the business-to-consumer form of e-commerce has made logistics a prominent determinant in gaining the competitive edge in this market. Logistcs service providers have to distinguish themselves both in prices and service quality, which is mainly identified with short delivery times. Carriers, in general, employ hub-and-spoke networks in their many-to-many distribution systems to meet customers’ high service level expectations (i.e., faster delivery, larger coverage and higher availability) without additional costs (monetary and environmental) that customers are quite reluctant to face (Joerss et al., 2016). Strategically located operations centers can drastically increase the efficiency of the transportation network by providing opportunities to
consolidate freight from different originating branch offices, disseminate it according to destined branch offices and enable a seamless intermodal integration. With these critical capabilities, hub networks are at the core of the emerging revolutionary logistic concepts, such as physical internet (Montreuil, 2011; Crainic and Montreuil, 2016) and mobility as a service (MaaS) (Hietanen, 2014; Maheo et al., 2017).

As a test bed for our suggested modelling approach, we investigate the next-day delivery network design problem for a national package express company in Turkey that is about to launch into the market. As we discuss in more detail in Section 5, where we present the results of our numerical experiments, this setting presents an interesting example where hubs do not solely function as consolidation points but are also required to facilitate mode shifts (transfers) in a multimodal transportation network and the classical assumption of cost reduction in inter-hub transfer is not valid. In what follows, we consider this practical example to explain the details of the problem setting we study in this paper.

With the existing highway network structure and due to the geographical disparity of the population centers in Turkey, it is not possible to provide next day delivery services for every origin-destination pair without resorting to air transportation. As is customary in the operations of the cargo sector, each branch office will be assigned to an operations center (hub), where packages can be transferred between the vehicles (i.e., from trucks to planes or vice versa). To benefit from economies of scale, stopovers will be permitted both in the spoke and hub networks. Service quality will be ensured by limiting both the spoke and hub segments in terms of time and/or in terms of number of intermediate nodes (hops). The resulting network will be intermodal, utilizing various ground and air transportation means. To this end, a parcel that is picked up from its originating branch office will first travel on some form of ground transportation, perhaps visiting other demand centers on the way, till its designated hub, will then get transferred to another vehicle (possibly plane) that is either bypassing through or originating from the designated hub, traverse the hub network, perhaps visiting some stopover hubs on the way, till its destination’s designated hub, where it will get transferred to a smaller vehicle, and travel till its final destination with possibly visiting other demand centers on the way. In the worst case, the parcel will travel on three different vehicles and there will be a transfer of vehicles only at the hub nodes upon entering and leaving the hub network. Routes from demand centers to their designated hubs are called access routes and routes traversing the hub network are called hub routes. Choosing the access and hub routes to operate and determining the types and numbers of vehicles to assign to those paths (to build enough capacity) are the decisions for the transportation network design. On the other hand, locations of the hubs, as strategic level decisions, need to be determined to support the mentioned intermodal structure of the transportation network. Clearly, transportation mode switches (change of vehicles) can only be performed in “hub” nodes with proper sorting and packaging capabilities as well as the facilities that can process involved vehicles (i.e., planes, ships, trains, trucks) together. As such, the hub deployment decisions are strongly connected with the transportation network design decisions and need to be considered jointly, as we do with our model.

As explained above, our methodology will aid in strategical, tactical, and operational level plans. In particular, our model provides the answers for the following questions.

- What are the locations of hubs?
- Which is the designated hub of each demand center?
• How should the demand centers of a particular hub be partitioned to a collection of access routes?

• What should be the course of each vehicle used on access and hub routes?

• How many vehicles of a certain type (with a given capacity) should be used on each route?

The classical hub location problem has some intrinsic as well as extrinsic assumptions that challenge its match to realistic transportation settings. Since the seminal paper of O’Kelly (1987), many variants of the hub location problem have been studied with an effort to relax some of the assumptions of the basic problem (Campbell, 1996) to make it more realistic. This is also our aim in this study. In particular, we eliminate all of the following assumptions.

**Complete hub network:** In the classical version, hubs are assumed to be interconnected via direct links. An efficiently designed incomplete hub network may be cost beneficial without sacrificing service quality. Various research works in the literature have relaxed this assumption and tackled the design problem of the hub network (O’Kelly and Miller, 1994; Nickel et al., 2001; Yoon and Current, 2008; Calık et al., 2009; Alumur et al., 2009; Conterras et al., 2010). Most of these studies focus on designing a connected topology in the strategic level without detailing the underlying transportation network, i.e., the actual routes. We on the other hand design routes that visit a set of hub nodes and carry the flow originating from hubs visited earlier on the route destined to hubs that are visited later. Hub routes are served by capacitated vehicles and consequently, it may be necessary to allocate more than one vehicle on a hub route depending on the traffic. These routes are also bounded in length and/or in hops. The resulting hub network does not have any particular property other than being connected.

**Star access network:** The access network is typically star shaped in the classical hub location problem. In the single allocation variant, each non-hub node is allocated to exactly one hub node and all its traffic transits through this hub. In the multiple allocation variant, a node’s traffic can transit through all hubs. In both cases, the traffic from a node to another node passes through at least one hub and no other non-hub nodes. Consistent with the literature utilizing more realistic access networks with stopovers (Yaman et al. (2007)) or ring topologies (Rodríguez-Martín et al. (2014)), we shall also benefit from the consolidation of flows on our access routes carrying traffic of several non-hub nodes to their designated hub and back. These access routes may start and end at the hub as in the hub location and routing problem, or can start at a non-hub node and end at a hub node as in the hub location problem with stopovers. Each access route is served by a vehicle and consequently the total traffic that can be picked-up or delivered on it cannot exceed the capacity of the vehicle. In addition, the length of an access route is bounded to ensure quality of service. This bound can be in terms of the distance, travel time and/or the number of non-hub nodes on the route (hops). The cost of an access route is the cost of allocating a vehicle to this route. If routes start at non-hub nodes, the access network of a particular hub is the union of paths that cover a subset of non-hub nodes and end at the hub. If routes start and end at the hub node then each access network is a union of cycles.

**Constant discount factor:** In the classical hub location problem, the unit shipment cost on the hub network is discounted by a constant factor independent of the actual flow. Campbell (2013) observes that in optimal solutions of the basic hub location problems, it happens quite frequently that the connections between non-hub nodes and hub nodes carry more flow than hub-to-hub connections. In other words, the assumption that there is more traffic flow between hubs is usually
not verified and modelling the economics of scale by using a constant discount factor for inter-hub flows might not be representative in most cases. Several studies in the literature challenged the validity of this fundamental assumption and suggested different means to relax it. Podnar et al. (2002) discount the transportation cost on a link if the flow on this link exceeds a threshold. O'Kelly and Bryan (1998), Horner and O'Kelly (2001), de Camargo et al. (2009) model economies of scale as a function of flow on the inter-hub links. Campbell et al. (2005a,b), to this end, study the problem of locating a given number of hub arcs with discounted costs rather than locating hubs. We directly consider the operational cost of the vehicle fleet that will be chosen to handle the transportation in the designed network, i.e., the actual cost that will be incurred in real life.

Link based dimensioning of vehicles: Be it an incomplete or a complete hub network design, the common underlying inherent assumption of classical hub location is that one or more vehicles will be reserved to each link that will carry out round-trips between the two hub endpoints with some frequency. Even in the rare cases (such as Yaman and Carello (2005), Yaman (2005), Corberán et al. (2016) and Serper and Alumur (2016)) where vehicles that will carry the flow are also dimensioned, this dimensioning might be unnecessarily conservative since this association is done without considering the operations on the transportation end. The same vehicle can carry the flow on two consecutive hub links, provided the time limit allows. One can argue that once the hub network is strategically designed, the transportation network can be laid over it, through a subsequent optimization, with minimum cost. However, since the two designs will not be jointly made, the costs will not be reflecting the reality. Our designs jointly dimension and route the appropriate fleet carrying the demand, and as such, closely mimic the daily operations on the field as well. The unrealistic implicit assumption of incomplete designs that demand can be transferred indefinitely between vehicles traversing the hub network is replaced by the operational simplicity of pairwise hub demand being transferred without any intermediate handling on the stopover hubs.

Single means of transportation: Most of the existing hub location literature assumes a single type of transportation mode. Within the bunch operating with multimodes, the networks of different modes are typically assumed to be disjoint (as in Alumur et al. (2012b)) or hubs are reserved for single modes (as in Alumur et al. (2012a)). Even if there are studies (such as Serper and Alumur (2016)) that root for intermodal designs that dimension different modal vehicles, they leave out the routing issue of such vehicles.

Single objective: Models appearing in the literature typically design hub networks solely with cost or service level objectives. Studies on time definite deliveries such as Alumur et al. (2012b) and Campbell (2009) combine both cost and service level dimensions. To ensure quality of service, our access and hub routes are bounded. This bound can be in terms of the distance, travel time and/or the number of hops on the route. The cost of an access or a hub route is simply the operational cost of the vehicle assigned to it.

There is a large amount of literature on hub location problems. We refer the reader to Alumur and Kara (2008), Campbell and O’Kelly (2012), Contreras (2015), and Farahani et al. (2013) for overviews of the existing hub location literature. Here, we focused on the few representative studies on variants relaxing the listed assumptions. To the best of our knowledge, there is no study in the literature that jointly relaxes all these assumptions in an effort to better design real-world logistics and transportation systems.

Through our novel modeling approach, we manage to alleviate the drawbacks resulting from the challenges of relaxing the simplifying assumptions. Even though we work with incomplete designs, we only suffer from the material handling burden of the basic complete hub network designs and
the transfer between origin and destination hubs is virtually direct. Even though we do multimodal transportation, we still can keep track of the particular hub pair demand at the level of partition to different vehicles on the hub network. Even though we exert a hub covering type of restraint to provide a service quality, we manage to identify the actual transportation cost that this service will incur, and to this end, manage to integrate the cost and service bifurcation in the hub location models.

Against all this, we contribute the Hub Location and Route Dimensioning (HLRD) problem to the literature. Figure 1 contrasts the proposed network structure with that of a classical hub network. In both figures, circles and squares correspond to demand and hub locations, respectively. In Figure 1a, we see a classical design where the hub network is complete and each access network is a star. Figure 1b on the other hand depicts a typical HLRD design. The access networks consist of stopover paths visiting several demand locations. The hub network may be incomplete. Several vehicles with different capacities may carry the demand in the resulting network. For example, the red and green paths might be the routes of two different types of trucks, the purple path might correspond to a two-segment plane route and the blue arc might correspond to rented belly capacity.

![Classical hub network.](image1.png) ![HLRD hub network.](image2.png)

Figure 1: Classical versus HLRD hub network designs.

The remainder of the paper is organized as follows. In Section 2, we formally define the problem and introduce our compact mathematical model to solve it. In Section 3, we discuss the projection of the continuous variables in the compact model. The branch and cut algorithm we propose to solve the resulting cut model is discussed in Section 4. In Section 5, we present and analyze the results of an extensive computational study. In Section 6, we conclude with some final remarks.

## 2 Preliminaries and HLRD Model

In this section we formally describe HLRD. We are given a set $N$ of nodes with pairwise traffic demand. We define the set of distinct pairs of nodes, $D = \{(i, j) : i \in N, j \in N \setminus \{i\}\}$, and denote by $w_{ij}$ the demand for pair $(i, j) \in D$. Our designs will select $p$ nodes from the set $N$ as hub locations and the remaining will be access nodes. As depicted with Figure 1b, these two sets of nodes will be unified in their respective networks connected to each other. We shall seek for effective routing and dimensioning of different vehicle types traversing these networks and carrying the demand with a promised quality level of service. In particular:

**Access networks:** Each access node is assigned to exactly one hub and its traffic transits through this hub. We do not impose a direct connection from each non-hub node to its hub;
instead we design routes that connect several non-hub nodes to their hub. An access route may be a simple path that starts at a non-hub node and ends at a hub node. In this case, a vehicle starts its trip at the first non-hub node, visits other non-hub nodes, each time picking up their outgoing traffic demand and brings this load to the hub. Then it picks up the incoming traffic demand of the non-hub nodes from the hub and starts its return trip towards the non-hub nodes. Alternatively, an access route may be a directed cycle visiting a set of non-hub nodes and one hub. In this case, pick up and delivery may be done simultaneously. In both cases, each non-hub node is served by exactly one access route. To ensure quality of service, we limit the travel time and the number of non-hub nodes visited on an access route.

We denote by $P$ the set of feasible access routes and by $P_k$ the set of feasible access routes ending at node $k \in N$. Note here that for a given access route $\rho \in P$, we know the order in which nodes are visited and the required capacity of a vehicle that can serve this route. Consequently, the cheapest vehicle type that meets the travel time requirement and that has sufficient capacity can be chosen a priory. We denote the corresponding cost by $c'_\rho$.

**Hub network:** We do not assume direct connections between hubs; we rather design and dimension the hub routes. A hub route is a simple directed path that visits a subset of hub nodes. As we did for access routes, to ensure quality of service, we impose restrictions on the number of hub nodes and on the travel time on a hub route. Different from the case of access routes, we do not know a priory the amount of traffic that is carried on a hub route as this depends on the assignment of nodes to hubs. Consequently, the choice of vehicles for hub routes are part of the decisions to be made by the model.

For hub routes, we use the following notation: We define $\Pi$ to be the set of feasible hub routes, $\Pi_{kl}$ to be the set of feasible hub routes in which node $k \in N$ comes before node $l \in N \setminus \{k\}$. We have a set of vehicles, each with a capacity, a fixed and variable cost and a speed. For a hub route $\pi \in \Pi$, we define $V_\pi$ to be the set of vehicles that can traverse route $\pi$ in the allowed amount of travel time. For $v \in V_\pi$, let $c'_\pi v$ be the cost of using one vehicle of type $v$ on route $\pi$ and let $Q_v$ be the capacity of this vehicle. Our model designs hub routes and decides how many of each type of vehicle to use on each route.

We use the following decision variables: $x_{ik}$ is 1 if node $i \in N$ is assigned to hub node $k \in N$ and 0 otherwise. If node $k$ is a hub then $x_{kk}$ is 1. For an access route $\rho \in P$, $z_\rho$ is 1 if this route is used and is 0 otherwise. The continuous variable $y_{ijkl}$ is the amount of flow of pair $(i, j) \in D$ that enters the hub network at hub $k \in N$ and leaves it at hub $l \in N$ and $f_{kl}^\pi$ is the amount of flow from hub $k \in N$ to hub $l \in N \setminus \{k\}$ routed on hub route $\pi \in \Pi_{kl}$. Finally, $u_v^\pi$ is the number of vehicles of type $v \in V_\pi$ used on hub route $\pi \in \Pi$. 

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Using these variables, we model HLRD as follows.

\[
\begin{align*}
\min & \sum_{\pi \in \Pi} \sum_{v \in V_\pi} c^v \pi u^v \pi + \sum_{\rho \in P} c^\rho \pi z_\rho \\
\text{s.t.} & \sum_{k \in N} x_{ik} = 1 \quad \forall i \in N, \\
& x_{ik} \leq x_{kk} \quad \forall i, k \in N, i \neq k, \\
& \sum_{k \in N} x_{kk} = p, \\
& x_{ik} = \sum_{\rho \in P : i \in \rho} z_\rho \quad \forall i, k \in N : i \neq k, \\
& \sum_{l \in N} y_{ijkl} \geq w_{ij} x_{ik} \quad \forall (i, j) \in D, k \in N, \\
& \sum_{k \in N} y_{ijkl} \leq w_{ij} x_{jl} \quad \forall (i, j) \in D, l \in N, \\
& \sum_{\pi \in \Pi_{kl}} f_{\pi}^{kl} \geq \sum_{(i,j) \in D} y_{ijkl} \quad \forall k, l \in N, k \neq l, \\
& \sum_{k,l \in N, k \neq l, a \in \pi} f_{\pi}^{kl} \leq \sum_{v \in V_\pi} Q_v u^v_{\pi} \quad \forall \pi \in \Pi, a \in \pi, \\
& u^v_{\pi} \leq M x_{kk} \quad \forall \pi \in \Pi, k \in \pi, v \in V_\pi, \\
& x_{ik} \in \{0, 1\} \quad \forall i, k \in N, \\
& z_\rho \in \{0, 1\} \quad \forall \rho \in P, \\
& u^v_{\pi} \geq 0 \text{ and integer} \quad \forall \pi \in \Pi, v \in V_\pi, \\
& f_{\pi}^{kl} \geq 0 \quad \forall k, l \in N, k \neq l, \pi \in \Pi_{kl}, \\
& y_{ijkl} \geq 0 \quad \forall (i, j) \in D, k, l \in N.
\end{align*}
\]

The objective function (1) minimizes the total operational cost (fixed and variable) of the vehicle fleet. Constraints (2), (3) and (4) ensure that each node either becomes a hub or is assigned to exactly one hub node and \( p \) hubs are chosen. Constraints (5) relate the assignments and the access routes: if node \( i \) is assigned to hub \( k \), then one access route that visits node \( i \) and ends at hub \( k \) is used. Otherwise, no such route can be part of the solution. For binary \( x \), constraints (6) and (7) imply that \( y_{ijkl} = w_{ij} x_{ik} x_{jl} \). The amount \( \sum_{(i,j) \in D} y_{ijkl} \) is the amount of traffic that enters the hub network at hub \( k \) and that leaves it at hub \( l \). Using constraints (8), we allocate this traffic to hub routes that visit \( k \) before \( l \). Note that constraints (6)-(8) can also be written as equalities without changing the optimal value. For a hub route \( \pi \in \Pi \), the required capacity is the maximum traffic carried on any of its arcs. The traffic on arc \( a \in \pi \) is the sum of the traffic from hubs \( k \) to hubs \( l \) so that \( a \) is on the subpath of \( \pi \) from \( k \) to \( l \). For example, if the hub route is 123, then arc (1,2) carries the traffic from hub 1 to hubs 2 to 3, whereas arc (2,3) carries the traffic from hubs 1 and 2 to hub 3. These capacity restrictions are imposed through constraints (9) where the left hand side is the traffic on arc \( a \) and the right hand side is the total capacity of vehicles allocated to route \( \pi \). Constraints (10) ensure that hub routes only go through hubs. In our case, as the costs satisfy triangle inequality, we drop these constraints. The remaining constraints are variable restrictions.
To better illustrate the hub routes and what they can carry as traffic, we refer the reader to Figure 2. In this figure, we have four hubs and five hub routes, each depicted with a different color. The red route 1234 can carry the traffic from hub 1 to hubs 2, 3 and 4 on arc (1,2). The traffic from hub 1 to hub 2 is unloaded at hub 2 and the traffic from hub 2 to hubs 3 and 4 is loaded on the vehicle. Hence on arc (2,3), the load on the vehicle is the traffic from hubs 1 and 2 to hubs 3 and 4. At node 3, the traffic from hubs 1 and 2 to hub 3 is unloaded and the traffic from 3 to 4 is loaded. These three commodities are routed to hub 4, which is the final hub of this route.

The total traffic that needs to travel from hub 1 to hub 2 can use the red route and the green route. However the traffic from hub 4 to hub 3 cannot first use the green route (4 → 1 → 2) and then the (2 → 3) segment of the red route since this would require an unloading and loading operation at hub 2. For ease of operations and synchronization, we do not allow such actions and enforce that each demand is carried in one vehicle through the hub network.

Figure 2: Hub transportation network.
3 Projection of continuous variables

As our model contains a large number of variables, in this section, we project out the continuous variables, namely variables $y$ and $f$. This gives us a formulation in the space of $x$, $z$ and $u$ variables. However, this formulation is not compact.

**Proposition 1.** A solution $(x, z, u)$ that satisfies (2)-(5) and (10)-(13) is feasible for HLRD if and only if the projection inequality

$$\sum_{\pi \in \Pi} \sum_{\alpha \in \pi} \sum_{v \in V} Q_v u^v \geq \sum_{(i,j) \in D} w_{ij} \left( \sum_{k \in N} \alpha_{ijk} x_{ik} - \sum_{l \in N} \beta_{ijl} x_{jl} \right)$$  \hspace{1cm} (16)

is satisfied for all $(\alpha, \beta, \gamma, \delta) \geq 0$ such that

$$\alpha_{ijk} - \beta_{ijl} - \gamma_{kl} \leq 0 \quad \forall (i,j) \in D, k, l \in N : k \neq l,$$

$$\alpha_{ijk} - \beta_{ijl} \leq 0 \quad \forall (i,j) \in D, k \in N,$$

$$\gamma_{kl} - \sum_{a \in \pi} \delta_{\pi a} \leq 0 \quad \forall k, l \in N : k \neq l, \pi \in \Pi_{kl}.$$  \hspace{1cm} (17)-(19)

**Proof.** A solution $(x, z, u)$ that satisfies (2)-(5) and (10)-(13) is feasible if and only if there exist $y$ and $f$ such that

$$\sum_{i \in N} y_{ijkl} \geq w_{ij} x_{ik} \quad \forall (i,j) \in D, k \in N,$$

$$- \sum_{k \in N} y_{ijkl} \geq -w_{ij} x_{jl} \quad \forall (i,j) \in D, l \in N,$$

$$\sum_{\pi \in \Pi_{kl}} f_{\pi} - \sum_{(i,j) \in D} y_{ijkl} \geq 0 \quad \forall k, l \in N, k \neq l,$$

$$- \sum_{k,l \in N : k \neq l} \sum_{a \in \pi_{kl}} f_{\pi} \geq - \sum_{v \in V} Q_v u^v \quad \forall \pi \in \Pi, a \in \pi,$$

$$y_{ijkl} \geq 0 \quad \forall (i,j) \in D, k, l \in N,$$

$$f_{\pi} \geq 0 \quad \forall k, l \in N : k \neq l, \pi \in \Pi_{kl}.$$  \hspace{1cm} (20)-(25)

We associate dual variables $\alpha_{ijk}$, $\beta_{ijl}$, $\gamma_{kl}$ and $\delta_{\pi a}$ to constraints (20)-(23), respectively. The feasibility problem (20)-(25) has a solution if and only if the dual problem

$$\max \sum_{(i,j) \in D} w_{ij} \left( \sum_{k \in N} \alpha_{ijk} x_{ik} - \sum_{l \in N} \beta_{ijl} x_{jl} \right) - \sum_{\pi \in \Pi} \sum_{a \in \pi} \sum_{v \in V} Q_v u^v$$

s.t. (17)-(19)

$$\alpha, \beta, \gamma, \delta \geq 0.$$  \hspace{1cm} (26)

is bounded. As the feasible set of the dual problem is a cone, inequalities (16) impose that no feasible direction is an improving direction. □
In an attempt to gain insight into the projection inequalities, we analyze the special cases of short hub routes. If each hub route consists of a single leg, then $\Pi_{kl}$ contains only one path which corresponds to arc $(k, l)$. For that reason, the $f$ variables in (22) and (23) can be eliminated and these constraints can be combined into

$$- \sum_{(i,j) \in D} y_{ijkl} \geq - \sum_{v \in V_{kl}} Q_{v} u_{kl}^v \quad \forall k \in N, l \in N \setminus \{k\}.$$  

In a similar manner, associating dual variables $\alpha_{ijk}$ and $\beta_{ijl}$ to constraints (20) and (21), respectively, and $\gamma_{kl}$ to the above inequalities, we can show that a solution $(x, z, u)$ that satisfies (2)-(5) and (10)-(13) is feasible if and only if

$$\sum_{k \in N} \sum_{l \in N \setminus \{k\}} \gamma_{kl} \sum_{v \in V_{kl}} Q_{v} u_{kl}^v \geq \sum_{(i,j) \in D} w_{ij} \left( \sum_{k \in N} \alpha_{ijk} x_{ik} - \sum_{l \in N} \beta_{ijl} x_{jl} \right)$$  

is satisfied for all $(\alpha, \beta, \gamma) \geq 0$ such that

$$\alpha_{ijk} - \beta_{ijl} - \gamma_{kl} \leq 0 \quad \forall (i, j) \in D, k, l \in N : k \neq l,$$

$$\alpha_{ijk} - \beta_{ijk} \leq 0 \quad \forall (i, j) \in D, k \in N.$$  

This projection has been studied in Labbé and Yaman (2004) and Labbè et al. (2005) for a single type of vehicle. Based on the results of these studies, we can see easily that, in the case of hub routes of single arcs, i.e., the classical complete hub networks, the projection inequalities

$$\sum_{v \in V_{kl}} Q_{v} u_{kl}^v \geq \sum_{(i,j) \in D} w_{ij} (x_{ik} + x_{jl} - 1)$$  

for all $D' \subseteq D$, $k \in N$ and $l \in N \setminus \{k\}$ are sufficient to have a valid formulation of the problem. Inequalities (29) can be separated by enumerating all pairs $k$ and $l$ and choosing $D' = \{(i, j) \in D : x_{ik} + x_{jl} > 1\}$. The more general class of inequalities (26) can be separated in polynomial time by solving a linear program; however, we are not aware of any polynomial time combinatorial algorithm for their separation.

Now we analyze the projection inequalities for the special case with hub routes of at most two legs. Observe that in this case we have hub routes of the form $k \rightarrow l$ and $k \rightarrow l \rightarrow m$. The traffic from hub $k$ to hub $l$ can be carried on paths $k \rightarrow l$, $k \rightarrow l \rightarrow m$, $m \rightarrow k \rightarrow l$ and $k \rightarrow m \rightarrow l$ for $m \in N \setminus \{k, l\}$.

**Proposition 2.** Suppose that hub routes can contain at most two arcs. Let $D' \subseteq D$, $S, T \subseteq N$ with $S \cap T = \emptyset$. The projection inequalities

$$\sum_{k \in S} \sum_{l \in T} \left( \sum_{v \in V_{kl}} Q_{v} u_{kl}^v + \sum_{m \in N \setminus \{k, l\}} \left( \sum_{v \in V_{klm}} Q_{v} u_{klm}^v + \sum_{v \in V_{mkl}} Q_{v} u_{mkl}^v \right) \right) + \sum_{m \in N \setminus (S \cup T)} \sum_{v \in V_{klm}} Q_{v} u_{klm}^v \geq \sum_{(i,j) \in D'} w_{ij} \left( \sum_{k \in S} x_{ik} + \sum_{l \in T} x_{jl} - 1 \right)$$  

are valid inequalities.
Proof. We first show that the following vector \((\alpha, \beta, \gamma, \delta)\) is feasible for the dual problem:

- \(\alpha_{ijk} = 1\) for \((i, j) \in D'\) and \(k \in S\),
- \(\beta_{ijl} = 1\) for \((i, j) \in D'\) and \(l \in N \setminus T\),
- \(\gamma_{kl} = 1\) and \(\delta_{kl,(k,l)} = 1\) for \(k \in S\) and \(l \in T\),
- \(\delta_{klm,(k,l)} = \delta_{mkl,(k,l)} = 1\) for \(k \in S\), \(l \in T\) and \(m \in N \setminus \{k, l\}\),
- \(\delta_{kml,(m,l)} = 1/2\) for \(k \in S\), \(l \in T\) and \(m \in N \setminus (S \cup T)\),
- and other entries are zero.

It is easy to check that \(\alpha, \beta\) and \(\gamma\) satisfy the first two sets of constraints.

For \(k, l \in N\) such that \(k \neq l\), if \(\gamma_{kl} = 1\), for each \(\pi \in \Pi_{kl}\), we need \(\sum_{a \in \pi} \delta_{\pi a} \geq 1\). The paths in set \(\Pi_{kl}\) are \(k \to l\), \(k \to l \to m\), \(m \to k \to l\) and \(k \to m \to l\) for \(m \in N \setminus \{k, l\}\). The associated constraints are:

\[
\begin{align*}
\delta_{kl,(k,l)} &\geq 1, \\
\delta_{klm,(k,l)} &\geq 1 & m &\in N \setminus \{k, l\}, \\
\delta_{mkl,(k,l)} &\geq 1 & m &\in N \setminus \{k, l\}, \\
\delta_{kml,(m,l)} + \delta_{kml,(m,l)} &\geq 1 & m &\in N \setminus \{k, l\}.
\end{align*}
\]

We can verify easily that the first three constraints are satisfied. For the fourth one, we observe that path \(k \to m \to l\) is also in \(\Pi_{km}\) and \(\Pi_{ml}\). We have three cases: If \(m \in S\) then \(\delta_{kml,(m,l)} = 1\), if \(m \in T\) then \(\delta_{kml,(m,l)} = 1\) and finally if \(m \in N \setminus (S \cup T)\) then \(\delta_{kml,(m,l)} = \delta_{kml,(m,l)} = 1/2\). In all three cases, the fourth constraint is satisfied. Hence, \((\alpha, \beta, \gamma, \delta)\) is feasible for the dual problem.

Next, we derive the projection inequality for this dual solution. It is easy to see that

\[
\sum_{(i,j) \in D} w_{ij} \left( \sum_{k \in N} \alpha_{ijk} x_{ik} - \sum_{l \in N} \beta_{ijl} x_{jl} \right) = \sum_{(i,j) \in D'} w_{ij} \left( \sum_{k \in S} x_{ik} + \sum_{l \in T} x_{jl} - 1 \right).
\]

We need to show that

\[
\sum_{\pi \in \Pi} \sum_{a \in \pi} \sum_{v \in V_{kl}} Q_v u_{\pi v} = \sum_{k \in S} \sum_{l \in T} \left( \sum_{v \in V_{kl}} Q_v u_{kl} + \sum_{m \in N \setminus \{k, l\}} \left( \sum_{v \in V_{kl}} Q_v u_{klm} + \sum_{v \in V_{kl}} Q_v u_{mkl} \right) \right).
\]

For a path \(\pi \in \Pi\) that consists of one arc, say \(\pi = k \to l\), \(\delta_{kl,(k,l)} = 1\) if \(k \in S\) and \(l \in T\).

For a path \(\pi \in \Pi\) that consists of two arcs, \(\pi = k \to l \to m\), \(\sum_{a \in \pi} \delta_{\pi a} = \delta_{klm,(k,l)} + \delta_{klm,(l,m)}\).

If \(k \in S\) and \(l \in T\) then \(\delta_{klm,(k,l)} = 1\) and \(\delta_{klm,(l,m)} = 0\), and if \(l \in S\) and \(m \in T\) then \(\delta_{klm,(l,m)} = 1\) and \(\delta_{klm,(k,l)} = 0\). Also if \(k \in S\), \(m \in T\) and \(l \in N \setminus (S \cup T)\) then \(\delta_{klm,(k,l)} = \delta_{klm,(l,m)} = 1/2\). In all other cases, \(\delta_{klm,(k,l)} = \delta_{klm,(l,m)} = 0\). As these events are mutually exclusive, we can conclude that for path \(\pi = k \to l \to m\), \(\sum_{a \in \pi} \delta_{\pi a} = 1\) if and only if one of the following is true:

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i. \( k \in S \) and \( l \in T \),

ii. \( l \in S \) and \( m \in T \),

iii. \( k \in S \), \( m \in T \) and \( l \in N \setminus (S \cup T) \),

and \( \sum_{a \in \pi} \delta_{\pi a} = 0 \) otherwise. \( \square \)

We do not know an efficient separation algorithm for inequalities (30). For given sets \( S \) and \( T \), the best set \( D' \) can be computed as

\[
D' = \{ (i,j) \in D : \sum_{k \in S} x_{ik} + \sum_{l \in T} x_{jl} > 1 \}.
\]

It is interesting to note that unlike the case of hub routes with single arcs, when hub routes can contain two legs, including all of inequalities (30) does not yield a valid formulation. To see this consider the following very simple example where we have three hubs, \( a \), \( b \) and \( c \). Each hub wants to send one unit of traffic to every other hub. There is a single type of vehicle and its capacity is one. For this problem instance the solution where one vehicle is allocated to each route \( abc \), \( bca \) and \( cab \) satisfies inequalities (30) for all subsets \( S \) and \( T \). However this solution is not feasible. Each route indeed requires two vehicles.

As inequalities (30) are not sufficient to obtain a formulation in the case of hub routes with at most two arcs, we separate inequalities (16). This can be done by solving the linear program

\[
\theta = \max \sum_{(i,j) \in D} w_{ij} \left( \sum_{k \in N} \alpha_{ijk} x_{ik} - \sum_{l \in N} \beta_{ijl} x_{jl} \right) - \sum_{\pi \in \Pi} \sum_{a \in \pi} \sum_{v \in V_\pi} Q_v u_{v}^\pi
\]

s.t. \( (17)-(19) \)

\[ 0 \leq \alpha, \beta, \gamma, \delta \leq 1. \]

where the variables are bounded above by one for normalization. If \( \theta \leq 0 \), then all projection inequalities are satisfied. Otherwise, an optimal solution gives a most violated projection inequality.

## 4 Branch and Cut Algorithm

The model (1)-(15) contains a large number of variables for two reasons. First we enumerate all feasible access routes and hub routes. Second we have a large number of continuous variables \( y_{ijkl} \) and \( f_{kl}^\pi \). Our preliminary analysis with the Turkish data showed that the biggest problem was due to the large number of continuous variables. For this reason, we developed a branch and cut algorithm, referred as BC, based on the projection of continuous variables.

To enhance the computational efficiency of BC, we consider a heuristic phase in addition to the exact method to solve the separation problem. In the heuristic phase we separate a special class of projection inequalities for which we have an efficient combinatorial algorithm. If this phase fails to identify a violated inequality, we resort to the exact method where we solve a linear program. With complete hub networks the heuristic phase can separate, for instance, inequalities (29); with two-leg hub routes, the heuristic separation can make use of inequalities (30). In our experiments we restrict our attention to the case where the hub routes have at most two legs and during the heuristic phase consider the projection inequalities (30) for singletons, i.e., when \( |S| \) and \( |T| \) are one, as lazy constraints, for which the separation is trivial. For the exact separation we solve the linear program to detect violated projection inequalities (16), as lazy constraints. It worths noting
that, in the general case where hub routes can have more than two segments, inequalities similar to (29) and (30) can be derived and separated in the heuristic phase.

To further increase the computational efficiency, we also consider two alternative methods (H1 and H2) as repair heuristics to process integer solutions detected by the separation method as having violated some projection inequalities with an aim to find high quality feasible solutions to update the upper bound and the incumbent. In H1, we compute the optimal cost for the given set of hubs in this solution, say $N'$, by solving model (1)-(15). Note the the resulting model is much smaller when the hub locations are fixed. We compare the optimal cost with the best upper bound and update the latter and the incumbent, if necessary. Then we also add the cut $\sum_{k \in N'} y_k \leq p - 1$ in order not to consider the same set of hubs again. In H2, this time we compute the optimal cost for the given set of assignments $A$, by solving model (1)-(15) after fixing the values for the assignment variables $x$ as:

$$x_{ij} = \begin{cases} 
1 & \text{if } (i, j) \in A \\
0 & \text{o.w.}
\end{cases} \quad \forall (i, j) \in N \times N$$

As before, we compare the optimal cost with the best upper bound and update the latter and the incumbent if necessary. In this case, we add the cut $\sum_{(i, j) \in A} x_{ij} \leq |N| - 1$ in order not to consider the same set of assignments again. Clearly, H2 requires to solve a simpler (more restricted) problem, which demands less computational effort. However, in this case the resulting integer feasible solutions can be worse and the no-good cuts are much weaker, compared to those obtained with H1. So, one needs to carefully consider the tradeoff between the level of computational effort to solve the restricted problems and the quality of the integer feasible solutions as well as the strengths of the no-good cuts. Our preliminary studies have shown that using H1 is better for the instances with small $p$ values where the differences between the quality of integer solutions and cuts of H1 and H2 justify the extra computational effort needed.

Finally, we also consider valid inequalities to strengthen our formulation to improve the computational efficiency of BC. Note that, with the removal of continuous variables $y_{ijkl}$ and $f_k^{kl}$ the cost of inter-hub transfers can be simply avoided by the optimizer, by setting all $u_v^k$ to zero at the root node at the initial relaxations, giving a quite loose LP relaxation bound. As such, we use the following valid inequalities to strengthen our model without the mentioned continuous variables.

$$\sum_{\pi \in \Pi_{kl}} \sum_{v \in V_\pi} u_v^k \geq \max_{v \in U_{\pi} \in V_\pi} \frac{Q_v}{w_{kl}} \left( x_{kk} + x_{ll} - 1 \right) \quad \forall (k, l) \in D$$

As a logical cut, inequalities (31) simply enforce that if there is a positive amount of traffic between a pair of hubs, then a set of hub routes, with a total capacity that is no less than the traffic between the two cities should be activated. Our numerical studies have shown that the inclusion of (31) can drastically increase the LP relaxation bound and speed up the convergence of the branch and cut algorithm.

## 5 Computational Study

In order to investigate the potential applications and the computational efficacy of our solution methodology we have conducted a comprehensive numerical study. In the following sections, we
discuss the goals of the study, the instances used, the analysis of the results, and the insights obtained.

5.1 Experimental Design

In this section we focus on a time constrained package delivery network design problem, faced by the package carriers to offer next day/next morning delivery services. As a test bed we consider the problem instances in Turkey where multiple transportation modes (vans, trucks, planes, rented belly capacity, etc.) with different costs, speeds and capacities need to be used in coordination to meet a stringent service time requirement over a large geographical area in a cost effective way (Yaman et al., 2012). With the apparent cost advantages, vans and trucks are preferred for short/medium distance access and hub routes whereas cargo planes and rented belly capacities are needed to execute the long distance hub routes to meet the delivery time constraints. In order to achieve load consolidation and facilitate mode switches, strategically located hubs are needed to be employed in the network. Clearly, imposing time limits on the access and hub routes is essential for meeting the service time requirements, achieving operational sustainability and abiding by the regulations (Barnhart and Schneur, 1996; Armacost et al., 2002; Yıldız and Savelsbergh, 2018).

We next present the details of the experimental setting that aims to answer, among others, the following questions:

- What are the basic characteristics of the (optimal) next day delivery networks?
- What are the benefits of allowing stopovers and multiple legs in the access and hub routes?
- What is the gain of using rented belly capacity?
- What is the computational efficiency of the proposed methodology to solve realistic problem instances?

To answer such questions we consider 48 different problem instances we generated by varying the basic problem parameters as we discuss below.

All instances we use in our numerical experiments consider next day delivery demand between 81 cities of Turkey. For the distances between the cities and the weights for the flow between them (normalized next day delivery demand volume) we use the Turkish Network data (Kara, 2009). Among the 81 cities, we consider 16 of them with relatively large populations, central locations and housing airports as the candidate locations for hubs. Figure 3 illustrates the locations of the cities (dots), candidate hub locations (red circles with province codes) and the normalized weight of next day delivery demand among them (edge thicknesses are proportional to the normalized flow weights).

We first present the basic problem parameters, the values of which stay the same in all problem instances. Using the cost structure suggested in Serper and Alumur (2016) we consider three types of vehicles for the access routes: vans, trucks and trailers with capacities 3.5, 15 and 25 tons; fixed costs 250, 309 and 364; variable costs (per km) 0.5, 1 and 1.2. The capacities for the owned cargo planes are assumed to be 18 tons, with a fixed operating cost of 5000 and variable cost of 6.4 (per km). The per kilometer cost of renting 5 tons of belly capacity is also assumed to be 6.4, which makes using belly capacity more advantageous for relatively small load amounts. Up to two slots of belly capacity (total of 10 tons) can be rented by the cargo company. Representing the real situation, belly capacity is available for all the inbound and outbound flights for Istanbul, Ankara
and Izmir airports (largest airports in Turkey with direct commercial flights to all other airports). For vans, trucks and trailers the vehicle speeds are assumed to be 90 km/hour (speed limit for the heavy vehicles in highways) and for the passenger and cargo planes we consider the ground speed of 750 km/hour. For each access route stop we add 45 minutes to the total travel time of the path (due to inner city traffic, loading/unloading operations). For the hub route stops we use 90 minutes for the needed loading/unloading, sorting and repackaging operations. Deducting six hours for inner city pick up, delivery, sorting and packaging operations, we consider the time limit of $T = 18$ hours to complete all next day transfers.

To generate different problem instances we consider the following variations of the respective problem parameters.

- For the number of hubs we consider $p \in \{3, 4, 5, 6\}$.
- For the total daily next day delivery demand (in tons) we consider $W \in \{100, 125, 150\}$.
- For the time allocations of the access and hub routes we consider the following four variants: $(T_a, T_h) \in \{(7.5, 3), (7, 4), (6.5, 5), (6, 6)\}$.

5.2 Implementation Details

Our algorithm is implemented in Java using IBM ILOG Concert technology with CPLEX 12.8 solver. All experiments are run on a 64-bit machine with 16GB RAM and Intel i7-4790 processor at 3.60 GHz. The time limit is set to three hours.

In our implementation we use the repair heuristic H1 for problem instances with $p = 3$ and H2 for the instances with larger $p$ values. Both methods are implemented with the ILOG Concert Technology HeuristicCallback class.

5.3 Analysis

In this subsection we present the results of our numerical experiments and discuss the computational and managerial insights we derive from them.
The detailed results about the computational performance of the BC algorithm (corresponding to 42 feasible instances out of 48) are presented in Table 1, in Appendix A, and the solution statistics are provided in Table 2, in Appendix B. In the following we summarize and discuss the critical insights.

![Graphs showing average optimality gaps and run times](image)

(a) Average optimality gaps.  
(b) Average run times (seconds).

Figure 4: Average gap and run time values for different $p$ values.

We first focus on the computational performance of the BC algorithm. Our results in Table 1 clearly indicate the computational effectiveness of the algorithm in solving realistic problem instances. We see that for more than 64% of the instances (27 out of the 42 feasible instances) the BC algorithm could find the optimal solution within the given time limit and for those instances where the algorithm stopped hitting the time limit, the average for the optimality gap is less than 4.2%. As we see in Figure 4, which illustrates the distribution of optimality gaps and run times averaged over all instances, for different $p$ values, problem instances with four hubs are hardest to solve with an average run time of 8117 seconds and the optimality gap of 3.8%.

Investigating Table 1, we can see that the high computational performance of the BC algorithm can be explained by two basic factors: relatively small number of (separation) cuts added during the execution of the branch and cut method and the high efficiency of BC to solve the separation problem with the suggested two phase approach. As we can observe from Table 1, the total number of cuts added during the algorithm (1327.6 on the average) is quite small (relative to the problem sizes) and most of the time (83.4% of the time) the heuristic approach can solve the separation problem, which eliminates the need for resorting to computationally (more) expensive exact separation. In Figure 5 we plot the number of violated cuts detected by the heuristic and the exact separation algorithms for different $p$ values. Inspecting figures 4 and 5, we clearly see that the more the need for the exact separation phase, the higher the solution times and the optimality gaps, which underlines the computational benefits of the two phase separation approach.

We next examine the structure of the optimal solutions obtained by our model. In Figure 6 we illustrate the particular designs for E0, E1, E2 and E3 instances (see Table 1 for the details of the instance parameters). For the access routes the line thicknesses are proportional to the total capacity of the vehicles assigned to the respective paths. For the hub routes the straight lines indicate the paths that use trucks, dotted curves are used to depict the rented belly capacity and dot-dashed (thicker) curves illustrate the routes run with owned cargo planes.

Inspecting Figure 6 we observe that the optimal designs favor hub locations that are close...
to each other, even though this may necessitate longer access routes. This interesting result is quite different than one would observe in classical hub networks without time restrictions and cost efficient inter-hub transportation (Tan and Kara, 2007; Alumur et al., 2009; Çetiner et al., 2010). In our setting, when the land transportation is considered, inter-hub transportation can still be less costly since larger vehicles with higher utilization levels can be used in hub routes. However, the time restriction does not allow using trucks between many inter-hub trips, hence a more expensive air transportation (provided by rented belly capacity or owned cargo aircraft) needs to be used. Moreover, since we allow stopovers, access routes already provide opportunities for load consolidation and better use of vehicle capacities. Because of these reasons the inter-hub transportation becomes more expensive (in general) and network designs with closely located hubs are favored. Related to this observation we also see in Table 2 that \( p = 3 \) typically gives cheaper solutions compared to higher \( p \) values. With smaller number of hubs higher levels of load consolidation become possible in inter-hub transportation and utilization rates of the vehicles increase. Considering the fact that many hub routes use air transportation with high operational costs, the importance of high capacity utilization in those paths is obvious. We can clearly see this relation in Figure 7, which presents the cost and vehicle utilization percentages for access and hub routes in the optimal solutions, for different \( p \) values. As we see in Figure 7a the cost of inter-hub transportation increases dramatically as \( p \) increases, whereas the reduction in the access route costs is rather limited, compared to this increase. We also see that the vehicle utilization levels drop significantly (from 82% to 24%) in the hub routes as \( p \) increases from three to six, where the access route vehicle utilizations stay almost the same. These results also underline the importance of availability of trucks and rented belly capacity, besides the owned aircraft, for the inter-hub transportation. As we see in Figure 6 and also observe in Table 2, optimal solutions frequently use these alternative transportation modes.

Different than classical hub network design models, our approach considers multiple stop access and hub routes as well as a multimodal structure enhanced by the inclusion of the belly capacity. We now investigate the impact of having these flexibilities on the total cost. In Table 3, in Appendix C, we present the detailed solution statistics for the instances E0, E1, E2 and E3, when we consider
Figure 6: Illustration of the solutions for E0, E1, E2 and E3 problem instances.
Figure 7: Access and hub route costs and vehicle utilization levels for different $p$ values.

(1) single leg access routes (no stopovers), (2) single leg hub routes, and (3) no belly capacities cases separately. However, the critical information is captured in Figure 8, where we illustrate the cost increases (compared to the base case with no restrictions, as reported in Table 2) for each restriction type (single leg access routes, single leg hub routes and no belly capacity), for each problem instance.

Figure 8: Cost increases with traditional hub location assumptions.

The first thing one notices looking at Figure 8 is the drastic cost increases when access or hub routes are restricted to one leg trips. For the hub routes this result is not very surprising, since the fixed costs of aircrafts are quite high and even for the relatively small $T_h$ values the benefit of using the same aircraft in multiple legs is obvious. However, it is quite interesting to see that restricting the access routes to single stops has a similar impact on the total cost, which supports our previous discussions that with stopovers access routes provide a significant load consolidation opportunity to reduce the costs (up to 35 % in the considered instances). We also see that the cost increase is higher for smaller $p$ values (i.e., for instances E0 and E1). For the access routes the reason for this result is mainly due to the fact that as the number of hubs increases the total length of the access routes, as well as their share in the total cost decreases, so the cost increase due to
the stop restriction for access routes becomes less important. For hub routes the reason for this trend can be understood by looking closer at the cost distribution in Table 3, which shows that as \( p \) increases the optimal solution starts to use belly capacity heavily (almost two times more than the base case with no restrictions) to compensate for the increasing cost of owned aircraft due to stopover restrictions. Focusing on the role of the belly capacity, we see from Figure 8 that it can provide solid cost benefits (7.35% on the average) especially for the cases with high \( p \) values (13.3% for \( p = 4 \)).

6 Final Remarks

As exploding demand and rising customer expectations push the current logistics systems to their limits (or beyond) the practical and academic interest in hub networks is on the rise. With their inherent capabilities to promote consolidation, coordination and the benefits of multimodal transportation, hub networks are in the core of many revolutionary logistic concepts and business models that are to shape the future of transportation. These new perspectives open up exciting opportunities and novel research directions for hub network design.

Besides their practical significance, hub network design problems are theoretically interesting and quite challenging problems. Squeezed between the strong practical motivation and theoretical complexity, the vast hub location research has developed through models with strong assumptions, which (generally) fit well with the practical situation at hand and yield high computational efficiency to handle realistic size problem instances. However, for many recently proposed applications those assumptions are either too restrictive or not valid at all. So to improve the practical relevance and exploit the new opportunities for hub network design research, it is crucial to develop models that can relax (at least some) of the common modeling assumptions that do not fit well with the practical situation considered especially in the novel implementations of hub network designs that have a huge potential to revolutionize the logistics practice. In this study we take a first step towards this challenging yet quite promising research direction, with which we hope to open the way for the much needed advances in hub network design research.

In this study we develop a new hub network design model that does not make any assumptions on the structure of the network (i.e., direct assignments, complete connectivity of hub nodes) and relaxes the assumption of a constant cost reduction factor (denoted by \( \alpha \) in the classical hub location literature). Another distinguishing aspect of this novel approach is its linkage of the strategic, tactical and operational level decisions related to hub layout and transportation network design to jointly determine the locations of the hubs, hub assignments, types and number of vehicles to use and routes to operate. As we show in our numerical experiments all these new features of the model bring important practical benefits to better use the vehicle capacities, improve load consolidation in both access and hub routes and take better advantage of multimodal transportation capabilities. Our results show that hubs can play critical roles not only by facilitating load consolidation but also functioning as transfer points to enable an end to end intermodal transportation. Besides the managerial insights we derive out of them, these experiments also show that with the recent advances in integer programming methodologies it is possible to solve realistic size problem instances without resorting to two stage solution heuristics or restrictive assumptions on the solutions. This suggests that developing new hub network design models that can answer the specific needs of the novel logistics and mobility concepts such as physical internet and mobility as a service provides rich opportunities for high impact research in hub network design, which we plan to follow in our
future studies.

Acknowledgment

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Appendix A  Computational Performance Results

Table 1 presents important statistics about the computational performance of the BC algorithm, for the considered problem instances. Following the columns listing the instance name and the basic parameter values, column gap indicates the optimality gap when the algorithm terminates and column time reports the run time of the BC for the respective problem instance. The total number of cuts added during the heuristic and exact separation phases of the BC are reported in columns h.cuts and e.cuts, respectively.

Appendix B  Solution Statistics

Table 2 presents the solution statistics for the problem instances we studied except for the problems E8, E12, E24, E28, E40 and E44 for which the problem is infeasible. The column t.cost refers to the best feasible solution value (total cost) found by the BC algorithm in the given time limit. The columns a.cost and h.cost indicate the total cost of the access and hub routes in the given solution. The columns truck, belly and plane give the total cost of truck, rented belly and owned cargo planes used by the solution, respectively. The columns a.util and h.util indicate the vehicle capacity utilizations for the access and inter-hub transfers. For the access routes it corresponds to the maximum utilization during the trip (getting larger at every pickup stop and getting smaller at every delivery stop) and for the hub routes this value reports the average utilization levels of the vehicles used in the hub routes. Finally, the column hubs indicates the cities chosen as the hub locations.
Table 1: Computational Results

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<th>$T_h$</th>
<th>$p$</th>
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<th>time</th>
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23
Appendix C  Results with restricted path legs and no belly capacity

Table 3 presents the results for the problem instances E0, E1, E2 and E3 for the cases of restricted number of access and hub route legs and no belly capacities. The column \textit{r.type} indicates the restriction type considered in the solution, where \textit{SA} denotes single leg access routes, \textit{SH} denotes single leg hub routes and \textit{NB} denotes the case with no belly capacity. The rest of the column headings are as described for previous tables.

Table 3: Solution results for restricted path legs and no belly capacity

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