The Nurse Rostering Problem in COVID-19 emergency scenario

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Abstract

Healthcare facilities are struggling in fighting the spread of COVID-19. While machines needed for patients such as ventilators can be built or bought, healthcare personnel is a very scarce resource that cannot be increased by hospitals in a short period. Furthermore, healthcare personnel is getting sick while taking care of infected people, increasing this shortage of qualified personnel. As a consequence, they are asked to work overtime in order to guarantee enough assistance to patients in critical conditions. This article has the scope to provide healthcare facilities with a flexible mathematical formulation for scheduling nurses' shifts in scenarios with insufficient number of nurses by introducing the possibility for nurses to work more than one shift per day. This will reduce the number of shifts with an insufficient number of nurses while minimizing stress for the healthcare personnel by defining balanced schedules. Numerical results carried on synthetic data show the effectiveness of the formulation here introduced. Finally, all the models described are implemented in Python and made available as open-source software to all those facilities struggling for this emergence in order to help them in scheduling nurses during this critical situation.

Keywords: Nurse Rostering Problem, Scheduling Problems, COVID-19, Mixed Integer Optimization, Emergency scenario.

1. Introduction

The spread of COVID-19 is requiring an unprecedented effort by healthcare facilities to assist patients with critical conditions. The healthcare system in the North of Italy is collapsing and many other centers might end up in similar conditions very soon. The steep increase of patients with critical conditions who need particular care is difficult to be satisfied by the medical staff. Resources are limited and hospitals are not ready to deal with a similar pandemic scenario. While machines such as ventilators can be purchased or built, an even more scarce resource is represented by physicians and nurses able to take care of infected patients. Besides, the lack of sufficient protective equipment to keep them safe is causing healthcare personnel to get sick while taking care of infected patients. Those who are infected must stay away from work for at least 14 days, depleting the already exhausted workforce. Up to the 20th of March, healthcare workers made up 9% of Italy's COVID-19 cases [10]. In order to take care of all the infected patients, more and more often nurses are asked to work overtime but standard scheduling formulations are not thought to consider this possibility since it is out of the scope of scheduling in ordinary scenarios.

In this optic, the definition of a mathematical formulation for the scheduling of nurses' shifts where nurses can work extra hours to cover some shifts more, will provide healthcare facilities with an intelligent-automated system able to reduce the uncovered shifts while minimizing the effort and stress for nursing personnel and guaranteeing as much as possible the satisfaction of the working condition regulations within the ward. Nurses deem fairness and balanced workloads essential to their well-being [11]. Properly scheduling the nursing staff could have a great impact on the quality of healthcare service offered by reducing unbalanced workloads or excessive stress which may lead to reduced efficiency of nurses and possible human errors.

The Nurse Rostering Problem

The Nurse Rostering Problem (NRP), also called Nurse Scheduling Problem (NSP), is the problem of assigning nurses to shifts so to satisfy some constraints such as guaranteeing the minimum level of assistance required to patients while optimizing an objective function such as the overall number of hours worked by each nurse. It is an NP-complete optimization problem [14] that has already been extensively investigated in the literature. For an overview of the topic, have a look at [3, 4, 7] and the reference therein. Different types of formulations have been developed such as cyclical formulations, where a pattern is specified for each nurse and then repeated for the rest of the time horizon [2], whereas, in non-cyclic scheduling a pattern is not defined [13]. From an algorithmic point of view, both heuristics and exact methods have been applied. Examples of the former approach can be found in [1, 6, 8] where respectively Genetic Algorithms, Tabu Search and ant colony bases algorithms have been applied, while concerning exact methods many approaches have tested such as [2, 9, 12, 13].

Aim of the paper

In this work, we define a flexible scheduling formulation able to deal with pandemic scenarios where the number of nurses might not be enough to guarantee the minimum level of assistance required. The main scope of this article is to provide healthcare facilities with a flexible and easy-to-adopt scheduling tool that, by taking into account the possibility for nurses to work overtime, will help facilities in dealing with the NRP problem by balancing the workload required to each worker while trying to satisfy shift and ward's constraint as much as possible. We focus on the definition of a non-cyclic formulation since this kind of formulations is more suited for dealing with emergency scenarios where schedules might change very fast. We solve the formulation by resorting to exact methods since the time horizon considered is short and consequently, the dimension of the problem is relatively small (in emergency scenarios, given the unreliability of the scenario under investigation, is useless to schedule for long time interval). This work represents a general framework to include overtime shifts within standard NRP formulations and many other complexities can be included which are not extensively treated here for the sake of clearness. To foster the adoption of these techniques and help healthcare facilities to manage this pandemic scenario, all the formulations defined here (together with some extensions and modifications) are implemented using open-source libraries except for the optimization routine, and the code is made available at the following public github repository: `https://github.com/RuggieroSeccia/Scheduling-nurses-shifts`.

In the following, in section 2 we define the basic scheduling problem under normal conditions which will be used as a basis for extending the model to the overtime shifts scenario. Then in section 3 we include in the model the possibility that the number of nurses is not enough to guarantee the minimum level of assistance required. We first introduce a variable α which allow us to estimate the number of missing nurses in each shift, then we introduce a new formulation that considers the possibility for nurses to work more than one shift per day, discuss constraints that might be included in the model and consider the flexibility of most of the constraints introduced. Finally, in section 4 we analyze the performance of the model on some synthetic problems when changing the number of nurses available and the time horizon considered.

2. NRP: basic formulation

Let us consider a department in a hospital with a given number of nurses N. We want to organize their shifts for the next T days, e.g. $T = 7$ one week or $T = 30$ next month, and for all the following so to minimize the effort required by the staff to take care of all the hospitalized patients. By contract, each nurse i has to work at least H_i hours over the time horizon T (e.g. each nurse must work at least 36 hours per week, $H = 36$ and $T = 7$). If the ith nurse works for a number of hours higher than H_i, then the hours in surplus are counted as overtime work and then paid more by the healthcare structure. However, to avoid nurses from burning out, a maximum number of hours worked by each nurse per period is set to $H^{\max}$. Each day three shifts need to be covered by the nurses: morning, afternoon and night. According to the number of patients hospitalized, each shift s requires R_s nurses and lasts h_s hours. Each nurse cannot cover more than one shift per day. Moreover, we have the further constraint that if a nurse covers a night shift then they need to rest and cannot work the following day so to ensure a good life balance for them.

To formulate this optimization problem, let us introduce the binary variable $x_{ist} \in \{0, 1\}$ such that

$$x_{ist} = \begin{cases} 1 & \text{if nurse } i\text{th covers shift } s\text{th on day } t\text{th} \\ 0 & \text{otherwise} \end{cases}$$

Moreover, let us consider the parameter p_i which brings information about the previous period. Namely, p_i is a boolean parameter such that

$$p_i = \begin{cases} 1 & \text{if the } i\text{th nurse worked on the last shift of the previous period} \\ 0 & \text{otherwise.} \end{cases}$$

We want to find the optimal schedule $x^\star$ that minimizes the number of hours worked by nurses and satisfies all the ward's constraints.

This management problem can be formulated as a linear binary optimization problem [5]:

$$\min_{x_{ist}\in\{0,1\}} \quad \sum_{i=1}^{N}\sum_{s=1}^{3}\sum_{t=1}^{T} x_{ist}h_s \tag{1}$$

$$\text{subject to} \quad \sum_{s=1}^{3} x_{ist} \leq 1 \qquad \forall i = 1, ..., N \; t = 1, ..., T \tag{1a}$$

$$\sum_{i=1}^{N} x_{ist} = R_s \qquad \forall s = 1, ..., 3 \; t = 1, ..., T \tag{1b}$$

$$\sum_{s=1}^{3}\sum_{t=1}^{T} x_{ist}h_s \geq H_i \qquad \forall i = 1, ..., N \tag{1c}$$

$$\sum_{s=1}^{3}\sum_{t=1}^{T} x_{ist}h_s \leq H^{\max} \qquad \forall i = 1, ..., N \tag{1d}$$

$$x_{i3t} + \sum_{s=1}^{3} x_{ist+1} \leq 1 \qquad \forall i = 1, ...N \; t = 1, ..., T-1 \tag{1e}$$

$$\sum_{s=1}^{3} x_{is1} \leq (1 - p_i) \qquad \forall i = 1, ..., N. \tag{1f}$$

The objective function (1) is asking to minimize the overall number of hours worked by all nurses within the period under consideration. Note that if $\sum_{s=1}^{3}\sum_{t=1}^{T} h_s x_{ist} > H$, then the nurse ith is working $\sum_{s=1}^{3}\sum_{t=1}^{T} h_s x_{ist} - H$ extra hours. That is, by minimizing the objective function (1), we are actually minimizing the number of overtime work required to each nurse but we are not taking into account the possibility for nurses to work more than one shift per day. Concerning the constraints, constraint (1a) implies that each person cannot cover more than one shift on the same day. Constraint (1b) imposes that the minimum number of personnel per each shift in each day is guaranteed. Constraint (1c) and (1d) require each nurse to work respectively a minimum and a maximum number of hours per period as imposed by contract (e.g. full-time and part-time workers). Finally, constraint (1e) implies that if a nurse covers a night shift, then the next day they cannot work, and constraint (1f) implies that each nurse cannot work on the first day of the new period if they worked on the last night of the previous period. Several additional constraints can be added to this formulation, however, for the sake of simplicity we do not include them at this moment, but we treat some of them in section 3.2.

3. NRP: formulation in emergency scenarios

The model defined so far presents a basic and general representation of a scheduling problem within a ward and will be used as a starting point for the definition of our NRP in emergency scenarios. Indeed, we are interested in studying the realistic possibility in emergency scenarios that the number of nurses N is not enough to satisfy the demand.

If the number of nurses N is not big enough, then constraint (1b) cannot be satisfied and problem (1) is unfeasible. To compute how many shifts cannot be covered with N nurses, we can introduce the variable $\alpha_{st} \in R_+$ which represents the number of nurses that are missing to satisfy the minimum demand of the sth shift on the tth day. In this way, the problem comes feasible again, we can change the objective function so to add the term $\rho_1 \sum_{s=1}^{3} \sum_{t=1}^{T} \alpha_{st}$ and modify constraint (1b) as folllows:

$$\sum_{i=1}^{N} x_{ist} + \alpha_{st} = R_s \qquad \forall s = 1, ..., 3 \; t = 1, ..., T \tag{2}$$

To enforce the model to set $\alpha_{st} > 0$ only when the nurses available are not enough to satisfy the constraint, we can set ρ_1 to any value bigger than $\max_s h_s$ so that choosing $\alpha_{st} > 0$ returns always worse solutions that setting $x_{ist} > 0$. Note that, even if α represents a discrete quantity, it is modeled as a continuous variable since at the optimum it will achieve only integer values (it is easy to see that non-integer values of α cannot be optimal). Once the optimization problem is solved and the optimal schedule $x^\star$ is found, $\alpha^\star_{st}$ tells us how many nurses are missing on shift s on day t to satisfy the minimum requirement, while $\max_{st} \alpha^\star_{st}$ give us an estimate of the number of nurses that need to be added to satisfy the demand of the ward (e.g. through interim nurses).

The introduction of α in the formulation allows us to understand how many shifts cannot be covered with the actual number of nurses, but does not fill the lack of nurses. In ordinary situations hospital facilities may consider employing further nurses so to satisfy constrain (1b) as suggested by the parameter $\alpha^\star$, however in emergency situations they need to deal with this issue by asking the employees to work extra hours. This problem is not usually contemplated in scheduling problems being overtime work considered as something that cannot be scheduled in advance. The following section will address this problem by modeling this possibility within an optimization problem.

3.1. Introduction of overtime shifts

Consider the possibility that each nurse is asked to work more than one shift per day but up to $H^{\max}$ hours per period. To this aim, we assume that each nurse can work for a fraction $(1-c)$ or c of the shift respectively before or after their shift. The definition of the parameter c is intentionally left undetermined so to allow more flexibility in the adoption of the framework. E.g. by fixing $c = 0.5$ we allow each nurse to work half shift more before or after their proper shift, while by fixing $c = 1$ we ask some nurses to cover their shift and the following as overtime work. That is, we introduce the two additional integer variables $z_{ist}, q_{ist} \in \{0, 1\}$ such that:

$$q_{ist} = \begin{cases} 1 & \text{if nurse } i\text{th works the last } (1-c)h_s \text{ hours of the shift } s\text{th on day } t\text{th} \\ 0 & \text{otherwise.} \end{cases}$$

$$z_{ist} = \begin{cases} 1 & \text{if nurse } i\text{th works the first } ch_s \text{ hours of the shift } s\text{th on day } t\text{th} \\ 0 & \text{otherwise.} \end{cases}$$

In the following, when not specified the indices i, s, t will range over all their potential values.

Assuming that the additional hours are joined to a regular shift, we must add the following constraints:

$$z_{ist} \leq x_{is-1t} \qquad q_{ist} \leq x_{is+1t} \qquad \forall i, s, t \tag{3}$$

where $x_{i0t} = x_{i3t-1}$ and $x_{i4t} = x_{i1t+1}$. Note that $x_{i01} = p_i$, while x_{i4T+1} is assumed to be zero (i.e. nobody works on the first day of the next period). Moreover, we need to consider also that nurses cannot cover extra shifts if they have already been assigned to work on that shift:

$$z_{ist} \leq 1 - x_{ist} \qquad q_{ist} \leq 1 - x_{ist} \qquad \forall i, s, t$$

Then, as in constraint (1d), we can consider a maximum number of hours $H^{\max}$ that a nurse can work during the period T without burning out:

$$\sum_{s=1}^{3} \sum_{t=1}^{T} h_s \left(x_{ist} + cz_{ist} + (1-c)q_{ist} \right) \leq H^{\max} \qquad \forall\, i$$

We add another constraint to ensure that each nurse can work at most one fraction of overtime shift per day (either ch_s of $(1-c)h_s$)

$$\sum_{s=1}^{3} (z_{ist} + q_{ist}) \leq 1 \quad \forall i,\ t$$

We ask the number of nurses covering the first and the second part of each shift to be the same. In particular, we require:

$$\sum_{i=1}^{N} z_{ist} = \sum_{i=1}^{N} q_{ist} \qquad \forall s, t \tag{4}$$

This constraint ensures us to avoid shifts only fractionally covered so that constraint (2) can be changed to consider the extra shifts covered:

$$\sum_{i=1}^{N} (x_{ist} + z_{ist}) = R_s \qquad \forall s,\ t$$

To ensure nurses to have enough rest between two working shifts, we add the constraint that if a nurse covers as extra hours the first part of a night shift, then they cannot cover the morning shift of the next day, and similarly, if they cover the second part of the night shift, then they cannot cover the afternoon shift of the previous day:

$$z_{i3t} + x_{i1t+1} \leq 1 \qquad \forall i,\ t$$

$$q_{i3t} + x_{i2t} \leq 1 \qquad \forall i,\ t$$

In this case, the objective function can be defined as the overall hours worked by each nurse (considering regular and overtime shifts) plus the extra term to take into account the possibility that some shifts might not be completely covered

$$\min_{x_{ist}, z_{ist} \in \{0,1\}} \quad \sum_{i=1}^{N} \sum_{s=1}^{3} \sum_{t=1}^{T} h_s \left(x_{ist} + c z_{ist} + (1-c) q_{ist}\right) + \rho_1 \sum_{s=1}^{3} \sum_{t=1}^{T} \alpha_{st} \tag{5}$$

where $\rho_1 = \max_s h_s$, so that covering a shift with a nurse is more convenient than not covering it. Indeed, we are interested first in covering shifts with the available nurses and then in minimizing the overall number of hours worked.

Finally, the optimization problem to solve in order to determine the optimal schedule can be written as:

$$\min_{x,z,q} \quad \sum_{i=1}^{N}\sum_{s=1}^{3}\sum_{t=1}^{T} h_s\left(x_{ist} + cz_{ist} + (1-c)q_{ist}\right) + \rho_1 \sum_{s=1}^{3}\sum_{t=1}^{T} \alpha_{st} \tag{6}$$

$$\text{s.t.} \quad \sum_{s=1}^{3} x_{ist} \leq 1 \qquad \forall i,t \tag{6a}$$

$$\sum_{i=1}^{N} (x_{ist} + z_{ist}) + \alpha_{st} = R_s \qquad \forall s,t \tag{6b}$$

$$\sum_{s=1}^{3}\sum_{t=1}^{T} x_{ist} h_s \geq H_i \qquad \forall i \tag{6c}$$

$$x_{i3t} + \sum_{s=1}^{3} x_{ist+1} \leq 1 \qquad \forall i,t \tag{6d}$$

$$\sum_{s=1}^{3} x_{is1} \leq (1-p_i) \qquad \forall i \tag{6e}$$

$$z_{ist} \leq x_{is-1t} \qquad \forall i,s,t \tag{6f}$$

$$q_{ist} \leq x_{is+1t} \qquad \forall i,s,t \tag{6g}$$

$$z_{ist} \leq 1 - x_{ist} \qquad \forall i,s,t \tag{6h}$$

$$q_{ist} \leq 1 - x_{ist} \qquad \forall i,s,t \tag{6i}$$

$$\sum_{s=1}^{3}\sum_{t=1}^{T} h_s\left(x_{ist} + cz_{ist} + (1-c)q_{ist}\right) \leq H^{\max} \qquad \forall\ i,s,t \tag{6j}$$

$$\sum_{s=1}^{3} (z_{ist} + q_{ist}) \leq 1 \quad \forall i,\ t = 1,...,T-1 \tag{6k}$$

$$\sum_{i=1}^{N} z_{ist} = \sum_{i=1}^{N} q_{ist} \qquad \forall s,t \tag{6l}$$

$$z_{i3t} + x_{i1t+1} \leq 1 \qquad \forall i,\ t = 1,...,T-1 \tag{6m}$$

$$q_{i3t} + x_{i2t} \leq 1 \qquad \forall i,\ t \tag{6n}$$

$$x_{ist}, z_{ist}, q_{ist} \in \{0,1\}, \alpha_{st} \in R \tag{6o}$$

Note that constraint (4) allow us to retain the meaning of α_{st} as the number of nurses missing to cover all the shifts without forcing us to define α as a discrete variable.

3.2. Considerations

Most of the parameters in the models defined above can be rearranged to include further scheduling details and deal with different situations. In the following, we discuss some of the main scenarios that are or might be included in this formulation.

In order to include nurses' preferences (e.g. a nurses that cannot work on some specific shifts or are on vacation on a specific day), the right hand side (RHS) in constraint (6a) can be changed with a boolean parameter $V_{ist} \in [0, 1]$: $V_{ist} = 0$ implies that the ith nurse is not available on the sth shift on day t; conversely $V_{it} = 1$ implies the ith nurse can work for one shift on day t as a normal working day.

To avoid solutions where the workload is unevenly distributed among the nurses, we can require both the number of extra hours worked and the overall number of hours worked by each nurse to be close enough to their mean according to some pre-specified parameter $\mathcal{K}_1$ and $\mathcal{K}_2$ respectively

$$\left| \sum_{s=1}^{3} \sum_{t=1}^{T} z_{ist} - \frac{\sum_{i=1}^{N} \sum_{s=1}^{3} \sum_{t=1}^{T} z_{ist}}{N} \right| \leq \mathcal{K}_1 \qquad \forall i$$

$$\left| \sum_{s=1}^{3} \sum_{t=1}^{T} q_{ist} - \frac{\sum_{i=1}^{N} \sum_{s=1}^{3} \sum_{t=1}^{T} q_{ist}}{N} \right| \leq \mathcal{K}_1 \qquad \forall i$$

$$\left| \sum_{s,t} h_s \left(x_{ist} c z_{ist} + (1-c) q_{ist} \right) - \frac{\sum_{i,s,t} h_s \left(x_{ist} + c z_{ist} + (1-c) q_{ist} \right)}{N} \right| \leq \mathcal{K}_2 \qquad \forall i$$

where in the last constraint the indices in the summations were omitted for the sake of cleanness.

We might also include a constraint to avoid uneven distributions of undesirable shifts, such as night shifts and weekend shifts. In particular, we can define $\mathcal{U}$ as the index set of unpleasant shifts

$$\mathcal{U} = \{(s, t) : s = 3 \text{ or } t \text{ is a weekend}\}$$

and add the constraint for each nurse to cover a similar number of unpleasant shifts

$$\left| \sum_{(s,t)\in\mathcal{U}} x_{ist} - \sum_{(s,t)\in\mathcal{U}} x_{ist} \right| \leq \mathcal{K}_1 \qquad \forall i = 1, ..., N$$

Furthermore, in order to get balanced solutions, instead of minimizing (5), we should minimize the worst-case scenario by minimizing the overall number of hours worked by the nurse that works the most and the highest number of nurses missing in the same shift, namely:

$$\min_{x_{ist}, z_{ist} \in \{0,1\}} \quad \max_i \left\{ \sum_{s=1}^{3} \sum_{t=1}^{T} h_s \left(x_{ist} + c z_{ist} + (1-c) q_{ist} \right) \right\} + \rho_2 \max_{st} \{\alpha_{st}\} \tag{7}$$

where ρ_2 is a big enough penalization term so to ensure that the second term is more important than the first one. By minimizing objective function (7) we end up with a schedule such that $\max \alpha^\star$ is minimized. As a further post-processing of the solution, we might solve the problem again changing the objective function to be:

$$\max_i \left\{ \sum_{s=1}^{3} \sum_{t=1}^{T} h_s \left(x_{ist} + c z_{ist} + (1-c) q_{ist} \right) \right\} + \rho_1 \sum_{s=1}^{3} \sum_{t=1}^{T} \alpha_{st}$$

and including the constraint $\alpha_{st} \leq \max_{st} \alpha^\star$ so to guarantee that the maximum number of nurses missing in the most violated shift is the same, but reducing the number of missing nurses on the other shifts (the same objective could be achieved by adding a penalization term in the objective function (6), however we prefer this second approach to avoid the introduction of other terms in the objective function).

So far, we have not discussed the choice of T. Formulation (6) can be easily modified to consider different time horizons by changing the parameter specification in some of the constraints. For instance, if we are scheduling the next K weeks but want to have balanced weeks for each worker, we can define H_i and $H^{\max}$ as the minimum and maximum number of hours worked by nurse i in one week and change constraints (6j) (6c) and (6e) as follows

$$\sum_{s=1}^{3} \sum_{t=7(k-1)}^{k+7} \left(x_{ist} + \bar{w}_s \left(z_{ist} + q_{ist} \right) \right) \leq H^{\max} \qquad \forall i = 1, ..., N \; k = 1, ..., K$$

$$\sum_{s=1}^{3} \sum_{t=7(k-1)}^{k+7} x_{ist} h_s \geq H_i \qquad \forall i = 1, ..., N \; k = 1, ..., K$$

$$\sum_{s=1}^{3} x_{is(7(k-1)+1)} \leq (1 - x_{i3(7(k-1))}) \qquad \forall i = 1, ..., N \; k = 1, ..., K$$

where $x_{i30} = p_i$. Similar considerations apply in the case we would like to schedule short time periods, e.g. day by day schedules.

4. Numerical results

In this section, we report and discuss the numerical results on synthetic problems when comparing formulation (1) with the addition of the slack variable α as described in section 3 and formulation (6). The first formulation is denoted as **Base** while the second as **Ext**. We analyze the solution found when changing the number of available personnel in the ward N and the time horizon T with all the other parameters fixed. In particular, let $k = \left\lfloor \frac{T}{7} \right\rfloor$, we set

$$H_i = 36k \qquad H^{\max} = 60k \qquad \forall i, \qquad c = 0.5$$

$$x_{i4T} = 0 \qquad x_{i01} = 0 \qquad \forall i$$

$$R = [6, 5, 4] \qquad h = [7, 8, 9] \qquad p = [1, 0, ..., 0].$$

In particular, we are interested in analysing the potential improvements obtained when allowing nurses to work extra hours.

The code has been implemented in Python with Jupyter notebooks by leveraging well-known open-source libraries, like `pandas` and `numpy`. The optimization model has been developed in Python with the IBM Decision Optimization `docplex` APIs, using IBM ILOG CPLEX 12.9. The numerical results are run over a Dell XPS 15 9570 with an Intel i7-8750H at 4.41GHz and a RAM of 16 GB.

Numerical results are reported in Table 1 where for each experiment we report:

- the running time in seconds;
- a boolean which specifies whether the algorithm finds the integer optimum (represented as 1) or not (represented as 0);

- $\mathcal{I}_{>0}(\alpha)$ which is defined as

$$\mathcal{I}_{>0}(\alpha) = \sum_{i=1}^{N} \mathcal{I}_{>0}(\alpha_i)$$

$$\mathcal{I}_{>0}(\alpha_i) = \begin{cases} 1 & \text{if } \alpha_i > 0 \\ 0 & \text{otherwise} \end{cases}$$

  namely $\mathcal{I}_{>0}(\alpha)$ denotes the number of cases in which the number of nurses is below the required quantity;

- $\max_{s,t} \alpha_s t$ which represent the maximum number of nurses missing in one shift;

- $\sum_{s,t} \alpha_{st}$ which represents the overall number of nurses missing to fulfill constraint (1b);

- $\max i, s, t \, (h_s x_{ist} + ch_s z_{ist} + (1-c)h_s q_{ist})$ denoted as $\max_{ist}$x_z_q

Overall, formulation **Base** is always able to find optimal solutions in a shorter amount of time if compared with **Ext** which in few cases stops only after having reached the specified time limit (i.e. 240 seconds). However, thanks to the possibility for nurses to cover extra shifts, formulation **Ext** drastically reduces the number of shifts with a number of nurses below the required quantity. Also comparing the maximum number of nurses missing in one shift and the overall number of missing nurses, **Ext** is able to reduce these figures. This phenomenon is particularly evident when considering the case $N = 15$, where the advantages of asking nurses to cover overtime shifts are clear with **Ext** able to cover all the shifts with the required number of personnel while **Base** still unable to cover more than 13 shifts in all the three different periods considered. When the number of available nurses is enough to satisfy the minimum number of nurses in each shift, $N = 20$, then the two models become very similar, with **Ext** able to find smaller values of $\max_{ist}$x_z_q thanks to how the objective function has been defined. Finally, we point out how the results obtained when changing the time horizon T are very similar, with the exception of the case $N = 10$ where for increasing values of T the solver is not able to find the optimal solution. Nevertheless, in these cases, the values found by **Ext** are still better than those returned by **Base** when considering the overall number of nurses missing in the planned shifts.

Table 1: Numerical results when changing the size of the ward and the time horizon.

N	T	Prob	Cpu Time	Optimal	$\mathcal{I}_{>0}(\alpha)$	$\max_{st}\alpha$	$\sum_{st}\alpha$	$\max_{ist}$x_z_q
10	7	**Base**	**0.01**	1	14	4	36	56
		Ext	5.77	1	14	**3**	**26**	**60**
10	14	**Base**	**0.01**	**1**	14	4	36	56
		Ext	244.47	0	14	4	**26**	**60**
10	28	**Base**	**0.02**	**1**	14	4	35	56
		Ext	240.07	0	14	4	**26**	**60**
15	7	**Base**	**0.02**	1	6	4	13	54
		Ext	0.14	1	**0**	**0**	**0**	**60**
15	14	**Base**	**0.02**	1	6	4	14	56
		Ext	0.21	1	**0**	**0**	**0**	**60**
15	28	**Base**	**0.05**	1	7	3	14	55
		Ext	0.50	1	**0**	**0**	**0**	**60**
20	7	**Base**	**0.03**	1	0	0	0	**53**
		Ext	0.12	1	0	0	0	47
20	14	**Base**	**0.08**	1	0	0	0	**54**
		Ext	0.15	1	0	0	0	49
20	28	**Base**	**0.13**	1	0	0	0	**53**
		Ext	0.37	1	0	0	0	49

5. Conclusion

In this work, we have extended the literature about the NRP problem by introducing a scheduling formulation that deals with the possibility that the number of available nurses N is not enough to guarantee the satisfaction of the working condition regulations within the ward. By allowing nurses to cover overtime shifts, we have defined a framework able to reduce the number of shifts without the required quantity of nurses while ensuring the definition of balanced shifts avoiding uneven schedules for some nurses compared to others.

We have shown the flexibility of the introduced framework by discussing some refinements to the problem. Further changes can be included so to consider more complicated shift rules and settings. However, for the sake of cleanness, we preferred avoiding further investigating these extensions with the aim to provide a clear and easy-to-follow framework.

Numerical results on synthetic problems show the effectiveness of the frameworks and their ability to solve in a short amount of time the scheduling problem under consideration. All the formulations proposed are described and implemented. The resulting frameworks are made available as open-source tools on github in order to help all the healthcare facilities in managing nurses' shifts during this emergency period.

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