Energy-Efficient Timetabling in a German Underground System

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Abstract

Timetabling of railway traffic and other modes of transport is among the most prominent applications of discrete optimization in practice. However, it has only been recently that the connection between timetabling and energy consumption has been studied more extensively. In our joint project with *VAG Verkehrs-Aktiengesellschaft*, the transit authority and operator of underground transport in the German city of Nürnberg, we develop algorithms for optimal timetabling to minimize the energy consumption of the trains via more energy-efficient driving as well as increasing the usability of recuperated energy from braking. Together with VAG, we have worked extensively to establish a broad basis of operational data, for example characteristic power consumption profiles as well as travel time and dwell time distributions for the trains running in the network, to serve as input to our optimization methods. On the collected data sets, our approach has already shown significant potential to reduce energy consumption and, as a consequence, electricity costs and environmental impact. Furthermore, mathematical analysis of the polyhedral and graph structures involved in the optimization approach have enabled us to compute high-quality solutions within short time. This positive outlook motivated VAG to extend this project to include further operational constraints in the model and to adopt the resulting software planning tool in practice afterwards. It will assist timetable planners at VAG in using the available degrees of freedom in their timetable drafts to optimize the energy-efficiency of the underground system.

Keywords: Timetabling, Energy, Clique Problem with Multiple-Choice Constraints, Combinatorial Optimization, Perfect Graph

Mathematics Subject Classification: 90C90 - 90C27 - 90C57 - 90C35

1 Industrial challenge and motivation

Traction energy consumption is among the most important cost factors in the electricity bill of a railway undertaking. It is significantly influenced by the manner in which the trains are driven. Thus, a significant reduction in energy consumption can be achieved by choosing energy-efficient velocity profiles. This includes making use of pure rolling phases, the so-called *coasting*, as far as possible, as a train consumes no traction energy at all in this phase and, due to the low rolling friction, only slowly looses speed. In Figure 2, the effects of choosing between different driving modes of a train are shown schematically for a train in an underground network.

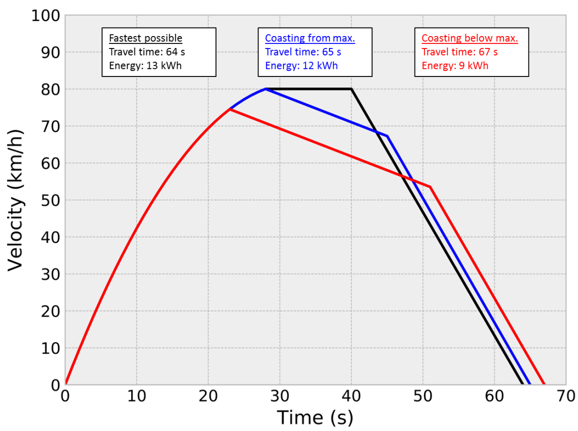


Figure 2: Schematic representation of the effect of choosing between one of the three driving modes "always driving as fast as possible" (*black*), "accelerating to maximum velocity followed by coasting" (*blue*) and "accelerating to below maximum velocity followed by extended coasting" (*red*) on a sample underground leg

These data show that by slightly slowing down the fastest possible speed profile on a given track, the train may consume up to 1/3 less in energy. In this respect, it is especially beneficial to extend the coasting phases of the train as much as possible. Altogether, choosing the optimal velocity profile for each train on each leg (= timetabled run between two stations) with respect to given total line travel times entails a huge leverage for bringing down the consumption of the overall underground system. This finding motivated our joint research project with *VAG Verkehrs-Aktiengesellschaft*, the local operator of public transport in the German city of Nürnberg. Its idea was to take a given timetable draft toward the end of the timetable planning phase and to use the remaining degrees of freedom to slightly shift train departures within fixed windows around their currently planned departure times. The aim is to create the necessary flexibility to enable choosing the best-possible velocity profile on each leg. At the same time, these shifts in the departures times allow for the better synchronization of departure and arrival events. This is important as a braking train is able to feed back recuperated energy to the grid. However, this energy can only be used if there is another train in the network which is accelerating at the same time, otherwise it is lost. Overall, there is a considerable potential for cost saving, as we have demonstrated in our collaboration. In the following, we will elaborate on the mathematical approach and present our case study for the underground system of Nürnberg.

2 Mathematical research

Based on a timetable draft created by expert planners, the studied task is to determine slight modifications in the train departure times as well as choosing velocity profiles for all trains in an energy-optimal way. However, these modifications shall retain the timetable structure established in the draft according to stated criteria, e.g. *dwell times* (= passenger interchange times) in the stations, *minimum headway times* (= safety distances) between trains and desired connections between trains. In order to construct a mixed-integer programming (MIP) model for this task, we discretized the time horizon into time steps of e.g. 5 seconds each and determined a suitable discrete set of (e.g. 3) alternatives for the velocity profiles for each train on a given leg. The profiles were initially chosen as heuristic solutions to an optimal control problem; later we changed them against measured profiles of actual train runs in the network. Furthermore, we allowed departure time shifts up to a given amount, e.g. 15 seconds around the draft departure time for each leg – a change that is hardly noticeable by the passengers but that can still allow for significant energy savings as we were able to show. With allowed shifts of ± 15 seconds in increments of 5 seconds and 3 profiles to choose from, there are already $3 \cdot 7 = 21$ possible choices for the combination of departure time and velocity profile for each leg. Given that there are 24,000 legs to be served in the Nürnberg underground each day, this means there are $24,000^{21} \approx 10^{92}$ possible timetables adjustments to choose from. No company planner could hope to evaluate all of them manually in order to determine the most energy-efficient one. Via the techniques of discrete optimization we have developed over the course of this project, however, we are able to produce near-optimal timetables within one hour or less.

To this end, we came up with a model formulation for the set of the feasible timetable adjustments as a special case of the clique problem with multiple-choice constraints on an undirected graph G (see [BGMS18, BGM20]). Its nodes represent possible combinations of departure time and velocity profile for the legs to be scheduled, while the edges model compatibilities between the departure configurations for different legs. Whenever the departure configurations for two specific legs do not violate any requirements for a feasible timetable, such as the above-mentioned ones, the corresponding nodes are connected by an edge. This results in an optimization model of the type

$$\begin{aligned} \min \quad & \sum_{t \in T} \max(P(x,t), 0) \\ \text{s.t.} \quad & x \in X, \end{aligned}$$

where $P(x,t)$ represents to total energy consumption at time step t, summed over all running trains, while X is the set of feasible timetable adjustments. Taking the maximum of $P(x,t)$ and 0 reflects that energy from a braking train can only be recuperated if it is used by other trains in the same time step. After linearizing the objective function with the help of additional auxiliary variables, the above model can be written as an MIP. We point out that all relevant types of timetabling constraints can indeed be expressed as pairwise node conflicts, which constitutes a very special structure. There are several ways to translate them into linear constraints. However, modelling the feasible region X in the most efficient way is very important as standard MIP solvers cannot solve the problem efficiently for real-world networks if a naive model formulation is used.

Our search for an adequate model formulation was inspired by the work of [SG97]. It was among the first to study the combined optimization of railway (or more precisely underground) timetables and energy consumption, giving a heuristic for reducing instantaneous power peaks. In [BMS17], we took up their basic idea and studied the effects of optimal timetabling for small subnetworks of German railway traffic under different objective functions relating to power consumption patterns. During this work, we realized that the problem contains an interesting structure to be exploited in order to reduce solution times. The nodes of the compatibility graph can be partitioned by the legs they belong to, and within each partition V_l they can be sorted by departure time. For the special (but still NP-hard) case of a single energy profile available for each leg, the compatibility structure then allows for a totally unimodular description of the timetabling polytope. It could be improved to an even more efficient dual-flow formulation by using the canonical ordering of the departure times for each leg. This special structure also comprises problems in other application contexts, such as the piecewise linearization of path flows – e.g. of natural gas in a pipeline (see [LM16]). We generalized the core properties of the compatibility structure to the abstract notion of *staircase compatibility* in [BGMS18]. There, the resulting model formulations were successfully employed on much larger subnetworks of *Deutsche Bahn AG (DB)*, up to the Germany-wide network, for minimizing peak power consumption. When using our improved formulations, we observed significant savings in solution times (over a factor of 100 in several cases), which allowed us to solve the problem for the Germany-wide network within a couple of minutes, but took hours to solve beforehand.

For multiple energy profiles per leg to choose from, the structure of the feasible set still tends to favour similar reformulations but cannot be perfectly described by staircase compatibility. We continued our polyhedral studies by considering a special case with respect to the *dependency graph* of the subsets in the node partition according to legs. This is the graph that encodes which pairs subsets of nodes directly impose *any* restrictions on each other. We showed that the feasible set in this case can be completely described by stable-set inequalities if the dependency graph is cycle-free. This leads to the following overall formulation:

$$\begin{aligned} \min \quad & \sum_{t \in T} \max(P(x,t),0) \\ \text{s.t.} \quad & \sum_{v \in V_l} x_v = 1 \quad \text{for all subsets } V_l \\ & \sum_{v \in S} x_v \leq 1 \quad \text{for all stable sets } S \text{ in } G \\ & x \geq 0, \end{aligned}$$

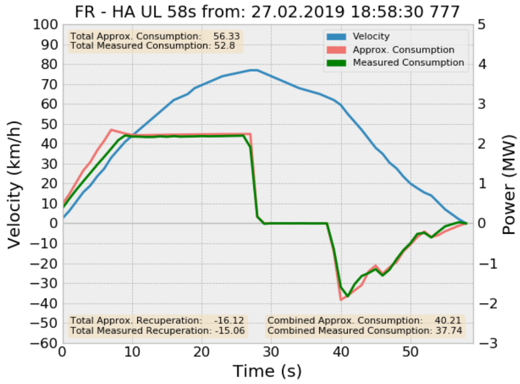
where $G = (V, E)$ is the compatibility graph and the subsets V_l for each leg l form a partition of V. Altogether, we want to choose exactly one departure configuration for each leg, as modelled by the variables x_v for each node $v \in V$. The stable sets in G represent exactly the subsets of nodes which are in pairwise conflict with each other. Note that the total number of stable sets is potentially large and difficult to generate in general. However, only stable sets involving nodes from just two subsets are needed, which significantly reduces the enumeration effort and the size of the formulation – especially if the number of departure configurations choices for each leg is small in comparison to the number of legs in the timetable.

We used this improved formulation to greatly reduce solution time for optimizing the timetable in the *Nürnberg* underground network, see [BGM20], also for more details on the aforementioned polyhedral results. In this preliminary computational study, an optimized shifted schedule of the longest line *U1* in the system reduced the overall energy consumption by about 18% during the morning rush hour interval between 5 a.m. and 9 a.m. when compared to the actual 2018 schedule. From there on, we have undertaken great efforts to broaden the available data base and to extend the results to all three Nürnberg underground lines over the whole day in order to see how much of these 18% in savings can be expected to be obtained in practice. These efforts and the findings we had will be described in the next two sections.

3 Implementation

The initial spark for this project was a cooperation with DB in project *E-Motion* (2013–2016), funded by the German Ministry of Education and Research (BMBF). Its aim was to come up with optimization algorithms to compute slight adjustments in the departure times of the trains to reduce peak power consumption for railway transport. The successful completion of this project led us to approach VAG in order to see if the same technique could be used to reduce peak consumption in the Nürnberg underground system. We soon learned that an even higher potential lies in reducing the overall power consumption by choosing energy-efficient driving patterns. So whereas in the project with DB we only adjusted departure times, the new task was to choose optimal travel times for each leg. After extending our timetabling model accordingly, we iteratively increased its performance. Firstly, the new degree of freedom, choosing travel times, added complexity to the mathematical model. An extensive study of its structure as described in [BGM20] enabled us to give a more compact problem formulation that was much easier to solve and still respected all necessary constraints. Secondly, we incorporated additional timetabling constraints to make sure that our solutions are not just energy efficient, but also real-world applicable. Note that these additions seamlessly fit into our new mathematical framework and therefore had no negative impact on solution times. Finally, we replaced the simulated velocity and power profiles, which were based on theoretical knowledge of train characeteristics, and which we used at the beginning, with profiles that were based on actual measurements for train runs in the system. For some time, we used profiles approximated from

(a) The DL350 device for measuring the power consumption of a train, i.e. the traction current used by the power trains (©Yokogawa Deutschland GmbH)



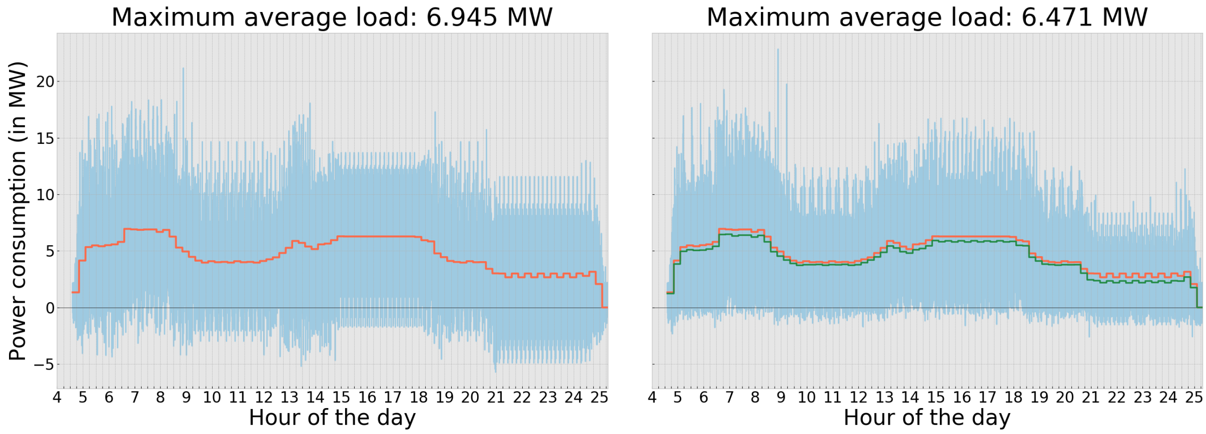
(b) A sample velocity profile (*blue*), approximated (*red*) and measured (*green*) power consumption profile on a given leg in the Nürnberg underground network

Figure 3: The company VAG has ordered and installed a dedicated device that measures the power consuption of a train running in the Nürnberg underground at a resolution of 200 ms. This allows for the derivation of precise velocity and power profiles as input to our timetable optimization model. As it turned out, the approximations of power consumption based on velocity and acceleration we used before were quite accurate in most cases.

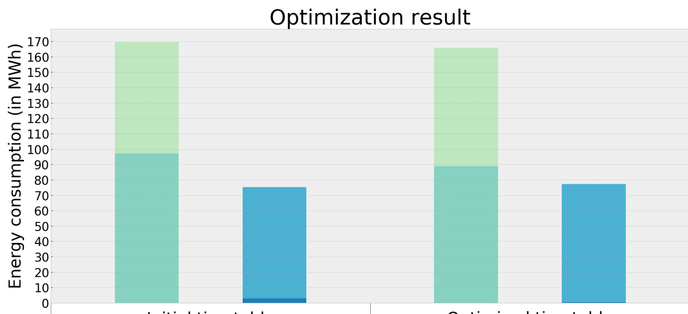
velocity and acceleration measurements as recorded by the tachographs in the wagons together with characteristic power consumption curves of the power trains. After confirming the potential of our approach on these approximate power profiles, VAG purchased and installed a dedicated device for measuring the traction current used by the powertrains in a wagon. Figure 3 shows this DL350 device as well as a sample profile recorded by it. These more precise recording recordings improved the accuracy of the model output further, and, as our database of sample power profiles grew, we could cooperate with the timetable experts at VAG on a related topic as well. Namely, the collected data allowed us to perform broad statistical evaluations to identify and study typical delays in the underground train operation. As a result, we were able to create a reference timetable for our optimization which more closely matches the actual underground traffic in the system. It can be used by VAG during schedule creation to improve both the reliability of future underground timetables and the reliability of the projected energy savings by our optimization procedures in practice. The next step in this ongoing project will be to refine our timetabling model further in order to integrate some more operational requirements. At the end of this process, VAG is going to adopt the timetable planning software we are implementing based on our mathematical approaches to support planners in creating energy-efficient underground timetables.

4 Industrial relevance and summary

The results we obtained exhibit a significant potential for both the reduction of energy consumption and for savings in electricity costs by optimized timetabling. We show a comparison of the unoptimized timetable draft with the optimized timetable in Figure 4.



(a) The power consumption pattern induced by the Nürnberg underground timetable before (left) and after (right) optimization. The *blue* curve shows the total power drawn by all trains in the network in a given second of the day, while the *red* curve shows the average power consumption over 15-minute intervals for the unoptimized timetable. The *green* curve shows the same for the optimized timetable. Energy consumption (as the area under the *red/green* curve) can be lowered throughout the whole day, with peak consumption in the morning rush hour decreasing by almost 7%.



	Initial timetable	Optimized timetable
Energy consumption without recuperation (in MWh)	169.6	165.8 (-2.3%)
Energy consumption with recuperation (in MWh)	97.2	88.9 (-8.5%)
Available recuperation energy (in MWh)	75.2	77.3 (+2.7%)
Unused recuperation energy (in MWh)	2.8	0.4 (-85.6%)

(b) Comparison between the initial (unoptimized) timetable with the optimized timetable for some statistics related to energy consumption. Overall, the optimized timetable saves 8.5% in energy. These savings stem from both a reduced net energy consumption from the external power supply system and from an increase in the use of recuperated braking energy.

Figure 4: The potential energy savings by our optimized underground timetable

Especially, Figure 4a shows the consumption pattern over time induced by the underground timetable of the Nürnberg underground system before and after optimization. We see at first sight that the optimization leads to a sizeable decline in wasted recuperation energy as there is much less overall “negative consumption”. It also turns out that the power consumption averages over consecutive 15-minute intervals, an important cost factor besides total consumption, can be lowered throughout the whole day, with peak consumption declining by almost 7%. Figure 4b shows that – according to the solution found by our optimization model – the total energy consumption can even be reduced by 8.5% through less net consumption by energy-efficient driving and an improved use of recuperation energy. These savings are twice as high as the internal threshold of 4% that VAG has set for the economic viability of this project. If realized, they would lead to an annual reduction in electricity costs of about 500,000 € per year under optimal conditions.

This outlook of not only reducing operational costs but also the environmental impact of the underground operations has motivated us to continue our joint project in order to assess the actual savings potential when implementing the approach in practice. As already mentioned, the aim is to provide planning experts with a decision support software which automatically produces proposals for energy-optimized, adjusted timetable drafts.

Acknowledgements

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