A Distributionally Robust Optimization Approach for Stochastic Elective Surgery Scheduling with Limited Intensive Care Unit Capacity

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Abstract

In this paper, we study the decision process of assigning elective surgery patients to available surgery blocks under random surgery durations, random length-of-stay in the intensive care unit (ICU), and limited capacity of ICU. The probability distribution of random parameters is assumed to be ambiguous, and only the mean and ranges are known. We propose a \textit{distributionally robust elective surgery scheduling} (DRESS) model that seeks optimal surgery scheduling decisions to minimize the cost of performing and postponing surgeries and the worst-case expected cost of overtime and idle time of operating rooms and lack of ICU capacity. We evaluate the worst-case over a family of distributions characterized by the known means and ranges of random parameters. We leverage the separability of DRESS in deriving an exact mixed-integer nonlinear programming reformulation. We linearize and derive a family of symmetry breaking inequalities to improve the solvability of the reformulation using an adapted column-and-constraint generation algorithm. Finally, we conduct extensive numerical experiments that demonstrate the superior performance of our DR approach as compared to the existing stochastic programming approach, and provide insights into DRESS.

\textit{Keywords:} OR in Health Services, Surgery Scheduling, Downstream Resource Constraint, Distributionally Robust Optimization, Column-and-Constraint Generation, Mixed-integer Programming

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1. Introduction

In this paper, we study the decision process of assigning elective surgery patients to available surgical blocks in a given decision period (e.g., a week). Each block is dedicated to only one type of surgical specialty, while there can be multiple blocks of the same specialty during a cycle of the surgery schedule. Each patient has a surgery type and can be assigned to any of the blocks dedicated to his/her surgery type. Surgery durations are random, and after surgery, a patient may need intensive care in the downstream intensive care unit (ICU) for an uncertain number of consecutive days. The quality of a schedule’s (i.e., assignments of surgeries to surgical blocks) performance is a function of the cost of performing or postponing surgery, overtime and idle time costs for operating rooms (ORs), and the insufficiency cost of ICU.

This specific elective surgery scheduling problem is challenging primarily due to the inherent uncertainty in surgery durations and patients’ length-of-stay (LOS) in the ICU. Surgery durations are hard to predict in advance, which may contribute to unpredictable OR overtime and idle times. OR overtime is costly and may lead to surgery cancellation and thus compromised quality of care, and idle time implies poor utilization of OR capacity (Bartek et al., 2019; Denton et al., 2010; Fügener et al., 2014; Shehadeh et al., 2019a; May et al., 2011; Wang et al., 2019).

Patient LOS in ICU is also hard to predict in advance, which may contribute to unpredictable availability of ICU beds (Min and Yih, 2010; Strand et al., 2010; Zhang et al., 2019). In the case of lack of ICU beds, OR manager may cancel surgeries, which generates extra costs and impact patient health, and/or transfer a patient from ICU to another hospital unit with a lower level of care (e.g., general ward) to free a bed for scheduled surgery. Several studies show that the rates of ICU readmission and risk of death of patients who leave ICU before they are considered fit for discharge are significantly higher than those who were discharged electively (Blunt and Burchett, 2001; Goldfrad and Rowan, 2000; Utzolino et al., 2010).

Stochastic programming (SP) is a common approach to address uncertainty in surgery scheduling. Existing SP models assume that the OR manager can fully characterize the distributions of uncertain parameters. Accordingly, SP models seek optimal surgery scheduling decisions to minimize patient-related costs and the expected cost of overtime, idle time, and lack of ICU capacity, where the expectation is taken with respect to a known joint probability distribution of uncertain parameters.

While SP is a powerful modeling approach, it also suffers from two shortcomings. First, data on uncertain parameters is often not available or inadequate to accurately estimate the distribution
of random parameters (Macario, 2009; Wang et al., 2019). If we calibrate an SP model to a data sample from a biased distribution, then the resulting (optimistically biased) optimal decisions may have a disappointing out-of-sample performance (Esfahani and Kuhn, 2018). Second, SP models are often computationally expensive and intractable (Birge and Louveaux, 2011).

In this paper, we address the distributional ambiguity of surgery durations and LOS in ICU. Specifically, we propose a distributionally robust elective surgery scheduling (DRESS) model that seeks optimal surgery scheduling decisions to minimize the cost of performing and postponing surgeries and the worst-case expected cost of overtime and idle time of ORs and lack of ICU capacity. We evaluate the worst-case over an ambiguity set (a family of distributions) characterized by the known means and ranges of random parameters. We leverage the separability of DRESS in deriving an exact mixed-integer nonlinear programming (MINLP) reformulation. We linearize and derive a family of symmetry breaking inequalities to improve the solvability of the reformulation using an adapted column-and-constraint generation (C&CG) algorithm.

With the intent of justifying the value of a DR approach, we conduct extensive numerical experiments using the practice configuration presented by Min and Yih (2010) (a well-known benchmark instance of surgery scheduling with ICU capacity constraints). Specifically, we demonstrate that our DR approach can solve large instances of DRESS in a reasonable time and produce surgery schedules that have superior operational performance under various probability distributions of random parameters as compared to the SP approach. In addition, we provide several managerial insights by examining trade-offs between costs, utilization, and capacity.

To the best of our knowledge, this paper is the first to propose a tractable DR approach for elective surgery scheduling with downstream ICU capacity constraints. We further contribute with the first symmetry breaking inequalities, which break symmetries in the solution space of surgery scheduling decisions. These inequalities are independent of the method of modeling uncertainty and thus they are valid for the SP and deterministic formulations.

The remainder of the paper is structured as follows. In Section 2, we review the relevant literature. In Section 3, we formally define DRESS and its reformulation. In Section 4, we present our C&CG algorithm and strategies to speed convergence. In Section 5, we present our computational results and managerial implications. Finally, we draw conclusions and discuss future directions in Section 6.
2. Relevant literature

In this section, we focus primarily on the literature most relevant to our problem: papers that apply stochastic optimization to address elective surgery scheduling with downstream capacity constraints. For comprehensive surveys of operating room scheduling problems, we refer the reader to (Cardoen et al., 2010; Hof et al., 2017; May et al., 2011; Samudra et al., 2016). Bai et al. (2018) provide a detailed survey of the application of operation research in managing intensive care units.

To the best of our knowledge, there are four SP approaches that are closely related to our work; Min and Yih (2010), Jebali and Diabat (2015), Jebali and Diabat (2017), and Zhang et al. (2019). We further limit the scope of this review to these papers (see (Appendix A) for a comparison). Min and Yih (2010) propose a two-stage SP model for elective surgery scheduling with the objective to minimize patient-related costs and the expected overtime cost. To solve their model, Min and Yih (2010) employ the sample average approximation (SAA) approach (see Shapiro and Homem-de Mello (2000), Kleywegt et al. (2002) for a detailed discussion on SAA). Min and Yih (2010) results demonstrate a superior performance of their SP approach over the deterministic approach.

Jebali and Diabat (2015) generalize Min and Yih (2010) model by incorporating the expected cost of OR idle time and lack of ICU capacity in the objective. Jebali and Diabat (2015) employ SAA to solve their model, and their results highlight the importance of considering the uncertainty in LOS in the downstream resources. Jebali and Diabat (2017) propose a two-stage chance-constrained SP model that minimizes patient-related costs and the expected costs of OR overtime, OR idle time, and exceeding ICU capacity. While Jebali and Diabat (2017) approach is superior to the deterministic approach, the robustness of their schedules is achieved at the expense of higher total costs and lower OR utilization.

Zhang et al. (2019) propose a two-level optimization model that combines MDP and 2-stage SP to minimize the cost incurred by maintaining open OR blocks, cost of performing and delaying surgeries, the expected costs of OR overtime and insufficient ICU capacity. At the first level (MDP), Zhang et al. (2019) determine the selection of patients to be operated; at the second level, they solve a 2-stage SP model that assigns selected patients to open OR blocks. Zhang et al. (2019) results show the importance of incorporating uncertainty of LOS in surgery scheduling.

Although SP is a powerful approach to model uncertainty, its applicability is limited to the case in which we know the distribution of uncertain parameters. Data on uncertain parameters is often not available to fully characterize their distributions (Rahimian and Mehrotra, 2019). If we calibrate an SP model to a data sample from a biased distribution, then the resulting (optimistically biased)
optimal decisions may have a disappointing out-of-sample performance (Esfahani and Kuhn 2018). SP models are also challenging to solve and generally intractable (Birge and Louveaux 2011).

Neyshabouri and Berg (2017) assume that surgery durations and LOS reside in the so-called uncertainty set of possible outcomes. Accordingly, they propose a two-stage robust optimization (RO) model, where optimization is based on the worst-case (maximum positive) deviations from the means of random parameters within the uncertainty set (see, e.g., Bertsimas and Sim (2004); Gorissen et al. (2015) for a thorough discussion on RO). Neyshabouri and Berg (2017) adopt the C&CG algorithm of Zeng and Zhao (2013) to solve their RO model. Neyshabouri and Berg (2017) results demonstrate that it is challenging to solve some small instances of the problem (e.g., 15 surgery) within a reasonable time. As argued by Chen et al. (2019), Delage and Saif (2018), Rahimian and Mehrotra (2019), and Thiele (2010), RO may yield over-conservative solutions and poor expected performances because it cannot capture the distributional information of uncertainty.

To bridge the gap between the conservatism of RO and the requirement of exact distribution in SP, distributionally robust (DR) optimization has been developed in recent years and has become a promising approach for addressing optimization problems contaminated with uncertain data. In DR optimization, one assumes that the distribution of uncertain parameters resides in a so-called ambiguity set, and optimization is based on the worst-case distribution within the ambiguity set. The ambiguity set is a family of all possible distributions of random parameters characterized by known properties of random parameters (Esfahani and Kuhn 2018; Delage and Ye 2010).

DR optimization alleviates the optimistic, and often unrealistic, assumption of the decision-maker’s complete knowledge of the distribution of uncertain parameters, and is often more computationally tractable than their SP counterparts (Delage and Ye 2010). One can use information that can be calculated from available data such as the mean and range of random parameters to construct the ambiguity sets and build DR models that better mimic reality and are less conservative than RO. We refer to Rahimian and Mehrotra (2019) for a survey of DR optimization.

2.1. Contributions

In this paper, we propose a DR model that helps OR managers schedule elective surgery by minimizing the cost of OR overtime, idle time, and lack of ICU capacity over the worst-case distribution occurring within an ambiguity set characterized by the known means and ranges of random parameters. We summarize our main contributions as follows:

- Using duality theory, we derive an equivalent MINLP reformulation of DRESS. We linearize
and propose an adapted C&CG algorithm to solve the resulting reformulation.

- We derive a family of symmetry breaking inequalities, which break symmetries in the solution space of scheduling decisions and thus improve the solvability of the master problem of C&CG algorithm and speed convergence. These inequalities are independent of the method of modeling uncertainty, and so they are valid for and can improve the solvability of the SP, RO, and deterministic formulations.

- In contrast to the RO model of Neyshabouri and Berg (2017), our model incorporates the cost of OR idle time in the objective and address the uncertainty in the distribution of random parameters. In addition, our DR model does not require the addition of inequalities for the OR block capacity recourse problem into the master problem of C&CG, which results in slower growth in the size of our master problem and faster convergence (see Section 3.4.1).

- We conduct extensive computational experiments that (1) demonstrate that our DR approach has a superior computational performance and can produce robust schedules that have excellent operational performance under various probability distributions of random parameters as compared to the SP approach, and (2) provide several managerial insights into DRESS.

- This paper is the first to propose a DR approach for surgery scheduling and downstream capacity planning. More broadly, our DR approach can be used in other applications where multiple entities share the same downstream resources. Thus, our paper expands the research dimension of DR optimization and stochastic scheduling with limited downstream resources.

3. DRESS Formulation and analysis

3.1. Definitions and assumptions

We consider a waiting list of $I$ elective surgery patients and a set of $B$ available blocks within an arbitrary planning horizon of $T$ days (e.g., a week). Each block is dedicated to only one type of surgical specialty, while there can be multiple blocks of the same specialty during a cycle of the surgery schedule. Each patient has a surgery type and can be assigned to any of the blocks dedicated to his/her surgery type during the planning horizon. Surgery durations are random and depend on surgery type. After surgery, a patient may need intensive care in the ICU for an uncertain number of consecutive days. The joint probability distribution of random parameters is assumed ambiguous, and only the mean and ranges are known.
Associated with each patient, there is a cost of performing or delaying his/her surgery to the next planning horizon, which depends on his/her waiting time on the list and clinical priority. The cost of delaying surgery is higher than performing surgery. Each block has a pre-allocated length of time $L_b$ on a specific day $t_b \leq T$. The ICU has a fixed number of beds $R_t$ on each day $t \leq T$.

We make the following assumption on DRESS based on prior SP studies:

A1. The set of patients, their surgery types, and priorities are known with certainty (a standard assumption in the elective surgery scheduling literature, see, e.g., Jebali and Diabat (2015); Min and Yih (2010); Neyshabouri and Berg (2017); Shehadeh (2019)).

A2. The planning horizon $T$ is an integer multiple of the surgery schedule cycle length.

A3. Surgical blocks and their assignments to operating rooms during a cycle of surgery schedule are previously determined, i.e., we consider elective surgery scheduling under the block scheduling policy (Min and Yih, 2010; Neyshabouri and Berg, 2017; Zhang et al., 2019).

A4. For ease of modeling, we assume that patients who are not assigned to any surgery block during the current planning horizon are assigned to a dummy block $b'$ (Min and Yih, 2010).

A5. There is a hospital unit with a lower level of care than the ICU that can accommodate patients transferred from the ICU (Min and Yih (2010); Zhang et al. (2019)).

A6. Emergency patients are treated in specialized units, and so we do not consider them in this study (a classical assumption in OR scheduling literature and SP studies that mimics OR practice, see, e.g., Cardoen et al. (2010); Guerriero and Guido (2011); Jebali and Diabat (2015); Min and Yih (2010); Neyshabouri and Berg (2017); Zhang et al. (2019)).

A7. Patients of the same type have identical probability distributions of surgery duration and LOS in ICU (a common assumption in prior studies, see, e.g., Zhang et al. (2019)).

**Notation**: For $a, b \in \mathbb{Z}$, we define $[a] := \{1, 2, \ldots, a\}$ and $[a, b]_\mathbb{Z} := \{c \in \mathbb{Z} : a \leq c \leq b\}$. The abbreviations “w.l.o.g.” and “w.l.o.o.” respectively represent “without loss of generality” and “without loss of optimality.” We define $(x)^+ := \max(x, 0)$. Finally, for notation brevity, we use $(I, B, T)$ to denote both the number and set of (patients, blocks, days).

### 3.2. Modeling surgery schedule under uncertainty

Let the binary decision variable $x_{i,b}$ represent the assignment of surgery $i$ to block $b$, for all $i \in I$ and $b \in B \cup \{b'\}$. The feasible region $\mathcal{X}$ of variables $x$ is defined in (1) such that each surgery...
i is assigned to one of the blocks dedicated to his/her surgery type or the dummy block \( b' \) (i.e., postponed to the next planning period). For ease of modeling, we set \( x_{i,b} = 0 \) for all blocks \( b \) that are not dedicated to surgery \( i \). Finally, if a surgeon requests to schedule surgery \( i \) in a block \( b \), we add \( x_{i,b} = 1 \) and \( x_{i,b'} = 0 \) to \( X \).

\[
X := \left\{ x : \sum_{b \in B \cup \{ b' \}} x_{i,b} = 1, \forall i \in [I], x_{i,b} \in \{0,1\}, \forall i \in [I], b \in B \cup \{ b' \} \right\} \quad (1)
\]

For all \( i \in [I] \) and \( t \in [T] \), let binary decision variable \( y_{i,t} \) equals one if patient \( i \) occupies an ICU bed on day \( t \), and zero otherwise. For all \( i \in [I] \), let non-negative parameters \( d_i \) and \( l_i \) respectively represent surgery duration and LOS in ICU of patient \( i \). Let continuous decision variables \( o_b \) and \( g_b \) respectively represent block \( b \) overtime and idle time, for all \( b \in [B] \). Let decision variable \( u_t \) represent the number of transfers from ICU on day \( t \), for all \( t \in [T] \). Table 1 summarizes these notation. Given \( x \in X \) and a joint realization of uncertain parameters \( \xi = [d, l]^T \), we formulate the following mixed-integer linear program (MILP) to compute the total cost incurred by overtime, idle time, and lack of ICU capacity.

\[
Q(x, \xi) = \min \left( \sum_{b \in B} (c_o^g o_b + c_g^g g_b) + \sum_{t \in T} c_t u_t \right) \quad (2a)
\]

s.t. \( y_{i,t} \geq x_{i,b}, \quad \forall i \in [I], b \in [B], t = t_b, \ldots, t_b + l_i - 1 \quad (2b) \)

\[
\sum_{i=1}^{I} y_{i,t} - R_t \leq u_t, \quad \forall t \in [T] \quad (2c)
\]

\[
o_b - g_b = \sum_{i=1}^{I} d_i x_{i,b} - L_b, \quad \forall b \in [B] \quad (2d)
\]

\[
y_{i,t} \in \{0,1\}, u_t \geq 0, (o_b, g_b) \geq 0 \quad \forall i \in [I], t \in [T], b \in [B] \quad (2e)
\]

where \( c_o^g \) and \( c_g^g \) are respectively the non-negative unit penalty costs of overtime and idle time, and \( c_t \) is the non-negative cost of inadequate ICU capacity. The objective function \( (2a) \) minimizes a linear cost function of overtime (first term), idle time (second term), and lack of ICU capacity (third term). Constraints \( (2b) \) ensure that each patient \( i \) assigned to block \( b \) stays in the ICU for \( l_i \) consecutive days upon performing the surgery on day \( t_b \). Constraints \( (2c) \), calculate the number of transfers from ICU or equivalently number of patients that need ICU beds for each day \( t \), but cannot be admitted to the ICU due to lack of capacity. Constraints \( (2d) \) yield either block overtime or idle time based on the durations of scheduled surgeries in this block. Finally, constraints \( (2e) \)
### Table 1: Notation.

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**Parameters and sets**

| I       | number, or set, of patients |
| B       | number, or, set of surgery blocks |
| \( c_{i,b} \) | cost of assigning patient \( i \) to block \( b \) |
| \( c^o_b \) | unit overtime cost for each block \( b \) |
| \( c^i_b \) | unit idle cost for each block \( b \) |
| \( c_t \) | cost of inadequate ICU bed capacity on day \( t \), i.e., cost of not having an ICU bed for patient. |
| \( R_t \) | number of available ICU beds on day \( t \), i.e., ICU capacity on day \( t \) |
| \( L_b \) | planned length of surgery block \( b \) |
| \( d_i \) | surgery duration of patient \( i \) |
| \( d_i/l_i \) | lower/upper bound of surgery duration of patient \( i \) |
| \( l_i \) | length-of-stay of patient \( i \) |
| \( l_i/l_i \) | lower/upper bound of length-of-stay of patient \( i \) |

**First-stage decision variables**

| \( x_{i,b} \) | binary assignment variable indicating whether patient \( i \) is assigned to block \( b \) |

**Second-stage decision variables**

| \( y_{i,t} \) | binary decision variable indicating if a patient needs an ICU bed on day \( t \) |
| \( o_b \) | continuous decision variable capturing overtime in block \( b \) |
| \( g_b \) | continuous decision variable capturing idle time in block \( b \) |
| \( u_t \) | number of transfers from ICU on day or extra beds required in the ICU |

specify feasible ranges of the decision variables. Formulation (2) generalizes that of Min and Yih (2010), Neyshabouri and Berg (2017), and Zhang et al. (2019), by incorporating OR idle time in \( Q(x, \xi) \). Given that \( y_{i,t} \) and \( R_t \) are integers, \( u_t \) will assume integer values. Thus, we do not need a constraint to force \( u_t \) to be an integer. Note that \( Q(x, \xi) \) is feasible for any feasible first-stage decision \( x \in \mathcal{X} \). Thus, our two-stage formulation has complete recourse.

### 3.3. DRESS formulation

Classical two-stage SP models seek optimal surgery assignments \( x \in \mathcal{X} \) to minimize the expectation of the random cost \( Q(x, d, l) \) subject to uncertainty \((d, l)\) with a known joint probability distribution denoted as \( \mathbb{P} \). In DRESS, we assume that \( \mathbb{P} \) is not fully known, and only the mean values, lower bounds \((d, l)\), and upper bounds \((\overline{d}, \overline{l})\) of \((d, l)\) are known. Thus, we consider support \( S = S^d \times S^l \), where \( S^d \) and \( S^l \) are respectively the supports of random parameters \( d \) and \( l \), and defined as follows:

\[
S^d := \{ d \geq 0 : \ d_i \leq d_i \leq \overline{d}_i, \forall i \in [I] \}.
\]
\[
S^l := \{ l \geq 0 : \ l_i \leq l_i \leq \overline{l}_i, \forall i \in [I] \}.
\]
We let \( \mu^d \) and \( \mu^l \) represent the mean values of \( d \) and \( l \), respectively, and denote \( \mu := [\mu^d, \mu^l]^\top \) for notational brevity. Then, we consider the following mean-support ambiguity set \( \mathcal{F}(S, \mu) \):

\[
\mathcal{F}(S, \mu) := \left\{ P \in \mathcal{P}(S) : \int_S dP = 1, E_P[\xi] = \mu \right\}
\]

where \( \mathcal{P}(S) \) in \( \mathcal{F}(S, \mu) \) represents the set of probability distributions supported on \( S \) and each distribution matches the mean values of \( d \) and \( l \). Note that we do not consider higher moments of random parameters in \( \mathcal{F}(S, \mu) \) for several reasons. First, from a clinical point of view and as reported in several studies, surgery durations and LOSs in ICU are independent (see, e.g., Min and Yih (2010); Zhang et al. (2019)). Second, even if we assume that these parameters are dependent, it’s not straightforward for OR managers to approximate the correlation between these uncertain parameters, especially with limited data. In contrast, \( \mu \) and \( S \) are easy to approximate or compute from available data. Third, incorporating higher moments of random parameters often undermine the computational tractability of the DR model, and therefore its applicability in practice (Mak et al., 2014; Shehadeh et al., 2019b). Using the ambiguity set \( \mathcal{F}(S, \mu) \), we formulate DRESS as the following min-max problem:

\[
\min_{x \in \mathcal{X}} \left( \sum_{i \in I} \sum_{b \in B \cup \{b'\}} c_{i,b} x_{i,b} + \sup_{P \in \mathcal{F}(S, \mu)} E_P[Q(x, \xi)] \right)
\]

which searches for a first-stage decision \( x \in \mathcal{X} \) that minimizes a measure of the total cost. The first term is the cost of performing or postponing surgeries. The second term is the worst-case expected cost of OR overtime, OR idle time, and inadequate ICU capacity. We evaluate the worst-case expected cost over all possible distributions of random parameters residing in \( \mathcal{F}(S, \mu) \).

### 3.4. Reformulation

In this section, we use duality theory and follow a standard approach in DR optimization to reformulate the min-max model in (4) to a solvable formulation via C&CG. We first consider the inner maximization problem \( \sup_{P \in \mathcal{F}(S, \mu)} E_P[Q(x, \xi)] \) for a fixed first-stage decision \( x \in \mathcal{X} \), where \( P \) is the decision variable, i.e., we are choosing the distribution that maximizes \( E_P[Q(x, \xi)] \). For a fixed \( x \in \mathcal{X} \), we formulate \( \sup_{P \in \mathcal{F}(S, \mu)} E_P[Q(x, \xi)] \) as the following linear functional optimization problem.

\[
\max_{P \in \mathcal{F}(S, \mu)} E_P[Q(x, \xi)]
\]

s.t. \( E_P[\xi] = \mu \),

\[
(5a) \quad (5b)
\]
\[ E_P[1_S(\xi)] = 1 \]  

(5c)

where \( I_S(\xi) = 1 \) if \( \xi \in S \) and \( I_S(\xi) = 0 \) if \( \xi \notin S \). As we show in the proof of Proposition 1 in Appendix B, problem (5) is equivalent to problem (6).

**Proposition 1.** For any \( x \in X \), problem (5) is equivalent to

\[
\min_{\rho, \alpha} \left\{ \sum_{i=1}^{I} \mu_i^d \rho_i + \mu_i^l \alpha_i + \max_{(d, l) \in S} \left\{ Q(x, d, l) + \sum_{i=1}^{I} - (d_i \rho_i + l_i \alpha_i) \right\} \right\}
\]

(6)

Note that formulation (6) cannot be solved in the presented form for two reasons. First, the recourse problem \( Q(x, \xi) \) is a minimization problem. Thus, in (6) we have an inner max-min problem. Second, it is impossible to solve \( \max_{(d, l) \in S} Q(x, \xi) \) with the current formulation of \( Q(x, \xi) \) because uncertain decision variable \( l \) is in the set of indexes of constraints (2b). Thus, in \( \max_{(d, l) \in S} Q(x, \xi) \), the number of constraints of type (2b) is uncertain and a decision variable. Therefore, we next reformulate \( \max_{(d, l) \in S} Q(x, \xi) \) to a solvable formulation.

Our reformulation of \( \max_{(d, l) \in S} Q(x, \xi) \) is inspired by the pioneering work of Neyshabouri and Berg (2017) and relies directly on the separability of the recourse problem \( Q(x, \xi) \). It is straightforward to see from the definition of \( Q(x, \xi) \) in (2) that block overtime and idle time variables \((o, g)\) are independent of the variables capturing the status of ICU capacity \((y, u)\). This observation helps us to decompose the recourse problem \( Q(x, \xi) \) into two independent recourse problems:

1. **Block Capacity (BC) problem**: \( \max_{d \in S^d} Q^{BC}(x, d) := \min \{ \sum_{b=1}^{B} (c_{ob}^o + c_{ob}^g) : (2d), (o, g) \geq 0, \forall b \in [B] \} \).

2. **ICU Capacity (ICU) problem**: \( \max_{l \in S^l} Q^{ICU}(x, l) := \min \{ \sum_{t \in T} c_t u_t : (2b) - (2c), y_{i,t} \in \{0, 1\}, \forall i \in [I], t \in [T], u_t \geq 0, \forall t \in [T] \} \).

Accordingly, problem (6) is equivalent to problem (7).

\[
\min_{\rho, \alpha} \left\{ \sum_{i=1}^{I} \mu_i^d \rho_i + \mu_i^l \alpha_i + \max_{d \in S^d} Q^{BC}(x, d) + \max_{l \in S^l} Q^{ICU}(x, l) + \max_{(d, l) \in S} \sum_{i=1}^{I} - (d_i \rho_i + l_i \alpha_i) \right\}
\]

(7)

### 3.4.1 Reformulation of BC recourse problem

In this section, we analyze the BC recourse problem \( \max_{S^d} Q^{BC}(x, d) \) for a fixed \( x \in X \). Letting \( \beta = [\beta_1, \ldots, \beta_B]^T \) be the dual variables associated with constraints (2d), we formulate the inner linear program \( Q^{BC}(x, d) := \min \{ \sum_{b=1}^{B} (c_{ob}^o + c_{ob}^g) : (2d), (o, g) \geq 0, \forall b \in [B] \} \) in its dual form as:

\[
Q^{BC}(x, d) = \max_{\beta} \sum_{b=1}^{B} \left( \sum_{i=1}^{I} d_{i,x_{i,b}} - L_{b} \right) \beta_b
\]

(8a)
s.t. \(-c_b^g \leq \beta_b \leq c_b^o, \quad \forall b \in [B] \) \quad (8b)

In view of (8), problem \( \max_{d \in S^4} Q^{BC}(x, d) = \max \{ \sum_{b=1}^{B} ( \sum_{i=1}^{I} \max_{d_i \in [d_i, \tilde{d}_i]} d_i x_{i,b} - \mathcal{L}_b) \beta_b : (8b) \} \) is equivalent to the minimization problem in (C.2) (see Appendix C for a detailed proof).

**Proposition 2.** For any \( x \in X \), \( \max \{ \sum_{b=1}^{B} ( \sum_{i=1}^{I} \max_{d_i \in [d_i, \tilde{d}_i]} d_i x_{i,b} - \mathcal{L}_b) \beta_b : (8b) \} \) is equivalent to

\[
\min_{\eta \geq 0} \sum_{b=1}^{B} \eta_b \quad \text{s.t. } \eta_b \geq ( \sum_{i=1}^{I} d_i x_{i,b} - \mathcal{L}_b)(-c_b^g), \quad \forall b \in [B] \quad (9b)
\]

\[
\eta_b \geq ( \sum_{i=1}^{I} d_i x_{i,b} - \mathcal{L}_b)(c_b^o), \quad \forall b \in [B] \quad (9c)
\]

Note that Neyshabouri and Berg (2017) reformulate their BC problem as a single (maximization) mixed-integer programming problem. Thus, Neyshabouri and Berg (2017) formulation requires incorporating optimality cuts for the BC recourse problem in the master problem of their C&CG approach. In contrast, our BC recourse problem (C.2) can be combined with the outer minimization problem in (7). As such, we do not need to incorporate inequalities for the BC recourse problem in the master problem of our C&CG algorithm, which results in slower growth in the size of our master problem and faster convergence of our algorithm than Neyshabouri and Berg (2017).

### 3.4.2. Reformulation of ICU recourse problem

In this section, we analyze the ICU recourse problem \( \max_{l \in S^j} Q^{ICU}(x, l) \). Thanks to the work of Neyshabouri and Berg (2017), we address the uncertainty in the number of constraints of type (2b) by transforming patient LOS in the ICU to an arrival/departure process from the ICU. In what follows, we slightly modify Neyshabouri and Berg (2017) idea to fit DRESS and refer to Neyshabouri and Berg (2017) for the original idea.

Let binary decision variable \( v_{i,t} = 1 \) if patient \( i \) enters the ICU “by” day \( t \), and \( v_{i,t} = 0 \) otherwise (note that we use “by” rather than “at” in the definition of these variables). Let binary decision variable \( w_{i,t} = 1 \) if patient \( i \) leaves the ICU by day \( t \), and \( w_{i,t} = 0 \) otherwise. Given that \( l_i \in [\underline{l}_i, \overline{l}_i] \) and w.l.o.g. \( \underline{l}_i \) and \( \overline{l}_i \) are integers, feasible regions \( \mathcal{V} \) and \( \mathcal{W} \) capture the arrival to and departure from ICU, respectively.
\[ \mathcal{V} := \left\{ \begin{array}{l}
(V1) \quad v_{i,t} \geq x_{i,b}, \quad t = t_b, \forall i \in [I], b \in [B], \\
(V2) \quad v_{i,t} \leq 1 - x_{i,b}, \quad t = 1, \ldots, t_b - 1, \forall i \in [I], b \in [B], \\
(V3) \quad v_{i,t} \leq v_{i,t+1}, \quad \forall i \in [I], t \in [T], \\
(V4) \quad v_{i,t} \in \{0, 1\}, \quad \forall i \in [I], t \in [T].
\end{array} \right\} \]  

\[ \mathcal{W} := \left\{ \begin{array}{l}
(W1) \quad w_{i,t} \geq x_{i,b}, \quad t = t_b + I_i - 1, \ldots, T, \forall i \in [I], b \in [B], \\
(W2) \quad w_{i,t} \leq 1 - x_{i,b}, \quad t = 1, \ldots, t_b + I_i - 1, \forall i \in [I], b \in [B], \\
(W3) \quad w_{i,t} \leq w_{i,t+1}, \quad \forall i \in [I], t \in [T], \\
(W4) \quad w_{i,t} \in \{0, 1\}, \forall i \in [I], t \in [T].
\end{array} \right\} \]  

Constraints (V1) ensure that each scheduled patient enters the ICU on the day of his/her surgery. Constraints (V2) prevent patients from going to the ICU before the day of the surgery. Constraints (V3) indicate that if a scheduled patient \( i \) arrives to the ICU by day \( t \), then s/he will be in the ICU in days after \( t \). Constraints (W1) ensure that a scheduled patient \( i \) stays in the ICU for at most \( I_i \) days. Constraints (W2) ensure that each scheduled patient \( i \) remains in the ICU for at least \( I_i \) days (i.e., a patient cannot leave ICU before her/his minimum LOS). Constraints (W3) indicate that if a scheduled patient \( i \) leaves the ICU by day \( t \), then s/he will leave the ICU by day after \( t \). Accordingly, LOS of each patient \( i \) equals to \( \sum_{t=1}^{T} (v_{i,t} - w_{i,t}) \) and the number of patients in the ICU on day \( t \) equals to \( \sum_{i=1}^{I} (v_{i,t} - w_{i,t}) \).

Note from constraints (V1) that variables \( x_{i,b} \) defines variables \( v_{i,t} \). Thus, variables \( v_{i,t} \) belong to the first stage of DRESS but do not contribute to the objective of the first stage. In contrast, \( w_{i,t} \) is a second stage variable. As such, we redefine the ambiguity set that describes LOS in the ICU based on patients’ departure from the ICU, while their arrival is an input parameter to the new ambiguity set \( W \). Given \( x \in \mathcal{X} \) and \( v \in \mathcal{V} \), we calculate the worst-case ICU costs by choosing the time that the patient leaves the ICU, \( w_{i,t} \), via the following alternative and equivalent formulation of the ICU recourse problem:

\[
\max_{w \in \mathcal{W}} \min_u \sum_{t \in T} c_t u_t \\
\text{s.t.} \quad \sum_{i \in I} (v_{i,t} - w_{i,t}) - R_t \leq u_t, \quad \forall t \in [T] \\
u_t \geq 0, \quad \forall t \in [T]
\]  

\[ (12) \]
As we show in the proof of Proposition 3 in Appendix D, problem \((12)\) (equivalently, ICU recourse problem \(\max_{l \in S^l} Q^{ICU}(x, l)\)) is equivalent to the maximization problem in \((13)\).

### Proposition 3

For any \(x \in X\) and \(v \in V\), \(\max_{l \in S^l} Q^{ICU}(x, l)\) is equivalent to

\[
\begin{align*}
\max_{\alpha, \rho, \eta} & \sum_{i \in I} \sum_{t = 1}^T v_{i,t} \lambda_t - \sum_{i \in I} \sum_{t = 1}^T q_{i,t} - \sum_{t \in T} R_t \lambda_t & (13a) \\
\text{s.t.} & \quad w \in W, \quad 0 \leq \lambda_t \leq c_t, \quad \forall t \in [T] & (13b) \\
& \quad q_{i,t} \geq 0, \quad q_{i,t} \geq \lambda_t - c_t(1 - w_{i,t}), \quad q_{i,t} \leq c_t w_{i,t}, \quad q_{i,t} \leq \lambda_t, \forall i \in [I], \forall t \in [T] & (13c)
\end{align*}
\]

#### 3.4.3 Solvable reformulation

Given that parameters \(d\) and \(l\) are independent, we can reformulate the last term in problem \((7)\)

\[
\begin{align*}
\min_{\alpha, \rho} \sum_{i = 1}^I \max_{i \in [d_i, \bar{d}_i]} (d_i \rho_i + \bar{d}_i \alpha_i) & \text{ as (see Appendix E for details)} \\
\begin{multlined}
\min_{\alpha, \rho} \sum_{i = 1}^I \left( \bar{d}_i \rho_i + (d_i - \bar{d}_i) \alpha_i \right) = L_i \alpha_i + (L_i - L_i) r_i) : k_i \geq \rho_i, r_i \geq \alpha_i, (k_i, r_i) \geq 0, \forall i \in [I]
\end{multlined}
\end{align*}
\]

In view of Propositions 2 and 3, problem \((7)\) is equivalent to

\[
\begin{align*}
\min_{\rho, \alpha, \eta, \delta} & \sum_{i \in I} \left( \mu_i^d \rho_i + \mu_i^l \alpha_i \right) + \sum_{b \in B} \eta_b + \sum_{i \in I} \left[-(\bar{d}_i \rho_i + (d_i - \bar{d}_i) \alpha_i) - (L_i \alpha_i + (L_i - L_i) r_i) \right] + \delta & (14a) \\
\text{s.t.} & \quad \text{as } (9b) - (9c), \quad \forall b \in [B] & (14b) \\
& \quad k_i \geq \rho_i, r_i \geq \alpha_i, \quad \forall i \in [I] & (14c) \\
& \quad (k_i, r_i) \geq 0, \quad \forall i \in [I] & (14d) \\
& \quad \delta \geq \max_{\lambda, \alpha, \eta, \delta} \sum_{t \in T} \sum_{i \in I} v_{i,t} \lambda_t - \sum_{t \in T} \sum_{i \in I} q_{i,t} - \sum_{t \in T} R_t \lambda_t : \text{as } (13b) - (13c) \}
\end{align*}
\]

Formulation \((14)\) assumes fixed surgery assignment decisions. To determine surgery assignment decisions, we first multiply the mean, lower bound, and upper bound of each random parameter in \((14a)\) with the associated surgery assignment decisions and then combine the inner problem in the form of \((14)\) with the outer minimization problem in \((4)\). This leads to the following equivalent reformulation of DRESS model in \((4)\).

\[
\begin{align*}
\min_{x, \rho, \alpha, \eta, \delta} \left( \sum_{i \in I} \sum_{b \in B} c_{i,b} x_{i,b} + \sum_{i \in I} \left( \sum_{b \in B} \mu_i^d \rho_i x_{i,b} + \sum_{b \in B} \mu_i^l \alpha_i x_{i,b} \right) \right) + \sum_{i \in I} \left[-(\sum_{b \in B} \bar{d}_i \rho_i x_{i,b} + \sum_{b \in B} (d_i - \bar{d}_i) k_{i,x_{i,b}}) - (\sum_{b \in B} L_i x_{i,b} x_{i,b} + \sum_{b \in B} (L_i - L_i) r_i x_{i,b}) \right] + \delta \quad (15a)
\end{align*}
\]
s.t. \( x \in \mathcal{X}, \ (14b) - (14e) \) \hspace{1cm} (15b)

Note that the objective function (15a) contains the interaction terms \( \rho_i x_{i,b}, \alpha_i x_{i,b}, k_i x_{i,b}, \) and \( r_i x_{i,b} \) with binary variables \( x_{i,b} \) and continuous variables \( \rho_i, \alpha_i, k_i, \) and \( r_i \). To linearize, we define \( \zeta_{i,b} = \rho_i x_{i,b}, \tau_{i,b} = \alpha_i x_{i,b}, \varphi_{i,b} = k_i x_{i,b}, \) and \( \gamma_{i,b} = r_i x_{i,b} \). We also introduce the following McCormick inequalities (16a)–(16d) for variables \( \varphi_{i,b}, \gamma_{i,b}, \zeta_{i,b}, \) and \( \tau_{i,b} \) for all \( i \in [I] \) and \( b \in [B] \). In Appendix \[\text{F}\] we derive tight upper \((\bar{k}, \bar{r}, \bar{\rho}, \bar{\alpha})\) and lower \((\bar{k}, \bar{r}, \rho, \alpha)\) bounds of variables \((k, r, \rho, \alpha)\) to strengthen inequalities (16a)–(16d).

\[
\begin{align*}
\varphi_{i,b} &\geq 0, \quad \varphi_{i,b} \leq k_i - \bar{k}_i (1 - x_{i,b}), \quad \varphi_{i,b} \leq k_i - 1 - \bar{k}_i (1 - x_{i,b}) \\
\gamma_{i,b} &\geq 0, \quad \gamma_{i,b} \leq r_i - \bar{r}_i (1 - x_{i,b}), \quad \gamma_{i,b} \leq r_i - 1 - \bar{r}_i (1 - x_{i,b}) \\
\zeta_{i,b} &\geq \rho_i x_{i,b}, \quad \zeta_{i,b} \leq \bar{\rho}_i x_{i,b}, \quad \zeta_{i,b} \leq \rho_i - \bar{\rho}_i (1 - x_{i,b}) \\
\tau_{i,b} &\geq \alpha_i x_{i,b}, \quad \tau_{i,b} \leq \bar{\alpha}_i (1 - x_{i,b}), \quad \tau_{i,b} \leq \alpha_i - \bar{\alpha}_i (1 - x_{i,b})
\end{align*}
\] (16)

Accordingly, formulation (15) (equivalently, DRESS model in (4)) is equivalent to:

\[
\begin{align*}
\min \left( \sum_{i \in I} \sum_{b \in B \setminus \{b'\}} c_{i,b} x_{i,b} + \sum_{i \in I} \sum_{b \in B} \left( \sum_{b' \in B} \mu_i^d \zeta_{i,b} + \sum_{b \in B} \mu_i^l \tau_{i,b} \right) + \sum_{b \in B} \eta_b 
+ \sum_{i \in I} \left[ - \left( \sum_{b \in B} \widehat{d}_i \zeta_{i,b} + \sum_{b \in B} (d_i - \widehat{d}_i) \varphi_{i,b} x_{i,b} \right) - \left( \sum_{b \in B} \widehat{l}_i \tau_{i,b} + \sum_{b \in B} (l_i - \widehat{l}_i) \gamma_{i,b} \right) \right] + \delta \right)
\text{s.t.} \quad x \in \mathcal{X}, \ (14b) - (14e), \ (16a) - (16d) \\
\delta \geq \max_{\lambda, q, \nu} \left\{ \sum_{t \in T} \sum_{i \in I} v_{i,t} \lambda_t - \sum_{t \in T} \sum_{i \in I} q_{i,t} - \sum_{t \in T} R_t \lambda_t : \ (13b) - (13c) \right\}
\end{align*}
\] (17)

4. Solution approach

Recall from Section 3.2 that the ICU recourse problem, and its equivalent reformulation in constraints (17c), are feasible for any feasible first-stage surgery assignment decision \( x \). This observation motivates us to apply a column-and-constraints generation (C&CG) based-algorithm to solve DRESS model in (17). In section 4.1 we present our C&CG algorithm. In Section 4.2 we present strategies to improve the convergence of the algorithm.

4.1. Column-and-constraints generation

Algorithm [1] presents DRESS–C&CG. This algorithm resembles the C&CG of Neyshabouri and Berg (2017), which is based on the original C&CG of Zeng and Zhao (2013) and is implemented in a master-sub problem framework (see also Chan et al. (2017); Xiao et al. (2018) for C&CG application...
in two-stage DR optimization). DRESS-C&CG is also motivated by Zeng’s intuition that only a few important scenarios of LOS from the worst-case distribution play a significant role in obtaining an optimal solution to DRESS. This is very different from the 2-stage SP in which every single scenario in the scenario set contributes to the optimal values, and thus it is necessary to consider the complete scenario set in deriving an optimal solution to the 2-stage SP model.

The master problem in Algorithm 1 employs the (scenario-based) ICU capacity constraints, and the ICU sub-problem generates LOS scenarios from the worst-case distribution. The intuition of the algorithm is as follows. At each iteration, we first solve the master problem, which includes only a subset of LOS scenarios that have a nonzero probability in the worst-case distribution of LOS for at least one of the previous iterations, and obtain the optimal surgery assignment and admission times to the ICU. Hence, by considering a subset of LOS scenarios, the master problem is a relaxation of DRESS and thus provides a lower bound on the optimal value of (17).

Second, using surgery assignment and admission times that we obtain from solving the master problem, we identify the worst-case distribution of departure time \( w \) from the ambiguity set \( \mathcal{W} \). Third, we compute and pass back the LOS scenarios supporting the worst-case distribution (obtained by solving the subproblem) to the master problem on as-needed basis, along with introducing relevant ICU status and capacity variables and constraints. Then, we solve the master problem again with the new information from the subproblem. This process continues until the gap between the lower and upper bound on the optimal value of DRESS obtained in each iteration satisfies a predetermined termination tolerance \( \epsilon \) (we use \( \epsilon = 0 \) in our experiments).

In contrast to the RO and C&CG approach of Neyshabouri and Berg (2017), our DR formulation (17) does not require the addition of inequalities for the BC recourse problem, which results in a slower growth in the size of our master problem and faster convergence. Given that the ICU subproblem is feasible for any feasible solution of the master problem, we do not need to add any feasibility cuts as in the original C&GG of Zeng and Zhao (2013). Additionally, given the complete recourse property of our sub-problem, DRESS-C&CG terminates in a finite number of iterations (we refer to Zeng and Zhao (2013) for a detailed proof). However, the complexity of the master problem may increase at each iteration. This is because at each iteration \( n \), we add \((I + 1)T\) new variables and \( T + 1 + \sum_{i \in I} l_i^n \) new constraints to the master problem, where \( l_i^n = \sum_{t \in T} (v_{i,t} - w_{i,t}^*) \) is the realization of the LOS for patient \( i \) at iteration \( n \). Hence, the size of the master problem may quickly grow for an instance of DRESS with a large number of patients and high variability in the length-of-stay.
**Algorithm 1**: DRESS column-and-constraint generation (DRESS-C&CG).

1. **Initialization.** Set $LB = -\infty$, $UB = +\infty$, $N = 0$, $O = \emptyset$

2. **Master Problem.** Solve the following master problem

$$
\text{opt} = \min \left( \sum_{i \in I} \sum_{b \in B(i)} c_{i,b}x_{i,b} + \sum_{i \in I} \left( \sum_{b \in B} \mu_i^b \xi_{i,b} + \sum_{b \in B} \mu_i^b \tau_{i,b} \right) + \sum_{b \in B} \eta_b \right)
+ \sum_{i \in I} \left[ - \left( \sum_{b \in B} \xi_{i,b} + \sum_{b \in B} \left( d_i - \bar{d}_i \right) \varphi_{i,b}x_{i,b} \right) - \left( \sum_{b \in B} \bar{t}_i \tau_{i,b} + \sum_{b \in B} \left( d_i - \bar{t}_i \right) \gamma_{i,b} \right) \right] + \delta 
$$

s.t. $\delta \geq \sum_{i \in T} c_i u_i^n$, $\forall n \in O$ (18a)

$$
x \in \mathcal{X}
$$

(14b) – (14d), (16a) – (16d) (18b)

$$
y_{i,t}^n \geq x_{i,b}, \forall i \in [I], b \in B(i), t = t_b, \ldots, t_b + l_i^n - 1, \forall n \leq N
$$

(18c)

$$
\sum_{i \in I} y_{i,t}^n - R_t \leq u_i^n, \forall t, \forall n \leq N
$$

(18d)

$$
y_{i,t}^N \in \{0,1\}, u_i^n \geq 0, \forall i \in [I], t \in [T], \forall n \leq N
$$

(18e)

Record the optimal solutions $x^*, \delta^*$ and set $LB = \text{opt}^*$ (i.e., optimal value of (18a)).

3. **ICU Recourse Problem**

3.1. construct the ICU problem: if $x_{i,b}^* = 1$, then set $v_{i,t} = 1, \forall t \geq t_b$, and $v_{i,t} = 0, \forall t < t_b$; else if $x_{i,b}^* = 1$, set $v_{i,t} = 0, \forall t$.  

3.2 solve the ICU problem (13a)–(13c) with $(x^*, v)$ and record the objective value $\text{opt}^{\text{ICU}}$ and the optimal $w_{i,t}^*$. Update $UB = \min \{UB, (LB - \delta^*) + \text{opt}^{\text{ICU}}\}$.  

3.3 if $UB - LB \leq \epsilon$ terminate and return $x^*$ as the optimal solution for DRESS; else go to step 4 (column-and-constraint generation routine).

4. **Column-and-Constraint Generation Routine**

4.1 use $w_{i,t}^*$ from step 3.2 to calculate the length-of-stay $l_i^{N+1} = \sum_{t \in T} (v_{i,t} - w_{i,t}^*)$ for each patient $i \in [I]$ at iteration $N + 1$.

4.2 add variables $y_{i,t}^{N+1}$, $\forall i \in [I], t \in [T]$, and $u_i^{N+1}$, $\forall t$, and the following constraints to the master problem:

$$
\delta \geq \sum_{i \in T} c_i u_i^{N+1}
$$

$$
y_{i,t}^{N+1} \geq x_{i,b}, \forall i \in [I], b \in [B], t = t_b, \ldots, t_b + l_i^{N+1} - 1
$$

$$
\sum_{i \in I} y_{i,t}^{N+1} - R_t \leq u_i^{N+1}, \forall t \in [T]
$$

update $N \leftarrow N + 1$, $O \leftarrow O \cup \{N + 1\}$ and go to step 2.
4.2. Structural properties

4.2.1. Linear relaxation of variables $y_{i,t}$

Thanks to the work of Min and Yih (2010), in Proposition 4, we observe that linear relaxation of variables $y_{i,t}$, for all $i \in [I]$ and $t \in [T]$, in the master problem of DRESS-C&CG algorithm will have optimal integer solutions whenever the right-hand side is integral. Given that $x_{i,b}$ is binary and $R_t$ is integer, we can relax constraints $y^n \in \{0, 1\}$ to $0 \leq y^n_{i,t} \leq 1$, $\forall i \in [I], t \in [T], \forall n \leq N$.

Proposition 4. The matrix defining the feasible region of variables $y^n_{i,t}$, for all $i \in [I], t \in [T], and n \in [N]$ in the master problem of C&CG is totally unimodular (TU).

Proof. We refer the reader to the proof of Proposition 1 in Min and Yih (2010).

4.2.2. Symmetry breaking inequalities

In this section, we derive symmetry breaking inequalities to break symmetry in the solution space of $x \in X$ in the master problem (18). Specifically, we aggregate the patients into classes, each consisting of patients having a common surgery type, distribution of surgery durations and LOS, and priority. Suppose that we have $C$ classes and $C_c$ be the set of patients in class $c$, $c = 1, \ldots, C$.

W.l.o.g, we assume that patients within each class are numbered sequentially. Additionally, w.l.o.g, we assume that the available blocks $B_c$ for class $c$ are numbered sequentially. Accordingly, we add the following inequalities to problem (18):

$$ x_{i,b} - \sum_{j=b}^{B_c} x_{i+1,j} - x_{i+1,b'} \leq 0, \forall b = 1, \ldots, B_c, \forall i, i + 1 \in C_c, c = 1, \ldots, C. \quad (19) $$

enforcing arbitrary assignments of surgery of the same class to surgical blocks. Inequalities (19) are novel in the sense that they can also break symmetries in the solution space of scheduling decisions of SP, RO, and even the deterministic formulation.

5. Computational results

In this section, we use a well-known case study to construct several instances of DRESS and compare the computational and solution performances of the DR and a benchmark SP model (see Appendix G for the formulation). In Section 5.1, we describe the set of DRESS instances that we construct and discuss other experimental setups. In Section 5.2, we compare the computational performance of DR and SP models. In Section 5.3, we compare the optimal schedules of the DR and SP. In Section 5.4, we compare the performance of optimal schedules of DR and SP via out-of-sample
Table 2: Block schedule (Min and Yih (2010); Neyshabouri and Berg (2017)).

<table>
<thead>
<tr>
<th>OR room</th>
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<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
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<td>ENT</td>
<td>ENT</td>
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</tr>
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<td>UROLOGY</td>
<td>UROLOGY</td>
<td>UROLOGY</td>
<td>UROLOGY</td>
</tr>
</tbody>
</table>

Table 3: Statistics for surgery duration (in minutes) and patient LOS (in days) based on surgery type. Notation: percent% is the percentage of patients needing a specific type of surgery, \((\mu^d, \mu^l)\) and \((\sigma^d, \sigma^l)\) are respectively the mean and standard deviations of \((d, l)\).

<table>
<thead>
<tr>
<th>Surgery type</th>
<th>percent%</th>
<th>(\mu^d)</th>
<th>(\sigma^d)</th>
<th>(\mu^l)</th>
<th>(\sigma^l)</th>
</tr>
</thead>
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<td>37</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>OBGYN</td>
<td>9.26</td>
<td>86</td>
<td>40</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>ORTHO</td>
<td>23.26</td>
<td>107</td>
<td>44</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>NEURO</td>
<td>5.04</td>
<td>160</td>
<td>77</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>GEN</td>
<td>22.12</td>
<td>93</td>
<td>49</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>OPTH</td>
<td>2.98</td>
<td>38</td>
<td>19</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>VASCULAR</td>
<td>8.20</td>
<td>120</td>
<td>61</td>
<td>3.5</td>
<td>3.5</td>
</tr>
<tr>
<td>CARDIAC</td>
<td>2.44</td>
<td>240</td>
<td>103</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>UROLOGY</td>
<td>5.36</td>
<td>64</td>
<td>52</td>
<td>0.8</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Simulation tests. In Section 5.4, we conduct sensitivity analysis, and provide managerial insights into DRESS, by examining trade-offs between costs, utilization, and capacity.

5.1. Description of experiments

Due to the lack of data from a real surgical setting, our computational study is based on the practice configuration of a regional hospital presented by Min and Yih (2010) (a well-known benchmark instance of stochastic elective surgery scheduling). Specifically, there are 9 different surgical specialties, 10 available ORs, and 32 surgical blocks. Table 2 presents the weekly assignments of surgical blocks to ORs. Each block is 8-hours long, and a patient can be assigned to any of the blocks allocated to his/her surgery type during the planning horizon. Table 3 provides statistics on the mean and standard deviation of surgery durations and patients’ LOS (by type).

We consider DRESS instances of 40-, 70-, and 140- surgeries, and we follow the same procedure as in the DR and other appointment scheduling studies (see. e.g., Jiang et al. (2017); Shehadeh et al. (2019b); Mak et al. (2014)) to generate random parameters for each instance as follows. First, we use the percentage of patients needing a specific type of surgery to generate lists of \(I = 40, 70,\) and \(140\) patients randomly. We set the mean and standard deviation of surgery duration \((\mu^d_i, \sigma^d_i)\) and patient LOS \((\mu^l_i, \sigma^l_i)\) of each patient \(i\) to their type-based values.

To approximate the lower \((d, l)\) and upper \((\bar{d}, \bar{l})\) bounds of random parameters \((d, l)\), we respec-
tively use the 20%-quantile and 80%-quantile values of the $N$ in-sample data. We generate the in-sample $N = 1000$ realizations of $(d_1^n, l_1^n), \ldots, (d_I^n, l_I^n)$, for all $n \in [N]$, by following lognormal (LogN) distributions for $d_i$ and $l_i$, for all $i \in [I]$, with $(\mu_{d_i}^d, \sigma_{d_i}^d)$ and $(\mu_{l_i}^l, \sigma_{l_i}^l)$, respectively. LogN is a common distribution to model service duration in the appointment scheduling literature (Gul et al. (2011); Min and Yih (2010); Shehadeh et al. (2019a)). We round each random parameter to the nearest integer.

We consider three different cost structures for the objective function: (1) Cost1: $c^o = \$26/\text{min}$, $c^g = c^o/1.5$, $c^t = 100$, (2) Cost2: $c^o = \$100/\text{min}$, $c^g = c^o/1.5$, $c^t = 100$, and (3) Cost3: $c^o = \$26/\text{min}$, $c^g = 0$, $c^t = 100$. An overtime cost of $26$ per minute is based on the work of Min and Yih (2010) and Stodd et al. (1998) who show that an hour of OR overtime costs $1560$. We fix the ratio $c^o/c^g$ to 1.5 as in prior surgery scheduling studies (Jebali and Diabat (2015); Liu et al. (2019); Shehadeh et al. (2019a)). Finally, we follow the same procedure as in Min and Yih (2010) to generate patient costs $c_{i,b}$ and $c_{i,b}^\prime$.

We perform the majority of our experiments with two ICU capacities, 5 and 10 beds. For each instance of DRESS, we optimize the SP model with the generated $N$ scenarios and the DR model with the generated mean and support of random parameters. We add symmetry breaking inequalities (19) to both models. We implemented the SP and C&CG algorithm using AMPL2016 Programming language calling CPLEX V12.6.2 as a solver with default settings. Preliminary tests revealed that CPLEX finds good integer solutions early for DRESS instances of 140-surgery at 1% gap, but takes additional long time to prove optimality. To speed up the solution process in such a case we change the relative optimality tolerance to 1%. We ran all experiments on a laptop with an Intel Core i7 processor, 2.5 GHz CPU, and 16 GB (1600MHz DDR3) of memory.

5.2. CPU time and computational details

In this section, we compare solution times of the DR and SP models. For each list of surgery, cost structure, and ICU capacity, we generate 5 random instances for a total of 90 DRESS instances and solve each instance using the two models. We imposed a time limit of 1000 seconds for each instance. For instances that take longer than 1000 seconds to solve, we instead report the optimality gap values (in %) achieved at the end of the computation process. Table 4 presents solution times.

We first observe that solution times increase as the number of surgeries increases. The DR model can quickly solve instances of 40- and 70- surgeries to optimality with solution times ranging from 1 to 12 seconds and from 2 to 117 seconds, respectively. When $I = 140$, the DR model can
Table 4: The minimum, average, and maximum CPU times (in seconds) using the DR and SP models.

<table>
<thead>
<tr>
<th>(I, R)</th>
<th>Model</th>
<th>(c^v = 26, c^s = 1\gamma, c_t = 100)</th>
<th>(c^v = 100, c^s = 1\gamma, c_t = 100)</th>
<th>(c^v = 100, c^s = 0, c_t = 100)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Avg</td>
<td>Max</td>
<td>Min</td>
</tr>
<tr>
<td>(40, 5)</td>
<td>DR</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>SP</td>
<td>19</td>
<td>40</td>
<td>57</td>
</tr>
<tr>
<td>(40, 10)</td>
<td>DR</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>SP</td>
<td>155</td>
<td>160</td>
<td>164</td>
</tr>
<tr>
<td>(70, 5)</td>
<td>DR</td>
<td>6</td>
<td>23</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>SP</td>
<td>5%</td>
<td>6%</td>
<td>7%</td>
</tr>
<tr>
<td>(70, 10)</td>
<td>DR</td>
<td>21</td>
<td>31</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td>SP</td>
<td>12%</td>
<td>26%</td>
<td>80%</td>
</tr>
<tr>
<td>(140, 5)</td>
<td>DR</td>
<td>164</td>
<td>235</td>
<td>320</td>
</tr>
<tr>
<td></td>
<td>SP</td>
<td>90%</td>
<td>90%</td>
<td>90%</td>
</tr>
<tr>
<td>(140, 10)</td>
<td>DR</td>
<td>210</td>
<td>435</td>
<td>605</td>
</tr>
<tr>
<td></td>
<td>SP</td>
<td>90%</td>
<td>91%</td>
<td>91%</td>
</tr>
</tbody>
</table>

obtain a near-optimal solution within 12 minutes under Cost1 and Cost2 and terminates with a 2% optimality gap under Cost3 and \(R = 10\) ICU beds. In contrast, the SP model can only solve instances of 40 surgeries and takes from 5 to 78 times longer than the DR to solve these instances. The optimality gap of the SP ranges from 5% to 80% and 90% to 91% with 70- and 140- surgeries, respectively.

Solution times of DR and SP models without symmetry breaking inequalities \([19]\) are significantly longer than those in Table 4. For example, average DR solution times of instance (70, 5) with and without inequalities \([19]\) are 23 and 990, respectively. The SP terminates without any feasible MIP solution (and thus no upper bound) for this instance without inequalities \([19]\).

Next, we discuss details of solving the DR model using DRESS–C&CG. Table H.1 in Appendix H illustrates the total number of branching nodes, the total number of MIP simplex iterations, the number of iterations \((N)\) in DRESS–C&CG before it converges to the optimum or reaches the time limit, and the average CPU seconds taken by the master (mast) and the sub (sub) problems in each iteration. From Table H.1, we observe that, for all cost structure and ICU capacity, the computational effort increases as the number of surgeries increase. For example, under Cost1 and \(R = 5\), (mast, \(N\)) increases from (0.5 sec, 4 iters) to (5 sec, 9 iters) as the number of surgeries increases from 40 to 70, respectively. Furthermore, the relatively small number of iterations (ranges from 3 to 53) support our hypothesis in Section 4.1 that only a subset of LOS scenarios from the worst-case distributions play a significant role in obtaining an optimal solution to DRESS.

To generalize our conclusions, we increase the mean and standard deviation of LOS by 1 day for each surgery and repeat the same steps to generate and solve DRESS instances of 70- and 140- surgeries. Table H.2 and Table H.3 in Appendix H present CPU times and the details of solving
these instances, respectively. Clearly, the computational effort increases under higher variability of LOS. However, the SP becomes more challenging to solve and terminates with a large optimality gap, while our DR approach maintains superior computational performance. For example, under Cost1 and (70 surgeries, 5 ICU beds), the average optimality gap of the SP increases from 6% (Table H.1) to 36% (Table H.2) and the average solution time of the DR increases from 23 to 177 seconds.

5.3. Optimal DR and SP scheduling decisions

In this section, we compare the optimal schedules of the DR and SP models in terms of the number of scheduled surgery and surgery assignment patterns to surgical blocks. For presentation brevity and illustrative purposes, hereafter, we focus on DRESS instances of 70-surgeries. Table I.1 in Appendix I presents the number of postponed surgeries. In general, the DR model schedules fewer surgeries than the SP model, especially when the cost of OR idle time is zero (under Cost3). For example, under Cost3 and when ICU capacity is 5 beds, the DR and SP schedule 48- and 56- out of 70 surgeries, respectively. Intuitively, by scheduling fewer surgeries, DR model intends to mitigate ICU congestion. In contrast, under (Cost1, Cost2) for which the OR idle time is costly, the DR and SP models respectively schedule (68, 69) and (69, 70).

Given that both the DR and SP schedule approximately the same number of surgeries under Cost1–2, we turn our attention to the optimal surgery assignment patterns of the DR and SP models under these costs. For illustrative purposes, we focus on the optimal assignment decisions of the ENT, ORTHO, and GEN surgeries as they represent the largest percentage of surgeries (46 out of 70 surgeries) and have multiple surgical blocks during the week. Table I.2 in Appendix I present the optimal surgery assignment decisions of the DR and SP models under Cost1 (assignment decisions under Cost2 are similar to those under Cost1).

From Table I.2, we observe that the DR model tends to schedule more surgeries at the start of the week (Monday), a fewer or same number of surgeries during Tuesday–Thursday, and more or same number of surgeries on Friday than the SP model. For example, when $R= 5$ ICU beds, DR schedules (4, 6) ENT and (3, 2) GEN surgeries and the SP schedules (0, 3) ENT and (1,2) GEN surgeries on (Monday, Friday). Note that GEN and ENT surgeries may not require ICU recovery and have short LOS (mean LOS are 0.1 and 0.05 days, respectively). Thus, it makes sense to schedule more of these surgeries at the beginning of the week with other types of surgery as in the DR to mitigate ICU congestion.
5.4. Out-of-sample performance

In this section, we compare the out-of-sample simulation performance of the optimal DR and SP schedules under “perfect” and “misspecified” distributional information. We simulate the optimal DR and SP schedules using the following two sets of $N'$ out-of-sample data.

1. **Perfect Information:** We use the same parameter settings that we use for generating the $N$ in-sample data to generate the $N'$ simulation data (i.e., LogN for $d$ and LogN for $l$ with mean and standard deviations as in Section 5.1). This is to simulate the in-sample performance.

2. **Misspecified Distribution:** We keep the same mean values ($\mu_d$, $\mu_l$) and ranges of random parameters ($d$, $l$), but we vary the distribution type of ($d$, $l$) to generate the $N'$ data. Specifically, for surgery durations, we follow positively correlated truncated normal and weibull distributions with support $[0, d]$ to generate realizations $d_1^n, \ldots, d_I^n$, for all $n \in [N']$. For patients’ LOS, we follow positively correlated truncated normal and uniform distributions with support $[0, l]$ to generate $l_1^n, \ldots, l_I^n$, for all $n \in [N']$. We follow the same standard statistical method in prior DR appointment scheduling literature (see, e.g., Jiang et al. (2017)) to design the parameters of the normal, weibull, uniform distributions to obtain positive data correlations and also to keep the mean and support of the $N'$ out-of-sample realizations the same as those of $N$ in-sample realizations. This is to simulate the out-of-sample performance of the DR and SP optimal schedules when the in-sample data is biased.

We evaluate the out-of-sample performance of the optimal DR and SP as follows. First, we fix the optimal assignment variables in the SP model. Then, we use the $N'$ data to compute OR overtime (OverT), number of transfers (TransF) from ICU, and the probability of having to transfer a patient from ICU (ProbTrans) which is equivalent to the probability of lack of ICU capacity.

Table J.1 in Appendix J presents means and quantiles of OverT (per day), TransF (per day), and ProbTrans under perfect distributional information. Clearly, the optimal schedules of the DR model outperforms those of the SP model with less OverT, a fewer TransF, and a lower ProbTrans. For example, under Cost1 and when the ICU capacity is 5 beds, the average (OverT, TransF) yielded by the optimal schedules of DR and SP are (54 min, 2.2 patients) and (70 min, 4.6 patients), respectively. These results imply that when the distributional information is accurate, the DR model yields a near-optimal schedule.

Tables J.2 and J.3 in Appendix J present simulation performance under misspecified distributional information. From these results, we observe that the optimal schedules of the DR model
yield a significantly shorter OverT, fewer TransF, and lower ProbTrans when the distributions of random parameters are misspecified. The significantly lower TransF of the DR schedules is a desirable property given that patients with premature discharge from the ICU are more likely to require readdmission to the ICU than those who are discharged electively. These results show how the optimal schedules of the SP can become sub-optimal when the distributions of \((d,l)\) are misspecified. In contrast, the optimal schedules of the DR model are more robust, i.e., maintain excellent operational performance under different probability distributions of \((d,l)\).

5.5. Sensitivity analysis and managerial insights

In this section, we provide managerial implications of our study by examining the sensitivity of the optimal DR schedules to ICU capacity and cost parameters. First, we increase the number of ICU beds from 3 to 15 and report: (1) percentage of scheduled surgeries, (2) OR utilization, i.e., number of scheduled blocks/total number of available blocks (3) number of transfers from the ICU under Cost1, and (4) ICU utilization under Cost1. The first metric reflects OR throughput, patient access to surgery, and hospital profit from a scheduled surgery. The second and fourth metrics evaluate the appropriate utilization of the given resources. The third metric measures ICU congestion. For illustrative purposes, we consider an instance of 70 surgeries.

Figure 1 presents the percentage of scheduled surgeries and OR utilization under Cost1 and Cost3 as a function of ICU capacity. Recall that under Cost3, for which the cost of idling is zero, the DR model tends to schedule a fewer number of surgeries. Figure 1 suggests that a slight expansion of ICU can increase the percentage of scheduled surgeries and OR utilization. For example, increasing the number of ICU beds from 5 to 7 increases the percentage of scheduled surgeries from 69% to 78% and OR utilization from 63% to 75%. The average overtime, number of transfers, and ICU utilization under this cost structure ranges from 1 to 10 minutes, 0 to 1 patients, and 50% to 70%, respectively.

In contrast, under Cost1, which incorporates the cost of OR idling, the DR schedules ~97% of surgeries irrespective of ICU capacity, which maintain high OR utilization but may result in a congested ICU when the ICU capacity is tight (e.g., 3 beds). Figure 2 presents the average number of transfers (TransF) from the ICU and ICU utilization under Cost1 as a function of ICU capacity. Clearly, increasing the ICU capacity from 3 to 7 beds results in reducing TransF from 3.8 to 1 while maintaining a high ICU utilization (~93% with 7 beds). ICU utilization drastically decreases with 8-15 beds. OR overtime ranges from 51 to 68 minutes under this cost. We obtain similar results,
Next, we investigate the effect of $c_t$, cost of inadequate ICU capacity, on the optimal number of scheduled surgery, OR utilization, and ICU utilization. In Figure K.3, we keep the OR overtime and idle time cost as in Cost1 and increase $c_t$ from 100 to 1000 and compute these metrics for the case of tight ICU capacity of 5 beds. From this figure, we observe that the percentage of scheduled surgery, OR utilization, and ICU utilization decrease as $c_t$ increases. The average number of transfers from ICU ranges from 0.5 ($c_t=1000$) to 2 (under $c_t=100$).

These results provide an example of how our approach can serve as a tool for elective surgery and downstream capacity planning under uncertainty. In addition, our results suggest that it is important to incorporate the cost of OR idling when planning elective surgeries. Finally, we note that although our results indicate that adding more ICU beds can improve patient access to surgery and thus increase OR utilization, several cost and other constraints may make ICU expansion seem impossible. However, the annual revenue generated by performing more surgeries can justify the one time cost of adding an ICU bed to the ICU (Hawkins (2019)). We refer to Angus et al. (2000).
and Kahn (2012) for a detailed discussion on the risks and rewards of extending ICU capacity.

6. Conclusion

In this paper, we study a DRESS problem, which seeks optimal surgery scheduling decisions to minimize patient-related costs and the worst-case expected cost of OR overtime, OR idle time, and lack of ICU capacity. We take the worst-case over an ambiguity set characterized by the known mean and range of the unknown distributions of random parameters. We propose an adapted C&CG algorithm to solve DRESS, and derive a family of novel symmetry breaking inequalities, which break symmetries in the solution space of surgery scheduling decisions and thus speed the convergence of C&CG. These inequalities are novel in the sense that they are valid for and can improve the computational performance of the SP, SO, and the deterministic formulations.

We conduct extensive numerical experiments that justify the value of our DR approach and provide managerial insights. Specifically, we demonstrate that this DR approach (1) can solve large instances of DRESS in a reasonable time, and (2) produce surgery schedules that have superior operational performance under various distributions of random parameters as compared to the SP approach. We close by providing an example of how our DR approach can be implemented to help OR managers study the trade-offs between access to surgery (in terms of the number of scheduled surgeries), OR utilization, and ICU capacity and utilization. The computational efficiency and promising operational performance of our DR approach suggest that practitioners can use this approach for elective surgery scheduling and capacity planning based on their practice configurations.

We suggest the following areas for future research. First, our results are based on the practice configuration of Min and Yih (2010) and assume that random parameters are unimodal. We would like to extend our approach by incorporating multi-modal probability distributions and higher moments of random parameters in a data-driven DR approach. Second, we aim to incorporate decisions of which OR to open and how to allocate OR capacity to surgical specialties in a DR model while considering the capacity of various downstream resources (e.g., general wards). Finally, we are interested in developing data-driven prediction models for patients’ LOS and clinical pathways to incorporate them into the scheduling process.

Acknowledgments

We are grateful to the four anonymous referees and the Editor for their constructive comments and helpful suggestions that allowed us to improve the paper. We are very thankful for the generous contribution of the anonymous reviewer to Proposition 2. Dr. Karmel S. Shehadeh would like to
dedicate her effort in this paper to all the special little girls and boys in the world who have dreams so big and so exciting—believe in your dreams, fight for them, and do whatever it takes to achieve them.

Appendix A. Summary of Related Papers and Contribution

Table A.1: History of approaches to stochastic elective surgery scheduling with ICU capacity. Notation: OverT is overtime, IdleT is idle time, SP is stochastic programming, CCSP is chance-constrained SP, SAA is sample average approximation, RO is robust optimization, DRO is distributionally robust optimization, and C&CG is column-and-constraint generation.

<table>
<thead>
<tr>
<th>Paper</th>
<th>Distributions of duration &amp; LOS</th>
<th>Random Metrics</th>
<th>Model</th>
<th>Sol. Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min and Yih (2010)</td>
<td>assumed known</td>
<td>✓</td>
<td>2-stage SP</td>
<td>SAA</td>
</tr>
<tr>
<td>Jebali and Diabat (2015)</td>
<td>assumed known</td>
<td>✓ ✓</td>
<td>2-stage SP</td>
<td>SAA</td>
</tr>
<tr>
<td>Jebali and Diabat (2017)</td>
<td>assumed known</td>
<td>✓ ✓ ✓</td>
<td>2-stage CCSP</td>
<td>SAA</td>
</tr>
<tr>
<td>Zhang et al. (2019)</td>
<td>c</td>
<td>✓ ✓</td>
<td>2-stage SP</td>
<td>SAA</td>
</tr>
<tr>
<td>Neyshabouri and Berg (2017)</td>
<td>unknown d</td>
<td>✓ ✓ ✓</td>
<td>2-stage RO</td>
<td>C&amp;CG</td>
</tr>
<tr>
<td>This paper</td>
<td></td>
<td>✓ ✓</td>
<td>2-stage DRO</td>
<td>C&amp;CG</td>
</tr>
</tbody>
</table>

A model includes constraints that ensure the respect of SICU capacity.

Consider the cost of keeping surgical blocks open.

Optimization is based on the maximum positive deviation of durations and LOS from the mean.

Optimization is based on the worst-case distribution occurring within an ambiguity set (i.e., a family of distributions).

Appendix B. Proof of Proposition 1

Proof. For a fixed $x \in \mathcal{X}$ we can formulate problem (5) as the following linear functional optimization problem.

$$\max_{\mathbb{P} \geq 0} \int_S Q(x, d, l) \, d\mathbb{P} \quad \text{(B.1a)}$$

subject to

$$\int_S d_i \, d\mathbb{P} = \mu_i^d \quad \forall i \in [I] \quad \text{(B.1b)}$$

$$\int_S l_i \, d\mathbb{P} = \mu_i^l \quad \forall i \in [I] \quad \text{(B.1c)}$$

$$\int_S d\mathbb{P} = 1 \quad \text{(B.1d)}$$

Where $[I] := \{1, 2, \ldots, I\}$. Letting $\rho = [\rho_1, \ldots, \rho_I]^T$, $\alpha = [\alpha_1, \ldots, \alpha_I]^T$, and $\theta$ be the dual variable associated with constraints (B.1b), (B.1c), and (B.1d), respectively, we present problem (B.1) in
its dual form:

\[
\min_{(\rho, \alpha) \in \mathbb{R}^I, \theta \in \mathbb{R}} \sum_{i=1}^{I} \mu_i^d \rho_i + \mu_i^l \alpha_i + \theta \\
\text{s.t.} \sum_{i=1}^{I} d_i \rho_i + l_i \alpha_i + \theta \geq Q(x, d, l) \quad \forall (d, l) \in S \tag{B.2a}
\]

where \(\rho, \alpha,\) and \(\theta\) are unrestricted in sign, and constraint \((B.2b)\) is associated with the primal variable \(P\). Under the standard assumptions that: (1) \(\mu_i^d\) belongs to the interior of the set \(\{\int_S d_i Q : Q\text{ is a probability distribution over support } S\}\), and (2) \(\mu_i^l\) belongs to the interior of the set \(\{\int_S l_i Q : Q\text{ is a probability distribution over support } S\}\), strong duality hold between \((B.1)\) and \((B.2)\) (see Bertsimas and Popescu (2005); Jiang et al. (2017); Mak et al. (2014) for a detailed discussion on these assumptions). Note that for fixed \((\rho, \alpha, \theta)\), constraints \((B.2b)\) are equivalent to \(\theta \geq \max_{(d, l) \in S} \{Q(x, d, l) - (\sum_{i=1}^{P} d_i \rho_i + l_i \alpha_i)\}\). Since we are minimizing \(\theta\) in \((B.2)\), the dual formulation of \((B.1)\) is equivalent to:

\[
\min_{\rho, \alpha} \left\{ \sum_{i=1}^{I} \mu_i^d \rho_i + \mu_i^l \alpha_i + \max_{(d, l) \in S} \left\{ Q(x, d, l) + \sum_{i=1}^{I} -(d_i \rho_i + l_i \alpha_i) \right\} \right\}
\]

Appendix C. Proof of Proposition 2

**Proof.** Note that problem \((8)\) is separable by each surgical block. Therefore, we can rewrite \(\max_{d \in S^d} Q^{bc}(x, d)\) as \(\sum_{b=1}^{B} \max_{d_i \in [d_i, \bar{d}_i]} Q^{bc}_b(x, d)\), where for each \(b \in [B]\)

\[
\max_{d_i \in [d_i, \bar{d}_i]} Q^{bc}_b(x, d) = \max_{\beta} \left( \sum_{i=1}^{I} \max_{d_i \in [d_i, \bar{d}_i]} d_i x_{i,b} - \mathcal{L}_b \right) \beta_b \tag{C.1a}
\]

\[
\text{s.t. } \mathcal{B} := \{-c^b_0 \leq \beta \leq c^b_0\}. \tag{C.1b}
\]

It is straightforward to verify that for a fixed \(x \in \mathcal{X}\) and \(d_i \in [d_i, \bar{d}_i]\), function \((C.1a)\) is convex in variables \(\beta\). Hence, problem \((C.1)\) is a convex maximization problem. It follows from the fundamental convex analysis (see, e.g., Boyd and Vandenberghe (2004)) that there exists an optimal solution \(\beta^*\) to problem \((C.1)\) at one of the extreme points \(\hat{\beta}\) of the polyhedron \(\mathcal{B}\) defined in \((C.1b)\).

In any extreme point, constraint \(-c^b_0 \leq \hat{\beta} \leq c^b_0\) is binding at either the lower bound or upper bound. Thus, it is easy to verify that the optimal objective value of problem \((C.1)\) is

\[
\eta^*_b = \max\{\sum_{i=1}^{I} d_i x_{i,b} - \mathcal{L}_b)(-c^b_0), (\sum_{i=1}^{I} d_i x_{i,b} - \mathcal{L}_b)(c^b_0)\}.
\]
Accordingly, we can formulate (C.1) as the following linear-program for a fixed $x \in X$

\[
\begin{align*}
\min & \quad \eta_b \\
\text{s.t.} & \quad \eta_b \geq \left( \sum_{i=1}^{I} d_i x_{i,b} - \mathcal{L}_b \right) (-c_b^i) \quad (C.2b) \\
& \quad \eta_b \geq \left( \sum_{i=1}^{I} d_i x_{i,b} - \mathcal{L}_b \right) (c_b^i) \quad (C.2c) \\
& \quad \eta_b \geq 0, \quad \eta_b \geq 0, \quad \eta_b \geq 0, \quad \eta_b \geq 0 \quad (C.2d) 
\end{align*}
\]

It follows that $\max_{d \in S^d} Q^{BC}(x, d)$ is equivalent to $\min\{ \sum_{b=1}^{B} \eta_b : [C.2b] - [C.2d], \forall b \in [B] \}$. This completes the proof. 

\[\square\]

**Appendix D. Proof of Proposition 3**

**Proof.** By strong duality, the inner-minimization problem $\min \{ \sum_{t \in T} c_t u_t : (12b)-(12c) \}$ is equivalent to the maximization problem $\max \{ \sum_{t \in T} \lambda_t \left[ \sum_{i \in I} (v_{i,t} - w_{i,t}) - R_t \right] : 0 \leq \lambda_t \leq c_t, \forall t = 1, \ldots, T \}$, where $\lambda_t$ is the dual variable associated with constraint (12b). Accordingly, problem (12) (equivalently, the ICU recourse problem) is equivalent to:

\[
\begin{align*}
\max & \quad \sum_{t \in T} \lambda_t \left[ \sum_{i \in I} (v_{i,t} - w_{i,t}) - R_t \right] \\
\text{s.t.} & \quad w \in \mathcal{W}. \quad \forall t \in [T] \\
& \quad 0 \leq \lambda_t \leq c_t, \quad \forall t \in [T] 
\end{align*}
\]

Note that the objective function (D.1a) contain the nonlinear term $w_{i,t} \lambda_t$. To linearize this mixed-integer nonlinear program (MINLP), as in Neyshabouri and Berg [2017], we define $q_{i,t} = w_{i,t} \lambda_t$ for all $i, t$. We also introduce the following McCormick inequalities for all $i, t$

\[
\begin{align*}
q_{i,t} & \geq 0, \quad q_{i,t} \geq \lambda_t - c_t (1 - w_{i,t}) \quad (D.2a) \\
q_{i,t} & \leq w_{i,t} c_t, \quad q_{i,t} \leq \lambda_t \quad (D.2b)
\end{align*}
\]

Letting $\sum_{t \in T} \lambda_t \left[ \sum_{i \in I} (v_{i,t} - w_{i,t}) - R_t \right] = \sum_{t \in T} \sum_{i \in I} v_{i,t} \lambda_t - \sum_{t \in T} \sum_{i \in I} q_{i,t} - \sum_{t \in T} R_t \lambda_t$, formulation (D.1) (or equivalently, the ICU recourse problem $\max_{l \in S^l} Q^{ICU}(x, l)$ is equivalent to the following MILP.

\[
\begin{align*}
\max & \quad \sum_{t \in T} \sum_{i \in I} v_{i,t} \lambda_t - \sum_{t \in T} \sum_{i \in I} q_{i,t} - \sum_{t \in T} R_t \lambda_t \quad (D.3a)
\end{align*}
\]
Appendix E. Derivation of the Last Term in Equation (7)

First, we observe that \( \max_{(d,l) \in S} \sum_{i=1}^{I} - (d_i \rho_i + l_i \alpha_i) \) is equivalent to \( \left( \sum_{i=1}^{I} \max_{d_i \in [\underline{d}_i, \overline{d}_i]} - d_i \rho_i + \sum_{i=1}^{I} \max_{l_i \in [\underline{l}_i, \overline{l}_i]} - l_i \alpha_i \right) \).

Second, we derive the following useful algebraic expression.

\[
\max_{d_i \in [\underline{d}_i, \overline{d}_i]} - d_i \rho_i = \begin{cases} 
-\overline{d}_i \rho_i & \text{if } \rho_i \leq 0 \\
-\underline{d}_i \rho_i & \text{if } \rho_i > 0 
\end{cases} = -(\overline{d}_i \rho_i + (\overline{d}_i - \underline{d}_i)(\rho_i)^+) 
\]

(E.1)

Third, we define variables \( k_i = \max\{\rho_i, 0\} \) and introduce constraint set \( \{k_i \geq \rho, k_i \geq 0, \forall i \in [I]\} \) for variables \( k_i \). Given the outer objective of minimizing \( \rho_i \) in equation (7), without loss of optimality, we can replace \(- (\overline{d}_i \rho_i + (\overline{d}_i - \underline{d}_i)(\rho_i)^+) \) with \(- (\overline{d}_i \rho_i + (\overline{d}_i - \underline{d}_i)k_i) \) in the objective of problem (7) and introduce constraints \( \{k_i \geq \alpha_i, k_i \geq 0, \forall i \in [I]\} \)

Using the same argument, one can easily verify that \( \max_{l_i \in [\underline{l}_i, \overline{l}_i]} - l_i \alpha_i = -(\underline{l}_i \alpha_i + (\underline{l}_i - \overline{l}_i)(\alpha_i)^+) \).

Then, we can replace \(- (\underline{l}_i \alpha_i + (\underline{l}_i - \overline{l}_i)(\alpha_i)^+) \) with \(- (\underline{l}_i \alpha_i + (\underline{l}_i - \overline{l}_i)r_i) \) and introduce constraints \( \{r_i \geq \rho, r_i \geq 0, \forall i \in [I]\} \).

Appendix F. Strengthening DRESS Formulation

In this section, we derive tight lower \((\underline{\rho}, \underline{\alpha}, \underline{r}, \underline{k})\) and upper \((\overline{\rho}, \overline{\alpha}, \overline{r}, \overline{k})\) bounds on the values of variables \((\rho, \alpha, r, k)\) in inequalities (16a)–(16d) of DRESS formulation in (17).

**Proposition 5.** \( \underline{\rho} = -c^\beta \) and \( \overline{\rho} = c^\beta \) are respectively valid lower and upper bounds on variables \( \rho \).

**Proof.** Observe from the objective of problem in (7) that variables \( \rho_i \) are multiplied by parameters \( \mu_i^d \) and variables \( d_i \), for all \( i = 1, \ldots, I \). And so, for fixed \( \beta \) and \( x \), the joint contribution \( \rho \) and \( d \) to the objective of problem (7) equals:

\[
\sum_{i=1}^{I} \mu_i^d \rho_i \sum_{i=1}^{I} - \max_{d_i \in [\underline{d}_i, \overline{d}_i]} d_i \rho_i + \sum_{b=1}^{B} \left( \sum_{i=1}^{I} \max_{d_i \in [\underline{d}_i, \overline{d}_i]} d_i x_{i,b} - \mathcal{L} \right) \beta_b \\
\equiv \sum_{i=1}^{I} \mu_i^d \rho_i \sum_{i=1}^{I} -(\overline{d}_i \rho_i + (\overline{d}_i - \underline{d}_i)(\rho_i)^+) + \sum_{b=1}^{B} \left( \sum_{i=1}^{I} d_i x_{i,b} - \mathcal{L}_b \right) \beta_b + \sum_{b=1}^{B} \left[ \sum_{i=1}^{I} d_i x_{i,b} - \sum_{i=1}^{I} d_i x_{i,b} \right] (\beta_b)^+ 
\]

(F.1)
Proposition 6. \( \rho = 0 \) and \( \alpha = c_i \) are respectively valid lower and upper bounds on variables \( \alpha \).

Proof. Observe from the objective of problem in (7) that variables \( \alpha_i \) are multiplied by parameters \( \mu_i \) and variables \( l_i \), for all \( i = 1, \ldots, I \). And so, for fixed \( x \), the joint contribution \( \alpha \) and \( l \) to the objective of problem (7) equals:

\[
\sum_{i=1}^{I} \mu_i l_i \alpha_i \sum_{i=1}^{I} (l_i \rho_i + (l_i - l_i)(\alpha_i)^+ + K
\]

where \( K \) is a non-negative term that equal to \( \max_{l \in S^I} Q^{\text{ICU}}(x, l) \)

Suppose that \( \alpha_i > c_i \). In this case, \( \alpha_i \) contributes to the objective value of problem (7) by \( (\mu_i^t - l_i)\alpha_i \). Let \( \alpha'_i = \alpha_i - \epsilon \) with \( \epsilon > 0 \). Since \( (\mu_i^t - l_i) \geq 0 \), then \( (\mu_i^t - l_i)\alpha'_i < (\mu_i^t - l_i)\alpha_i \), i.e., \( \alpha'_i \) improves the objective value of problem (7). It follows that without loss of optimality \( \bar{\alpha} = c_i \) is a valid upper bound on \( \alpha \).

Conversely, suppose that \( \alpha_i < 0 \). In this case, \( \alpha_i \) contributes to the objective value of problem (7) by \( (\mu_i^t - l_i)\alpha_i \). Let \( \alpha'_i = \alpha_i + \epsilon \) with \( \epsilon > 0 \). Since \( (\mu_i^t - l_i) \leq 0 \), then \( (\mu_i^t - l_i)\alpha'_i < (\mu_i^t - l_i)\alpha_i \), i.e., \( \alpha'_i \) improves the objective value of problem (7). It follows that without loss of optimality \( \alpha = 0 \) is a valid lower bound on \( \alpha \).

\[
\sqrt{t_i} = \rho_i, \ r_i = \alpha_i \] are valid upper bounds on variables \( (k_i, r_i) \), for all \( i \in [I] \).

Proof. By constraints (14c) and the objective of minimizing \( (k_i, r_i) \), \( k_i = \max(0, \rho_i) \) and \( r_i = \max(0, \alpha_i) \), for all \( i \in [I] \).
Appendix G. Stochastic Mixed-Integer Linear Program

The following stochastic mixed-integer linear program (SP) minimizes a measure of patient-related cost and the total expected cost of OR overtime, OR idle time, and lack of ICU capacity via the sample average approximation (SAA) approach (see, e.g., Kim et al. (2015); Kleywegt et al. (2002) and references therein for detailed information on SAA).

\[
\min_x \left[ \sum_{i \in I} \sum_{b \in B \cup \{\ell\}} c_{i,b} x_{i,b} + \frac{1}{N} \sum_{n=1}^{N} \left( \sum_{b \in B} \left( c_{o,b}^n + c_{g,b}^n \right) + \sum_{t \in T} c_t u_t^n \right) \right] \\
\text{s.t. } x \in \mathcal{X} \\
y_{n,i,t} \geq x_{i,b}, \quad \forall i \in [I], b \in [B], t = t_b, \ldots, t_{b + l_i - 1}, n \in [N] \\
\sum_{i=1}^{I} y_{n,i,t} - R_t \leq u_t^n, \quad \forall t \in [T], n \in [N] \\
o_b^n - g_b^n = \sum_{i=1}^{I} d_{i,b}^n x_{i,b} - L_{b,n} \quad \forall b \in [B], n \in [N] \\
u_t^n \geq 0, (o_b^n, g_b^n) \geq 0, \quad \forall t \in [T], b \in [B], n \in [N] \\
x_{i,b} \in \{0, 1\}, \quad \forall i \in I, b \in B \cup \{\ell\} \\
0 \leq y_{n,i,t} \leq 1, \quad \forall i \in [I], t \in [T], n \in [N]
\]
## Appendix H. Computational Details of DR model

Table H.1: Computational details of solving DRESS instances using C&CG Method. Notation: nodes is total number of branching nodes, MIPiter is total number of MIP simplex iterations, $N$ is average number of iterations of C&CG, mast is average CPU seconds taken by the master problem in each iteration, and sub is average CPU seconds taken by the subproblem in each iteration.

<table>
<thead>
<tr>
<th>$(I, R)$</th>
<th>(c^o = 26, c^g = 17, c_t = 100)</th>
<th>(c^o = 100, c^g = 17, c_t = 100)</th>
<th>(c^o = 100, c^g = 0, c_t = 100)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>nodes</td>
<td>MIPiter</td>
<td>$N$</td>
</tr>
<tr>
<td>(40, 5)</td>
<td>30</td>
<td>184153</td>
<td>4</td>
</tr>
<tr>
<td>(40, 10)</td>
<td>51</td>
<td>8532</td>
<td>8</td>
</tr>
<tr>
<td>(70, 5)</td>
<td>1212</td>
<td>77675</td>
<td>9</td>
</tr>
<tr>
<td>(70, 10)</td>
<td>940</td>
<td>74412</td>
<td>6</td>
</tr>
<tr>
<td>(140, 5)</td>
<td>31638</td>
<td>1862474</td>
<td>10</td>
</tr>
<tr>
<td>(140, 10)</td>
<td>46131</td>
<td>3243585</td>
<td>15</td>
</tr>
</tbody>
</table>
Table H.2: The minimum, average, and maximum solution times (in seconds) using the DR and SP models. The relative optimality gap is reported for those instances that were not solved in one hour. Results are for DRESS instances when we increase the mean and standard deviation of LOS in Table 3 by 1 day.

<table>
<thead>
<tr>
<th>(I, R)</th>
<th>Model</th>
<th>$c^O = 26, c^S = 17, c_t = 100$</th>
<th>$c^O = 100, c^S = 17, c_t = 100$</th>
<th>$c^O = 100, c^S = 0, c_t = 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Min</td>
<td>Avg</td>
<td>Max</td>
</tr>
<tr>
<td>(70, 5)</td>
<td>DR</td>
<td>57</td>
<td>177</td>
<td>188</td>
</tr>
<tr>
<td></td>
<td>SP</td>
<td>36%</td>
<td>36%</td>
<td>37%</td>
</tr>
<tr>
<td>(70, 10)</td>
<td>DR</td>
<td>43</td>
<td>111</td>
<td>232</td>
</tr>
<tr>
<td></td>
<td>SP</td>
<td>38%</td>
<td>38%</td>
<td>38%</td>
</tr>
<tr>
<td>(140, 5)</td>
<td>DR</td>
<td>0.04%</td>
<td>6%</td>
<td>14%</td>
</tr>
<tr>
<td></td>
<td>SP</td>
<td>90%</td>
<td>90%</td>
<td>90%</td>
</tr>
<tr>
<td>(140, 10)</td>
<td>DR</td>
<td>2%</td>
<td>2%</td>
<td>2%</td>
</tr>
<tr>
<td></td>
<td>SP</td>
<td>90%</td>
<td>91%</td>
<td>91%</td>
</tr>
</tbody>
</table>

Table H.3: Computational details of solving DRESS instances using C&CG Method with . Results are for DRESS instances when we increase the mean and standard deviation of LOS in Table 3 by 1 day.

<table>
<thead>
<tr>
<th>(I, R)</th>
<th>$c^O = 26, c^S = 17, c_t = 100$</th>
<th>$c^O = 100, c^S = 17, c_t = 100$</th>
<th>$c^O = 100, c^S = 0, c_t = 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>nodes</td>
<td>MIPiter</td>
<td>N</td>
</tr>
<tr>
<td>(70, 5)</td>
<td>13494</td>
<td>1155614</td>
<td>12</td>
</tr>
<tr>
<td>(70, 10)</td>
<td>8221</td>
<td>723678</td>
<td>10</td>
</tr>
<tr>
<td>(140, 5)</td>
<td>27235</td>
<td>2798988</td>
<td>14</td>
</tr>
<tr>
<td>(140, 10)</td>
<td>41955</td>
<td>2797164</td>
<td>4</td>
</tr>
</tbody>
</table>
### Appendix I. Scheduling Patterns

#### Table I.1: Number of Postponed Surgery as a Function of Cost and ICU Capacity

<table>
<thead>
<tr>
<th>(I,R)</th>
<th>Cost Model</th>
<th># of Postponed</th>
<th>(I,R)</th>
<th>Cost Model</th>
<th># of Postponed</th>
</tr>
</thead>
<tbody>
<tr>
<td>(70, 5)</td>
<td>Cost1 DR</td>
<td>2</td>
<td>(70, 10)</td>
<td>Cost1 SP</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Cost2 DR</td>
<td>1</td>
<td></td>
<td>Cost2 SP</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Cost3 DR</td>
<td>22</td>
<td></td>
<td>Cost3 SP</td>
<td>4</td>
</tr>
</tbody>
</table>

#### Table I.2: Surgery assignment decisions under Cost 1

<table>
<thead>
<tr>
<th>Type</th>
<th>Model</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th># of scheduled</th>
</tr>
</thead>
<tbody>
<tr>
<td>ENT</td>
<td>DR</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>6</td>
<td>15/15</td>
</tr>
<tr>
<td></td>
<td>SP</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>6</td>
<td>3</td>
<td>15/15</td>
</tr>
<tr>
<td>ORTHO</td>
<td>DR</td>
<td>4</td>
<td>6</td>
<td>1</td>
<td>0</td>
<td>4</td>
<td>15/16</td>
</tr>
<tr>
<td></td>
<td>SP</td>
<td>2</td>
<td>6</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>16/16</td>
</tr>
<tr>
<td>GEN</td>
<td>DR</td>
<td>3</td>
<td>6</td>
<td>4</td>
<td>0</td>
<td>2</td>
<td>15/15</td>
</tr>
<tr>
<td></td>
<td>SP</td>
<td>1</td>
<td>5</td>
<td>5</td>
<td>2</td>
<td>2</td>
<td>15/15</td>
</tr>
</tbody>
</table>

5 ICU beds per day

<table>
<thead>
<tr>
<th>Type</th>
<th>Model</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th># of scheduled</th>
</tr>
</thead>
<tbody>
<tr>
<td>ENT</td>
<td>DR</td>
<td>6</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>6</td>
<td>15/15</td>
</tr>
<tr>
<td></td>
<td>SP</td>
<td>1</td>
<td>0</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>15/15</td>
</tr>
<tr>
<td>ORTHO</td>
<td>DR</td>
<td>4</td>
<td>6</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>16/16</td>
</tr>
<tr>
<td></td>
<td>SP</td>
<td>3</td>
<td>6</td>
<td>1</td>
<td>4</td>
<td>2</td>
<td>16/16</td>
</tr>
<tr>
<td>GEN</td>
<td>DR</td>
<td>4</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>4</td>
<td>15/15</td>
</tr>
<tr>
<td></td>
<td>SP</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>5</td>
<td>4</td>
<td>15/15</td>
</tr>
</tbody>
</table>

10 ICU beds per day
## Appendix J. Out-of-Sample Performance

Table J.1: Out-of-sample simulation performance of optimal schedules given by DR and SP models under perfect distributional information.

<table>
<thead>
<tr>
<th>ICU Beds</th>
<th>Metric</th>
<th>Model</th>
<th>$c^o = 26, c^8 = 17, c_t = 100$</th>
<th>$c^o = 100, c^8 = 17, c_t = 100$</th>
<th>$c^o = 100, c^8 = 0, c_t = 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>OverT</td>
<td>TransF</td>
<td>ProbTrans</td>
</tr>
<tr>
<td>5</td>
<td>Mean</td>
<td>DR</td>
<td>54</td>
<td>2.2</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SP</td>
<td>70</td>
<td>4.6</td>
<td>0.43</td>
</tr>
<tr>
<td>75%–q</td>
<td>DR</td>
<td>57</td>
<td>2.2</td>
<td>0.27</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SP</td>
<td>71</td>
<td>4.6</td>
<td>0.43</td>
</tr>
<tr>
<td>95%–q</td>
<td>DR</td>
<td>58</td>
<td>2.4</td>
<td>0.28</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SP</td>
<td>73</td>
<td>4.6</td>
<td>0.43</td>
</tr>
<tr>
<td>10</td>
<td>Mean</td>
<td>DR</td>
<td>60</td>
<td>0</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SP</td>
<td>51</td>
<td>1</td>
<td>0.08</td>
</tr>
<tr>
<td>75%–q</td>
<td>DR</td>
<td>62</td>
<td>0</td>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SP</td>
<td>55</td>
<td>1.2</td>
<td>0.10</td>
</tr>
<tr>
<td>95%–q</td>
<td>DR</td>
<td>62</td>
<td>0</td>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SP</td>
<td>55</td>
<td>1.4</td>
<td>0.10</td>
</tr>
</tbody>
</table>
Table J.2: Out-of-sample simulation performance of optimal schedules given by DR and SP models under misspecified distributional information. Surgery durations and patients LOS were sampled from normal distributions.

<table>
<thead>
<tr>
<th>ICU Beds</th>
<th>Metric</th>
<th>Model</th>
<th>$c^d = 26, c^g = 17, c_t = 100$</th>
<th>$c^d = 100, c^g = 17, c_t = 100$</th>
<th>$c^d = 100, c^g = 0, c_t = 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>OverT</td>
<td>TransF</td>
<td>ProbTrans</td>
</tr>
<tr>
<td>5</td>
<td>Mean</td>
<td>DR</td>
<td>79</td>
<td>3.8</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SP</td>
<td>106</td>
<td>6</td>
<td>0.50</td>
</tr>
<tr>
<td>75–q%</td>
<td>DR</td>
<td>79</td>
<td>4</td>
<td>0.38</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>SP</td>
<td>108</td>
<td>6</td>
<td>0.50</td>
<td>73</td>
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<tr>
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<td>SP</td>
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Table J.3: Out-of-sample simulation performance of optimal schedules given by DR and SP models under misspecified distributional information. Surgery durations were sampled from weibull distributions, and LOS from uniform distributions.

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<th>Model</th>
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</table>
Appendix K. Sensitivity Analysis

Figure K.1: Percentage of scheduled surgery and OR utilization rate as a function of ICU capacity.

(a) percentage of Scheduled Surgery

(b) OR Utilization

Figure K.2: Average number of transfers from ICU and ICU utilization under Cost1.

(a) average number of transfers from ICU per day

(b) ICU Utilization

References


Figure K.3: Percentage of scheduled surgery and OR utilization rate as a function of cost of premature transfer under Cost1: $c^o = 26$, $c^g = c^o / 1.5$.


Chen, Z., Sim, M., Xiong, P., 2019. Robust stochastic optimization made easy with rsome.


Macario, A., 2009. Truth in scheduling: is it possible to accurately predict how long a surgical case will last?


