Distributionally Robust Chance-Constrained Building Load Control under Uncertain Renewables

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Aggregation of heating, ventilation, and air conditioning (HVAC) loads can provide reserves to absorb volatile renewable energy, especially solar photo-voltaic (PV) generation. However, the time-varying PV generation is not perfectly known when the system operator decides the HVAC control schedules. To consider the unknown uncertain PV generation, in this paper, we formulate a distributionally robust chance-constrained (DRCC) building load control problem under two typical ambiguity sets: moment-based and Wasserstein ambiguity sets. We derive mixed integer linear programming (MILP) reformulations for DRCC problems under both sets. Especially, for the DRCC problem under Wasserstein ambiguity set, we utilize the right-hand side (RHS) uncertainty to derive a more compact MILP reformulation than the commonly known big-M MILP reformulations. All the results also apply to general individual chance constraints with RHS uncertainty. Furthermore, we propose an adjustable chance-constrained formulation to achieve a reasonable trade-off between operational risk and costs. We derive tractable reformulations/algorithm under both ambiguity sets. Using real-world data, we conduct computational studies to demonstrate the effectiveness of the solution approaches and the efficiency of the solutions.

Key words: Building Load Control, Renewable Energy, Distributionally Robust Optimization, Chance-Constrained Program, Binary Program

1. Introduction

With growing environmental consciousness and government regulations, renewable energy sources (RESs) are expected to account for 29% of the total electricity consumption by 2040 (Conti et al. 2016). Given that the renewable energy generally cannot adjust their output to reflect changes of demand, higher penetration of renewable energy may cause electrical supply and demand imbalance.
issues and can be challenging with variability in output and stress on electricity grids’ balance, e.g., network frequency and voltage stability (Teodorescu et al. 2011).

With the advanced development of smart sensing, one solution can be utilizing heating, ventilation, and air conditioning (HVAC) systems as grid-responsive flexible load resource. Given their large amount of power consumption, enormous thermal mass and considerable resistances, the flexible HVAC loads can be employed as virtual storage resources to compensate high frequent fluctuations in renewable energy such as solar photo-voltaic (PV) (Yin et al. 2016).

In Dong et al. (2017, 2018), they study the problem of using aggregated HVAC systems to absorb solar PV generation. However, the inherent uncertainties of the problem are ignored in their deterministic models, such as uncertainties of the thermal controlled loads (TCLs) and renewable resources, which are mainly determined by factors of weather and consumer behavior (Zhang et al. 2018a). The uncertainties can be further intensified by missing samples and low resolution information in HVAC data collection (Wijayasekara and Manic 2015, Žáčeková et al. 2014). In this paper, we take the uncertainties into consideration by employing distributionally robust chance-constrained (DRCC) programs.

1.1. Relevant Literature

The aggregated HVAC loads as a virtual storage have become a key player in providing grid services including load balancing (see, e.g., Lu 2012, Dong et al. 2017, Barooah 2019, Wang et al. 2020). Hao et al. (2014) provide a virtual storage model to characterize aggregate energy flexibility from building loads. It is followed by virtual storage model identification and flexibility quantification in Hughes et al. (2015) and Stinner et al. (2016), respectively. Deterministic optimization models have been proposed to orchestrate the aggregated virtual storage devices (see, e.g., Hao et al. 2017, Dong et al. 2018). Both robust optimization and stochastic programming techniques have also been introduced to account for modeling and disturbance uncertainties (see, e.g., Chen et al. 2012, Nguyen and Le 2014, Zhang et al. 2019, Kocaman et al. 2020).

Recently, the distributionally robust optimization (DRO) techniques have gained wide interest. Instead of assuming a specific probability distribution of the system uncertainties, the DRO
approaches consider a family of probability distributions with prior knowledge of the uncertainties, termed as ambiguity set. The DRO approaches have been applied to many problems in power systems, such as energy storage operation (see, e.g., Yang 2019), optimal power flow (see, e.g., Zhang et al. 2016, Duan et al. 2018), and unit commitment (see, e.g., Zhao and Jiang 2018). Two typical groups of the ambiguity sets employed in DRO are moment-based (e.g., Delage and Ye 2010) and distance-based (e.g., Wasserstein metric Esfahani and Kuhn 2018) ambiguity sets. For example, considering a moment-based ambiguity set, Zhang et al. (2019) employ a DRCC program to enable more effectively use of uncertain renewables with HVAC systems. A similar formulation has been proposed in Guo et al. (2020) to solve optimal pump coordination in water distribution networks under uncertain water demand. The authors of Guo et al. (2020) consider a distributionally robust two-stage stochastic program under a Wasserstein ambiguity set. The stochastic model predictive control, an approach for energy efficiency in HVAC units (Dong et al. 2018), has been considered with distributionally robust chance constraints in Mark and Liu (2020). They use conditional value-at-risk to approximate the chance constraints.

1.2. Summary of Main Contributions

In this paper, in addition to the moment-based ambiguity set used in our prior work of Zhang et al. (2019), we further consider DRCC programs under Wasserstein ambiguity set. Specifically, under the Wasserstein ambiguity set, by exploiting the right-hand side (RHS) uncertainty of chance constraints, we derive an exact mixed integer linear programming (MILP) reformulation, which is more compact than the commonly known MILP reformulation provided in Xie et al. (2019), Chen et al. (2018). The results can be applied to general individual chance constraints with RHS uncertainty (Regarding improved formulations and valid inequalities for joint distributionally robust chance constraints with RHS uncertainty, we refer the readers to Ho-Nguyen et al. (2020).). Furthermore, to better balance the operational cost and PV utilization, we propose an adjustable chance-constrained formulation which considers the risk level of chance constraint as a decision variable than a given parameter in the DRCC formulations. Exact reformulations and algorithms
are developed for the adjustable DRCC formulations under both ambiguity sets. We summarize our contributions as follows:

1. We formulate the building load control (BLC) problem of HVAC units using DRCC optimization under two types of ambiguity sets: moment-based and Wasserstein ambiguity sets. We also propose their variants with adjustable chance constraints to balance operational cost and performance.

2. We provide exact reformulations and solution methods for the non-adjustable and adjustable DRCC formulations, which can also be applied to general individual chance constraints with RHS uncertainty.

3. We conduct computational tests on various instances and demonstrate the efficiency and effectiveness of the proposed reformulations and solution methods via real-world data.

The remainder of the paper is organized as follows. Section 2 presents the mathematical formulations of the deterministic and stochastic chance-constrained BLC problems. In Section 3, we present DRCC models under both moment-based and Wasserstein ambiguity sets and develop exact reformulations. In Section 4, we further extend the DRCC models under both ambiguity sets by considering adjustable chance constraints. We derive their tractable reformulations and develop solution approaches. In Sections 5–7, we conduct extensive numerical studies on both non-adjustable DRCC models and adjustable chance-constrained models. Finally, we draw conclusions in Section 8.

2. Model Formulation

The BLC problem utilizes an aggregated HVAC load of \( N_s \) units, i.e., buildings, to absorb the solar PV generation (collected from \( N_{PV} \) PV panels) locally while delicately maintaining desired indoor temperature for each unit throughout the day. We discretize the day-time duration into \( N_p \) periods with a time interval of \( \Delta t \) (e.g., 10 minutes). For each period \( t = 1, \ldots, N_p \), we denote a binary decision variable \( u_{t,j} \in \{0, 1\} \) for HVAC unit \( j \) to indicate its scheduled mode: if \( u_{t,j} = 1 \), ON; otherwise \( u_{t,j} = 0 \), OFF.
For each HVAC unit, to characterize the dynamics of room temperature and outdoor temperature, we consider a widely used building thermal model (Mathieu et al. 2013), where the system state is the room temperature $T$, the system input is HVAC ON/OFF status, $\text{Mode}_{\text{HVAC}}$, and the system disturbances include outdoor temperature $T_{\text{out}}$ and solar irradiance $Q_{\text{out}}$. Based on the building thermal model, a continuous-time linear time invariant (LTI) system for each HVAC unit in the state-space form is as follows.

$$\dot{T} = \frac{1}{RC}T_{\text{out}} + \frac{1}{RC}T + \frac{1}{C}Q_{\text{out}} + \frac{\text{Mode}_{\text{HVAC}}}{C}Q_{\text{HVAC}},$$

(1)

where $R$ is the building’s thermal resistance, $C$ is the building’s thermal capacity, and $Q_{\text{HVAC}}$ is cooling capacity of the building. In this paper, for building $j = 1, \ldots, N_s$, we consider a discrete-time building thermal model with a sampling interval of $\Delta t$ as

$$x_{t,j} = A_jx_{t-1,j} + B_ju_{t,j} + G_jv_j,$$

(2)

where $x_{t,j}$ is the room temperature of period $t$, $u_{t,j}$ is the binary mode decision variable, $v_j$ is the system disturbance, and the parameters $A_j, B_j, G_j$ can be computed from the continuous-time model (1).

In this study, we focus on the single-period models, where we solve for the optimal ON/OFF mode decisions at the current period given an initial room temperature resulted from previous periods’ decisions. To solve the control problem for all $N_p$ periods, we sequentially solve $N_p$ small optimization problems, each for one period. The problem could also be formulated as a multi-period model, which solves one monolithic optimization problem for all the mode decision variables over $N_p$ periods. In our preliminary results in Zhang et al. (2019), via extensive computational studies, we demonstrate that the multi-period models have similar solutions as those of single-period models. However, the multi-period models can suffer from computational difficulty especially with larger-sized problems. Therefore, we do not include them in this paper. For the rest of this section, we present two optimization formulations of the single-period models: a deterministic formulation, which assumes that the solar PV generation is deterministic and perfectly known, and a chance-constrained formulation, which assumes the PV generation is stochastic.
2.1. Deterministic Formulation

At period $t$, we denote $x_t = (x_{t,j}, \ j = 1, \ldots, N_s)^T$ the vector of room temperature of $N_s$ buildings, and denote $u_t = (u_{t,j}, \ j = 1, \ldots, N_s)^T$ the mode vector of ON/OFF decisions of $N_s$ buildings. We assume that the solar PV generation at period $t$ is perfectly known as $P_{PV,t} \in \mathbb{R}^{N_{PV}}$. At time $t$, given an initial room temperature $x_{t-1,j}, \ j = 1, \ldots, N_s$, the BLC problem can be formulated as the following MILP (Dong et al. 2018, Zhang et al. 2019).

$$\min_{u_t} \ c_{sys} \sum_{j=1}^{N_s} \beta_{t,j} + c_{\text{switch}} \sum_{j=1}^{N_s} u_{t,j} + c_{PV} \eta$$

s.t. (2)

$$-\eta \leq \sum_{j=1}^{N_s} P_j u_{t,j} - \sum_{i=1}^{N_{PV}} P_{PV,i,t} \leq \eta$$

$$-\beta_{t,j} \leq x_{t,j} - x_{\text{ref}} \leq \beta_{t,j}, \ j = 1, \ldots, N_s$$

$$x_{\text{min}} \leq x_{t,j} \leq x_{\text{max}}, \ j = 1, \ldots, N_s$$

$$u_t \in \{0, 1\}^{N_s},$$

where $|\cdot|$ takes the absolute value of a real number. The objective (3a) minimizes the total penalty cost of i) room temperature deviation from the set-point $x_{\text{ref}}$, ii) switching times, and iii) signal deviation between the total control signal (over all $N_s$ buildings) and the total PV signal (over $N_{PV}$ PV panels), with unit cost parameters $c_{sys}, c_{\text{switch}}, c_{PV}$. Constraints (3d) require that the room temperature $x_{t,j}$ is maintained in the comfort band $[x_{\text{min}}, x_{\text{max}}]$ for building $j = 1, \ldots, N_s$. The last constraint (3e) enforces binary decision $u_t$.

2.2. Chance-Constrained Formulation

An accurate prediction of the solar PV output $P_{PV,t}$ is critical to the performance of the deterministic formulation (3). However, in practice, a good prediction of the PV output may not be available, due to the fluctuating nature of solar energy, which can be introduced by cloud shadows, wind speed, and other factors, and thus can be uncertain. In this section, we introduce a chance-constrained formulation to take into account uncertain PV output.
Instead of penalizing the signal deviation $\eta$ in the objective to enforce all PV generation being consumed, the chance-constrained formulation (4) employs a soft constraint (4b), the chance constraint. The chance constraint ensures that, with a high probability, the solar PV output is consumed by the HVAC fleet. The chance-constrained formulation is

$$
\min_{u_t} \ c_{\text{sys}} \sum_{j=1}^{N_s} \beta_{t,j} + c_{\text{switch}} \sum_{j=1}^{N_s} u_{t,j}
$$

$$
s.t. \ P \left( \sum_{j=1}^{N_s} P_j u_{t,j} - \sum_{i=1}^{N_{\text{PV}}} \tilde{P}_{\text{PV},t,i} \geq 0 \right) \geq 1 - \alpha_t
$$

where $\tilde{P}_{\text{PV},t,i}$ denotes the uncertain PV generation at period $t$ of panel $i$. Constraint (4b) ensures that the PV generation is absorbed by the HVAC fleet with probability $1 - \alpha_t$. The risk level $\alpha_t \in (0,1)$ is pre-defined and reflects the system operator’s risk preference, which is usually a small number.

To solve the chance-constrained model, we employ the Sample Average Approximation (SAA) approach (see, e.g., Luedtke and Ahmed 2008) to derive bounds and obtain feasible solutions. Via the Monte Carlo sampling method, we generate a set of finite samples of the uncertainty $\tilde{P}_{\text{PV},t}$. and enforce $\sum_{j=1}^{N_s} P_j u_{t,j} - \sum_{i=1}^{N_{\text{PV}}} \tilde{P}_{\text{PV},t,i} \geq 0$ for sufficiently many samples.

Specifically, we generate $N$ i.i.d. scenarios of the uncertain PV output $\tilde{P}_{\text{PV},t}$, denoted by $P_{\text{PV},t,1}, \ldots, P_{\text{PV},t,N}$. Each scenario $P_{\text{PV},t}^n$ is associated with a probability $p_{t,n} \geq 0$, such that $\sum_{n=1}^{N} p_{t,n} = 1$. For each scenario $n$, we associate a binary variable $\rho_n$ such that $\rho_n = 0$ indicates

$$
\sum_{j=1}^{N_s} P_j u_{t,j} - \sum_{i=1}^{N_{\text{PV}}} P_{\text{PV},t,i}^n \geq 0
$$

and $\rho_n = 1$ indicates constraint (5) may be violated. The chance constraint (4b) is approximated by

$$
\sum_{j=1}^{N_s} P_j u_{t,j} - \sum_{i=1}^{N_{\text{PV}}} P_{\text{PV},t,i}^n \geq -M \rho_n, \ n = 1, \ldots, N
$$

$$
\sum_{n=1}^{N} \rho_n \leq \alpha_t
$$

$$
\rho_n \in \{0,1\}, \ n = 1, \ldots, N,
$$
where $M$ is a big-M coefficient. Constraint (6b) ensures that the probability of violating (5) is no more than $\alpha_t$. By replacing the chance constraint (4b) with (6a)–(6c), we obtain an MILP approximation of the chance-constrained model (4).

3. DRCC Formulations

In the stochastic chance-constrained formulation (4), full knowledge of the PV’s probability distribution is required. However, an accurate probability distribution can be challenging to obtain especially when the underlying distribution (while ambiguous) is time-varying. As a consequence, the solution obtained from the chance-constrained model might be sensitive to the choice of probability distribution and thus results in poor performance. This phenomenon is called the optimizer’s curse (Smith and Winkler 2006) of solving stochastic programs. To address the curse, a natural way is to employ a set of plausible probability distributions, denoted as $D_t$, rather than assuming a specific probability distribution. Specifically, we consider the DRCC formulation as follows

\[
\begin{align*}
\min_{u_t} & \quad c_{\text{sys}} \sum_{j=1}^{N_s} \beta_{t,j} + c_{\text{switch}} \sum_{j=1}^{N_s} u_{t,j} \\
\text{s.t.} & \quad \inf_{f \in D_t} \mathbb{P} \left( \sum_{j=1}^{N_s} P_j u_{t,j} - \sum_{i=1}^{N_{\text{PV}}} \tilde{P}_{\text{PV},t,i} \geq 0 \right) \geq 1 - \alpha_t \\
& \quad (2), (3c) - (3e).
\end{align*}
\]

Constraint (7b) ensures that the probability of absorbing the PV generation locally by the HVAC fleet is guaranteed at least $1 - \alpha_t$ for any probability distribution $f \in D_t$. That is, for all probability distributions in $D_t$, the worst-case probability of coordinating the HVAC fleet to consume the PV generation is no less than $1 - \alpha_t$. We note that constraint (7b) is an individual DR chance constraint with RHS uncertainty.

3.1. Ambiguity Sets

One critical question of the DRCC formulation is how to choose the ambiguity set $D_t$. A good choice of $D_t$ should take into account the characteristics of the underlying probability distribution and the tractability of the DRCC formulation. Two types of ambiguity sets have been widely studied:
(i) moment-based and (ii) distance-based ambiguity sets. In this paper, we consider a moment-based ambiguity set containing moment constraints on the first- and second-order moments (see, e.g., Delage and Ye 2010) and a distance-based ambiguity set using Wasserstein metric (e.g., Esfahani and Kuhn 2018)). Given a series of independent samples, \( \{P_{PV,t}^n\}_{n=1}^N \), sampled from the true underlying distribution of the PV generation, we consider the following two distributional ambiguity sets.

(i) Moment-based Ambiguity Set. The empirical mean and covariance matrix can be calculated as
\[
\mu_t = \frac{1}{N} \sum_{n=1}^N P_{PV,t}^n, \quad \Sigma_t = \frac{1}{N} \sum_{n=1}^N (P_{PV,t}^n - \mu_t)(P_{PV,t}^n - \mu_t)^\top.
\]
The ambiguity set based on the two moment estimates \( \mu_t, \Sigma_t \), first proposed by Delage and Ye (2010), is as follows.
\[
\mathcal{D}_1^t = \left\{ f : \mathbb{P}_f(\tilde{P}_{PV,t} \in \mathbb{R}^{NPV}) = 1, \frac{1}{N} \sum_{n=1}^N (P_{PV,t}^n - \mu_t)(P_{PV,t}^n - \mu_t)^\top \preceq \gamma_1 \Sigma_t, \left( \mathbb{E}_f[\tilde{P}_{PV,t} - \mu_t] \right)^\top \Sigma_t^{-1} \left( \mathbb{E}_f[\tilde{P}_{PV,t}] - \mu_t \right) \leq \gamma_2 \Sigma_t \right\},
\]
where \( \gamma_1 \geq 0 \) and \( \gamma_2 \geq \max\{\gamma_1, 1\} \). The three constraints guarantee that (1) the true mean of \( \tilde{P}_{PV,t} \) lies in an ellipsoid centered at \( \mu_t \) and (2) the true covariance of \( \tilde{P}_{PV,t} \) is bounded above by \( \gamma_2 \Sigma_t \). The two parameters \( \gamma_1 \) and \( \gamma_2 \) reflect the system operator’s tolerance of the moment and distributional ambiguity: the larger the two parameters are, the more the tolerance towards ambiguity and the more robustness of the optimal solutions are. The values of \( \gamma_1 \) and \( \gamma_2 \) depend on the samples size, support size, and confidence level (See more details in Definition 2 in Delage and Ye 2010).

(ii) Wasserstein Ambiguity Set. Given a positive radius \( \delta_t > 0 \), the Wasserstein ambiguity set defines a ball around the discrete empirical distribution based on the \( N \) samples,
\[
\mathbb{P}_{\tilde{P}_{PV,t}^N} \left[ \tilde{P}_{PV,t} = P_{PV,t}^n \right] = \frac{1}{N},
\]
in the space of probability distributions as follows.
\[
\mathcal{D}_2^t = \left\{ f : \mathbb{P}_f(\tilde{P}_{PV,t} \in \mathbb{R}^{NPV}) = 1, W(\mathbb{P}_f, \mathbb{P}_{\tilde{P}_{PV,t}^N}) \leq \delta_t \right\},
\]
where the Wasserstein distance is defined as

\[ W(P_1, P_2) = \inf_Q \left\{ \int_{\mathbb{R}^{N_{PV} \times \mathbb{R}^{N_{PV}}}} \|P^1_{PV,t} - P^2_{PV,t}\|_Q(dP^1_{PV,t}, dP^2_{PV,t}) : Q \text{ is a joint distribution of } \tilde{P}^1_{PV,t} \text{ and } \tilde{P}^2_{PV,t} \text{ with marginals } P_1 \text{ and } P_2, \right\} \]

The radius of the ambiguity set controls the degree of the conservatism of the DRCC model. If we set \( \delta_t = 0 \), the ambiguity set \( D^2_t \) only contains the empirical distribution and we can recover a chance-constrained model.

### 3.2. DRCC Reformulation under Moment-based Ambiguity Set \( D^1_t \)

In this section, we present an MILP reformulation of the DRCC model (7) under the moment-based ambiguity set \( D_t = D^1_t \). Let \( \theta_t = 1^\top \mu_t \) and \( \sigma_t = 1^\top \Sigma_t 1 \), where \( 1 \in \mathbb{R}^{N_{PV}} \) is a vector with all ones. According to Theorem 2 in Zhang et al. (2018b), under the ambiguity set \( D^1_t \), the DR chance constraint (7b) is equivalent to a linear constraint as

\[ \sum_{j=1}^{N_s} P_j u_{t,j} \geq \theta_t + \Omega_t \sigma_t, \]

where \( \Omega_t = \begin{cases} \sqrt{\gamma_1} + \sqrt{(1 - \alpha_t)(\gamma_2 - \gamma_1)/\alpha_t}, & \gamma_1/\gamma_2 \leq \alpha_t \\ \sqrt{\gamma_2/\alpha_t}, & \gamma_1/\gamma_2 > \alpha_t \end{cases} \). Then the DRCC formulation (7) under \( D^1_t \) is equivalent to the following MILP problem

\[ \min_{u_t} \left\{ c_{sys} \sum_{j=1}^{N_s} \beta_{t,j} + c_{switch} \sum_{j=1}^{N_s} u_{t,j} : (2), (3c), (3e), (8) \right\} . \]

### 3.3. DRCC Reformulations under Wasserstein Ambiguity Set \( D^2_t \)

Denote the total PV output \( P_{total,t}^n = 1^\top P_{PV,t}^n \). According to Corollary 2 in Xie (2019) (Theorem 3 in Chen et al. 2018), the DR chance constraint (7b) under the Wasserstein ambiguity set \( D^2_t \) is feasible if and only if the following constraints are satisfied

\[ \delta_t - \alpha_t \gamma \leq \frac{1}{N} \sum_{n=1}^{N} z_n \]

\[ - \max \left\{ \sum_{j=1}^{N_s} P_j u_{t,j} - P_{total,t}^n, 0 \right\} \leq -z_n - \gamma, \quad n = 1, \ldots, N \]

\[ z_n \leq 0, \quad n = 1, \ldots, N \]

\[ \gamma \geq 0. \]
To linearize the nonlinear constraints (9b), we introduce big-M coefficients for \( n = 1, \ldots, N \),
\[
M_n^1 = \max_{u_t} \left\{ \left| \sum_{j=1}^{N_s} P_j u_{t,j} - P_{n}^{n_{\text{total},t}} \right| \right\} = \max \left\{ \left| \sum_{j=1}^{N_s} P_j - P_{n}^{n_{\text{total},t}} \right|, P_{n}^{n_{\text{total},t}} \right\}.
\]
The constraints (9b) are equivalent to the following MILP constraints.
\[
\begin{align*}
z_n + \gamma & \leq s_n, \ n = 1, \ldots, N \quad (10a) \\
s_n & \leq \sum_{j=1}^{N_s} P_j u_{t,j} - P_{n}^{n_{\text{total},t}} + M_n^1 (1 - y_n), \ n = 1, \ldots, N \quad (10b) \\
s_n & \leq M_n^1 y_n, \ n = 1, \ldots, N \quad (10c) \\
y_n & \in \{0, 1\}, \ n = 1, \ldots, N \quad (10d) \\
s_n & \geq 0, \ n = 1, \ldots, N \quad (10e)
\end{align*}
\]
where \( s_n, y_n \) are auxiliary variables. Therefore, the DRCC formulation (7) under \( D_t^2 \) is reformulated as an MILP problem as follows.
\[
\text{MILP1: } \min_{u_t} \left\{ c_{\text{sys}} \sum_{j=1}^{N_s} \beta_{t,j} + c_{\text{switch}} \sum_{j=1}^{N_s} u_{t,j} : (2), (3c)-(3e), (9a), (9c)-(9d), (10a)-(10e) \right\}.
\]
We remark that the MILP formulation involves \( 3N + 1 \) constraints and \( N \) binary variables, which may pose computational challenges as \( N \) grows large. Next, by exploring the RHS uncertainty, we derive a more compact MILP reformulation for the DR chance constraint with only \( \lceil \alpha_t N \rceil + 3 \) constraints and \( \lceil \alpha_t N \rceil + 1 \) binary variables. The problem can be significantly reduced when \( \alpha_t \) is of a small value.

We first sort \( \{P_{n_{\text{total},t}}^{n}\}^{N}_{n=1} \) such that \( P_{n_{\text{total},t}}^{(1)} \geq P_{n_{\text{total},t}}^{(2)} \geq \cdots \geq P_{n_{\text{total},t}}^{(N)} \) and obtain the non-increasing permutation \( \{1, 2, \ldots, n\} \) of \( \{1, 2, \ldots, N\} \). We denote \( P_{n_{\text{total},t}}^{(0)} = \sum_{t=1}^{N_s} P_t \), the maximum load provided by turning on all HVAC units. We make the following assumption for the more compact MILP reformulation.

**Assumption 1.** \( P_{n_{\text{total},t}}^{(0)} > P_{n_{\text{total},t}}^{(N)} \).

This is a mild assumption as the smallest value of the solar PV generation realization is required to be smaller than the maximum HVAC load. That is, the smallest PV generation can be absorbed fully when all the HVAC units are ON.
Theorem 1. Under Assumption 1, the DR chance constraint (7b) under the Wasserstein ambiguity set $\mathcal{D}_t^2$ is feasible if and only if the following linear constraints are feasible with auxiliary variables $x_n \in \{0, 1\}$, $\rho_{n\ell} \in \mathbb{R}$, $n = 1, \ldots, k + 1$, $\ell = 1, \ldots, N_s$, where $k$ is an index such that $k/N \leq \alpha_t < (k + 1)/N$.

\[
- \frac{1}{N} \sum_{n=1}^{k+1} \sum_{i=1}^{n} x_i \left( P^{(k+1)}_{\text{total},t} - P^{(n)}_{\text{total},t} \right) + \left( \alpha_t - \frac{\sum_{n=1}^{k+1} nx_n - 1}{N} \right) P^{(k+1)}_{\text{total},t} - \left( \alpha_t + \frac{1}{N} \right) \sum_{\ell=1}^{N_s} P^{(n)}_{\text{total},t} u_{t,\ell} \leq -\delta
\]

\[
\frac{1}{N} \sum_{n=1}^{k+1} \sum_{\ell=1}^{N_s} P^{(n)}_{\text{total},t} \rho_{n\ell} \leq -\delta \tag{11a}
\]

\[
P^{(n)}_{\text{total},t} - P^{(0)}_{\text{total},t} (1 - x_n) \leq \sum_{\ell=1}^{N_s} P^{(n-1)}_{\text{total},t} u_{t,\ell} \leq P^{(0)}_{\text{total},t} (1 - x_n), \; n = 1, \ldots, k + 1 \tag{11b}
\]

\[
\sum_{n=1}^{k+1} x_n = 1 \tag{11c}
\]

\[
\rho_{n\ell} \geq x_n + u_{t,\ell} - 1, \; n = 1, \ldots, k + 1, \; \ell = 1, \ldots, N_s \tag{11d}
\]

\[
\rho_{n\ell} \leq u_{t,\ell}, \; n = 1, \ldots, k + 1, \; \ell = 1, \ldots, N_s \tag{11e}
\]

\[
\rho_{n\ell} \leq x_n, \; n = 1, \ldots, k + 1, \; \ell = 1, \ldots, N_s \tag{11f}
\]

\[
\rho_{n\ell} \geq 0, \; n = 1, \ldots, k + 1, \; \ell = 1, \ldots, N_s \tag{11g}
\]

\[
x_n \in \{0, 1\}, \; n = 1, \ldots, k + 1. \tag{11h}
\]

Proof: According to (9), the DR chance constraint (7b) with $\mathcal{D}_t = \mathcal{D}_t^2$ is satisfied if and only if the optimal value of the following linear program is no more than $-\delta$.

\[
\min - \frac{1}{N} \sum_{n=1}^{N} z_n - \alpha_t \gamma \tag{12a}
\]

s.t. \[-a_n \leq -z_n - \gamma, \; n = 1, \ldots, N \tag{12b}\]

\[
z_n \leq 0, \; n = 1, \ldots, N \tag{12c}
\]

\[
\gamma \geq 0, \tag{12d}
\]

where $a_n = \max \left[ \sum_{\ell=1}^{N_s} P^{(n)}_{\text{total},t} - P^{(n)}_{\text{total},t} \right]$. We associate dual variables $\pi_n \geq 0, \; n = 1, \ldots, N$ with constraints in (12b) and obtain the dual problem as follows.

\[
\max - \sum_{n=1}^{N} \pi_n a_n \tag{13a}
\]
\[ \begin{align*}
\text{s.t. } & \pi_n \leq \frac{1}{N}, \ n = 1, \ldots, N \quad (13b) \\
& \sum_{n=1}^{N} \pi_n \geq \alpha_t \quad (13c) \\
& \pi_n \geq 0, \ n = 1, \ldots, N. \quad (13d)
\end{align*} \]

Due to strong duality, the optimal value of (13) equals to that of (12), which needs to be no more than \(-\delta\). The problem (13) is always feasible as \(0 < \alpha_t < 1\). We remark that if Assumption 1 does not hold, the DR chance constraint (7b) is infeasible. Because when the Assumption 1 does not hold, \(a_n = 0, \ n = 1, \ldots, N\), for any solution of \(u_t\) and thus the optimal value of the dual problem (13) is zero. As the optimal value is more than \(-\delta\), the DR chance constraint (7b) is then infeasible and so is the DRCC problem (7).

We claim that one optimal solution to (13) is

\[
\pi(n) = \begin{cases} 
\frac{1}{N}, & n = 1, \ldots, k \\
\alpha_t - \frac{k}{N}, & n = k + 1 \\
0, & n = k + 2, \ldots, N.
\end{cases} \quad (14)
\]

Recall that \(\{1, 2, \ldots, (n)\}\) is a permutation such that \(P_{\text{total},t}^{(1)} \geq P_{\text{total},t}^{(2)} \geq \cdots \geq P_{\text{total},t}^{(N)}\), or equivalently, \(0 \leq a_{(1)} \leq a_{(2)} \leq \cdots \leq a_{(N)}\) (because for any \(n \leq m\), \(\sum_{t=1}^{N} P_t u_{t,t} - \sum_{t=1}^{N} P_{\text{total},t}^{(n)} \leq \sum_{t=1}^{N} P_t u_{t,t} - P_{\text{total},t}^{(m)}\) and thus \(a_{(n)} \leq a_{(m)}\)) and \(k\) is an index such that

\[
k/N \leq \alpha_t < (k + 1)/N. \quad (15)
\]

In the case where \(k = 0\), the optimal solution is \(\pi_{(1)} = \alpha_t\) and \(\pi_{(n)} = 0, \ n = 2, \ldots, N\).

To see the optimality of the solution (14), we notice that the dual problem (13) can be reformulated as a relaxed knapsack problem by replacing the decision variable \(\pi_n\) with its negative \(\pi'_n = -\pi_n\). It is well known that the relaxed knapsack problem can be effectively solved by the greedy algorithm:

1. Order the decision variables \(\pi'_n, \ n = 1, \ldots, N\) in a non-decreasing order of their objective coefficients \(a_n\) (Note that the non-decreasing order is provided by the permutation \(\{1, 2, \ldots, (n)\}\), i.e., \(0 \leq a_{(1)} \leq a_{(2)} \leq \cdots \leq a_{(N)}\).);
2. Find the index $k$ based on the condition (15);

3. Let $\pi'_{(n)} = -1/N$, $n = 1, \ldots, k$, $\pi'_{(k+1)} = \alpha_t - k/N$, and $\pi'_{(n)} = 0$, $n = k + 2, \ldots, N$.

Now, we obtain the optimal value of the problem (13), $-1/N \sum_{n=1}^{k} a_{(n)} - (\alpha_t - k/N) a_{(k+1)}$. For a given solution $u_t$, we are interested in a critical index $j$ such that

$$P_{\text{total},t}^{(j)} < \sum_{\ell=1}^{N_s} P_{\ell} u_{\ell,t} \leq P_{\text{total},t}^{(j-1)}$$

and thus $a_{(1)} = \ldots = a_{(j-1)} = 0 < a_{(j)} \leq \ldots \leq a_{(N)}$. In the case where for all $n = 1, \ldots, N$, $a_n > 0$, or equivalently $P_{\text{total},t}^{n} < \sum_{\ell=1}^{N_s} P_{\ell} u_{\ell,t}$, let $j = 1$. We note that $j \leq k + 1$, because, otherwise, the optimal value of (13) is zero and thus the DR chance constraint (7b) under the Wasserstein ambiguity set $D_t^2$ is infeasible.

To find the critical index $j$, we denote binary variables $x_n$, for $n = 1, \ldots, k + 1$. If $j = n$, $x_n = 1$ and 0 otherwise. We require that

$$\sum_{n=1}^{k+1} x_n = 1 \quad \text{and} \quad P_{\text{total},t}^{(n)} - M(1 - x_n) < \sum_{\ell=1}^{N_s} P_{\ell} u_{\ell,t} \leq P_{\text{total},t}^{(n-1)} + M(1 - x_n), \quad n = 1, \ldots, k + 1,$$

where the big-M coefficient $M$ can be $P_{\text{total},t}^{(0)}$. The optimal value of (13) can be written as

$$-\frac{1}{N} \sum_{n=j}^{k} a_{(n)} - \left(\frac{\alpha_t}{N} - \frac{k}{N}\right) a_{(k+1)} = -\frac{1}{N} \sum_{n=j}^{k} \left(\sum_{\ell=1}^{N_s} P_{\ell} u_{\ell,t} - P_{\text{total},t}^{(n)}\right) - \left(\frac{\alpha_t}{N} - \frac{k}{N}\right) \left(\sum_{\ell=1}^{N_s} P_{\ell} u_{\ell,t} - P_{\text{total},t}^{(k+1)}\right)$$

$$= -\frac{1}{N} \sum_{n=j}^{k} \left(\sum_{\ell=1}^{N_s} P_{\ell} u_{\ell,t} - P_{\text{total},t}^{(n)}\right) - \left(\frac{\alpha_t}{N} - \frac{k}{N}\right) \left(\sum_{\ell=1}^{N_s} P_{\ell} u_{\ell,t} - P_{\text{total},t}^{(k+1)}\right)$$

$$= -\frac{1}{N} \sum_{n=1}^{k} \sum_{i=1}^{n} x_i \left(\sum_{\ell=1}^{N_s} P_{\ell} u_{\ell,t} - P_{\text{total},t}^{(n)}\right) - \left(\frac{\alpha_t}{N} - \frac{k}{N}\right) \left(\sum_{\ell=1}^{N_s} P_{\ell} u_{\ell,t} - P_{\text{total},t}^{(k+1)}\right)$$

$$= -\frac{1}{N} \sum_{n=1}^{k} \sum_{i=1}^{n} x_i \left(\sum_{\ell=1}^{N_s} P_{\ell} u_{\ell,t} - P_{\text{total},t}^{(n)}\right) + \left(\frac{\alpha_t}{N} - \frac{k}{N}\right) \left(\sum_{\ell=1}^{N_s} P_{\ell} u_{\ell,t} - P_{\text{total},t}^{(k+1)}\right)$$

$$= \left(\alpha_t + \frac{1}{N}\right) \sum_{\ell=1}^{N_s} P_{\ell} u_{\ell,t} + \frac{1}{N} \sum_{n=1}^{k+1} \sum_{i=1}^{n} x_i \sum_{\ell=1}^{N_s} P_{\ell} u_{\ell,t},$$

(17)

where we introduce $\rho_{n\ell} := x_n u_{t,\ell}$ using McCormick inequalities (McCormick 1976): (11d)–(11g).

Now, we show that the DR chance constraint (7b) under $D_t^2$ is feasible, if and only if: (i) (16) holds and (ii) the optimal value of the dual problem (13), which is equal to (17), is no more than $-\delta$.

Therefore, we conclude the proof. □
Now, we obtain the second (more compact) MILP reformulation of the DRCC formulation (7) under the Wasserstein ambiguity set $\mathcal{D}_i^2$ as follows.

\[
\text{MILP2: } \min_{u_t} \left\{ c_{\text{sys}} \sum_{j=1}^{N_s} \beta_{t,j} + c_{\text{switch}} \sum_{j=1}^{N_s} u_{t,j} : (2), (3c) - (3e), (11a) - (11h) \right\}.
\]

4. Adjustable Chance-Constrained Formulations

In the DRCC formulation (7), $\alpha_t$ is a pre-defined risk level, which is often chosen based on operators’ experience and is usually a small number to guarantee high PV generation utilization. However, a too small $\alpha_t$ may result in infeasibility of the problem and can lead to high operational cost (see, e.g., Ma et al. 2019). It is challenging for the system operator to decide the value of $\alpha_t$ for optimally trading off between the risk and cost. Therefore, following Ma et al. (2019), Qiu et al. (2016), Wang et al. (2018), we consider $\alpha_t$ as an adjustable decision variable and modify the DRCC model (7) into the following adjustable DRCC problem.

\[
\begin{align*}
\min_{u_t, \alpha_t} & \quad c_{\text{sys}} \sum_{j=1}^{N_s} \beta_{t,j} + c_{\text{switch}} \sum_{j=1}^{N_s} u_{t,j} + c_t \alpha_t \quad (18a) \\
\text{s.t.} & \quad \inf_{f \in \mathcal{D}_t} \mathbb{P} \left( \sum_{j=1}^{N_s} P_{j,t,j} - \sum_{i=1}^{N_{PV}} \tilde{P}_{PV,i,t,i} \geq 0 \right) \geq 1 - \alpha_t \quad (18b) \\
& \quad 0 \leq \alpha_t \leq 1 \quad (18c)
\end{align*}
\]

where $c_t > 0$ is the coefficient weight on the risk level $\alpha_t$. If $c_t$ is close to zero, the utilization of solar PV output is low. In Section 7.2, we study the impact of $c_t$ on the optimal risk level and operational cost.

4.1. Adjustable DRCC Reformulations under Moment-based Ambiguity Set $\mathcal{D}_i^1$

According to (8), the choice of the coefficient $\Omega_t$ depends on the values of $\alpha_t$ and $\gamma_1/\gamma_2$. In this section, we present 0-1 second-order conic programming (SOCP) reformulations under the two cases: (i) $\gamma_1/\gamma_2 \leq \alpha_t$ and (ii) $\gamma_1/\gamma_2 > \alpha_t$.

Theorem 2. If $\gamma_1/\gamma_2 \leq \alpha_t$, the adjustable DR chance constraint (18b) under $\mathcal{D}_i^1$, or equivalently,

\[
\sum_{j=1}^{N_s} P_{j,t,j} u_{t,j} \geq \theta_t + \left( \sqrt{\gamma_1} + \sqrt{\frac{1 - \alpha_t}{\alpha_t} (\gamma_2 - \gamma_1)} \right) \sigma_t,
\]

(19)
is equivalent to the following 0-1 SOCP constraints

\[
\begin{align}
\left\| 2\sigma_t \sqrt{\gamma_2 - \gamma_1} \right\| \leq \alpha_t + d & \quad (20a) \\
\alpha_t - d & \quad \leq \alpha_t - v \\
2q & \quad \leq 2 \left\| q - w \right\|_2 \quad (22d)
\end{align}
\]

where the operator \( \cdot \) represents the Frobenius inner product, \( P \in \mathbb{R}^{N_s \times N_s} \) with \( P_{ij} = P_i P_j \) and \( g \in \mathbb{R}^{N_s \times N_s} \).

**Proof:** See Appendix A. \( \square \)

**Theorem 3.** If \( \gamma_1 / \gamma_2 > \alpha_t \), the adjustable DR chance constraint (18b), or equivalently,

\[
\sum_{j=1}^{N_s} P_j u_{t,j} \geq \theta_t + \sqrt{\gamma_2} \sigma_t
\]

is equivalent to the following 0-1 SOCP constraints

\[
\begin{align}
\sum_{j=1}^{N_s} P_j u_{t,j} \geq \theta_t + \sqrt{\gamma_2} \sigma_t & \quad (21) \\
\sum_{j=1}^{N_s} P_j u_{t,j} \geq \theta_t + v \sigma_t \sqrt{\gamma_2} & \quad (22a) \\
\alpha_t + v & \quad \geq \left\| \alpha_t - v \right\|_2 \quad (22b) \\
v & \quad \geq w^2 \quad (22c) \\
q + w & \quad \geq \left\| q - w \right\|_2 \quad (22d)
\end{align}
\]

**Proof:** See Appendix B. \( \square \)
Therefore, to solve the adjustable DRCC model (18) under the moment-based ambiguity set $\mathcal{D}_t^1$, we solve the following two 0-1 SOCP problems, separately.

**SOCP1:**

$$\min_{u_t, \alpha_t} \left\{ c_{\text{sys}} \sum_{j=1}^{N_s} \beta_{t,j} + c_{\text{switch}} \sum_{j=1}^{N_s} u_{t,j} + c_t \alpha_t : 1 \geq \alpha_t \geq \frac{\gamma_1}{\gamma_2}, (2), (3c) - (3e), (20a) - (20c) \right\},$$

**SOCP2:**

$$\min_{u_t, \alpha_t} \left\{ c_{\text{sys}} \sum_{j=1}^{N_s} \beta_{t,j} + c_{\text{switch}} \sum_{j=1}^{N_s} u_{t,j} + c_t \alpha_t : 0 \leq \alpha_t \leq \frac{\gamma_1}{\gamma_2}, (2), (3c) - (3e), (22a) - (22d) \right\}.$$

After obtaining the two optimal values, we compare them and let the solution with the higher optimal value be the optimal solution to the adjustable DRCC model (18).

Note that the reformulation SOCP1 incorporates $N_s^2$ auxiliary variables $o_{ij}, i,j = 1, \ldots, N_s$, which can result in computational burden when $N_s$ is large. Below, inspired by Xie et al. (2019), we present a more compact approximation that incorporates only four auxiliary variables if $\gamma_1/\gamma_2 \leq \alpha_t \leq 0.75$.

**Theorem 4.** If $\gamma_1/\gamma_2 \leq \alpha_t \leq 0.75$, the adjustable DR chance constraint (18b) is outer approximated by the following 0-1 SOCP constraints

\begin{align*}
2r & \geq v, \quad (23a) \\
\alpha_t + v & \geq \left\| \alpha_t - v \right\|_{2q}, \quad (23b) \\
q + w & \geq \left\| q - w \right\|_2, \quad (23c) \\
v & \geq w^2. \quad (23d)
\end{align*}

**Proof:** See Appendix C. \qed

Therefore, when $\gamma_1/\gamma_2 \leq \alpha_t \leq 0.75$, to solve the adjustable DRCC model (18) under the moment-based ambiguity set, we can implement a branch-and-cut algorithm, which solves the following 0-1 SOCP problem.

**SOCP3:**

$$\min_{u_t, \alpha_t} \left\{ c_{\text{sys}} \sum_{j=1}^{N_s} \beta_{t,j} + c_{\text{switch}} \sum_{j=1}^{N_s} u_{t,j} + c_t \alpha_t : 0.75 \geq \alpha_t \geq \frac{\gamma_1}{\gamma_2}, (2), (3c) - (3e), (37), (23a) - (23d) \right\}.$$
At each iteration, obtaining the current solution \((\hat{\alpha}_t, \hat{r}, \hat{u}_t)\), if constraint (38) is satisfied, we claim that \((\hat{\alpha}_t, \hat{r}, \hat{u}_t)\) is optimal. Otherwise, we generate the following supporting hyperplane as a valid inequality.

\[
r \geq \left( -\frac{1}{2} (1 - \hat{\alpha}_t)^{-\frac{1}{2}} \hat{\alpha}_t^{-\frac{3}{2}} \right) \alpha_t + (1 - \hat{\alpha}_t)^{-\frac{1}{2}} \hat{\alpha}_t^{-\frac{1}{2}} \left( \frac{3}{2} - \hat{\alpha}_t \right).
\]

If the decision maker chooses to only consider \(\alpha_t\) such that \(0 \leq \alpha_t \leq 0.75\) for the adjustable DRCC model, he first solves SOCP2 for \(0 \leq \alpha_t \leq \gamma_1/\gamma_2\) and implement the branch-and-cut algorithm for \(\gamma_1/\gamma_2 \leq \alpha_t \leq 0.75\). Then comparing the two optimal values, let the solution with the higher optimal value to be the optimal solution to the adjustable DRCC model.

4.2. Adjustable DRCC Reformulations under Wasserstein Ambiguity Set \(D_t^2\)

In the section, we first show that the adjustable DRCC model under the Wasserstein ambiguity set \(D_t^2\) can be reformulated as a 0-1 MILP formulation with big-M coefficients based on the work of Xie (2019). Then we present a big-M free MILP reformulation by exploiting the RHS uncertainty.

We denote \(Z = \{ u_t : (9a) - (9d) \}\) the feasible region (see Section 3.3) described by the DR chance constraint (7b) under Wasserstein ambiguity set \(D_t^2\). Now, we consider the risk level \(\alpha_t\) as a decision variable in the adjustable formulation. The product \(\alpha_t \gamma\) in (9a) becomes a bilinear term. In the following proposition, we derive an equivalent set to \(Z\) by eliminating the bilinear term.

**Proposition 1.** The set \(Z\) is equivalent to the following set:

\[
Z_1 = \left\{ u_t : \begin{align*}
\delta_t \lambda - \alpha_t &\leq \frac{1}{N} \sum_{n=1}^N z_n, \\
- \max \left[ \sum_{j=1}^{N_x} P_j \lambda u_{t,j} - P_{total,t} \lambda, \ 0 \right] &\leq -z_n - 1, \ n = 1, \ldots, N, \\
z_n &\leq 0, \ n = 1, \ldots, N, \\
\lambda &\geq 0 \end{align*} \right\}. \tag{24a}
\]

**Proof:** See Appendix D. □

By introducing suitable big-M coefficients \(M_n^2, \ n = 1, \ldots, N\), the set \(Z_1\) can be further reformulated as a mixed integer set below.

\[
Z_1 = \left\{ u_t : \begin{align*}
\delta_t \lambda - \alpha_t &\leq \frac{1}{N} \sum_{n=1}^N z_n \tag{25a}
\end{align*} \right\}
\]
\[ z_n + 1 \leq s_n, \ n = 1, \ldots, N \quad (25b) \]
\[ s_n \leq \sum_{j=1}^{N_a} P_j \lambda u_{t,j} - P_{\text{total},t}^n \lambda + M^2_n (1 - y_n), \ n = 1, \ldots, N \quad (25c) \]
\[ s_n \leq M^2_n y_n, \ n = 1, \ldots, N \quad (25d) \]
\[ \lambda \geq 0 \quad (25e) \]
\[ s_n \geq 0, \ z_n \leq 0, \ y_n \in \{0, 1\}, \ n = 1, \ldots, N \} \quad (25f) \]

We denote \( w_{t,j} := \lambda u_{t,j} \) and constraint (25c) becomes
\[ s_n \leq \sum_{j=1}^{N_a} P_j w_{t,j} - P_{\text{total},t}^n \lambda + M^2_n (1 - y_n), \ n = 1, \ldots, N. \quad (26) \]

According to the McCormick inequalities, we introduce the following linear constraints
\[ w_{t,j} \geq 0 \quad (27a) \]
\[ w_{t,j} \geq \lambda - (1 - u_{t,j}) \lambda^U \quad (27b) \]
\[ w_{t,j} \leq \lambda^U u_{t,j} \quad (27c) \]
\[ w_{t,j} \leq \lambda \quad (27d) \]

where \( \lambda^U \) is an upper bound of \( \lambda \). Therefore, the adjustable DRCC model (18) under the Wasserstein ambiguity set is equivalent to the following 0-1 MILP formulation.

**MILP 3:**
\[
\min_{u, \alpha_t} \left\{ c_{\text{sys}} \sum_{j=1}^{N_s} \beta_{t,j} + c_{\text{switch}} \sum_{j=1}^{N_s} u_{t,j} + c_t \alpha_t : \ (2), \ (3c) - (3e), \ (25a), \ (25b), \ (25d) - (25f), \ (26), \ (27), \ (18c) \right\}.
\]

MILP 3 is however difficult to solve in certain cases, according to Xie (2019) and Chen et al. (2018). Additionally, a bad choice of too large values for the two big-M parameters (i.e., \( M^2_n \) and \( \lambda^U \)) may lead to weak linear relaxations and thus can be detrimental to efficient computation. Taking into account the RHS uncertainty exploited in Theorem 1, we derive a big-M free reformulation as follows.
Theorem 5. Under Assumption 1, the DR chance constraint (18b) under the Wasserstein ambiguity set $\mathcal{D}_t = \mathcal{D}^t_\kappa$ is feasible if and only if the following MILP constraints with auxiliary variables are feasible.

$$\Delta_{jk} \in \{0, 1\}, \varepsilon_{jk} \in \mathbb{R}, \tau_{tjk} \in \mathbb{R}, \alpha_{tjk} \in \mathbb{R}$$

$$\sum_{j=1}^{N} \sum_{k=1}^{N-1} \left[ -\frac{1}{N} \sum_{i=j}^{k} (P_{\text{total},t}^{(k+1)} - P_{\text{total},t}^{(i)}) \Delta_{jk} - \sum_{\ell=1}^{N_s} P_{t} \left( \alpha_{tjk} - \frac{j-1}{N} \tau_{tjk} \right) + P_{\text{total},t}^{(k+1)} \left( \varepsilon_{jk} - \frac{j-1}{N} \Delta_{jk} \right) \right] \leq -\delta$$ (28a)

$$\sum_{j=1}^{N} \sum_{k=1}^{N-1} \Delta_{jk} = 1$$ (28b)

$$\sum_{j=1}^{N} \sum_{k=1}^{N-1} k\Delta_{jk} \leq \sum_{j=1}^{N} \sum_{k=1}^{N-1} \alpha_{t}N \leq \sum_{j=1}^{N} \sum_{k=1}^{N-1} (k+1) \Delta_{jk}$$ (28c)

$$\sum_{j=1}^{N} \sum_{k=1}^{N-1} P_{\text{total},t}^{(j)} \Delta_{jk} \leq \sum_{j=1}^{N} \sum_{k=1}^{N-1} \sum_{t \in \mathcal{T}} P_{t} u_{t} \Delta_{jk} \leq \sum_{j=1}^{N} \sum_{k=1}^{N-1} P_{\text{total},t}^{(j-1)} \Delta_{jk}$$ (28d)

$$\varepsilon_{jk} \leq \Delta_{jk}, \ v_{jk} \geq 0, \ v_{jk} \geq \alpha_{t} + \Delta_{jk} - 1, \ v_{jk} \geq 0, \ 0 \leq j - 1 \leq k \leq N - 1$$ (28e)

$$\alpha_{tjk} \leq \varepsilon_{jk}, \ \alpha_{tjk} \leq u_{t}, \ \alpha_{tjk} \geq \tau_{tjk} + u_{t} - 1, \ \alpha_{tjk} \geq 0, \ 0 \leq j - 1 \leq k \leq N - 1, \ 1 \leq \ell \leq N_s$$ (28f)

$$\tau_{tjk} \leq \Delta_{jk}, \ \tau_{tjk} \leq u_{t}, \ \tau_{tjk} \geq \Delta_{jk} + u_{t} - 1, \ \tau_{tjk} \geq 0, \ 0 \leq j - 1 \leq k \leq N - 1, \ 1 \leq \ell \leq N_s$$ (28g)

$$\Delta_{jk} \in \{0, 1\}, \ 0 \leq j - 1 \leq k \leq N - 1.$$ (28h)

Proof: According to Theorem 1, we obtain that for a given pair of $u_{t}$ and $\alpha_{t}$ which is feasible for the adjustable DR chance constraint (18b), there exists a $(j, k)$ pair such that

$$k/N \leq \alpha_{t} < (k+1)/N$$ (29)

$$P_{\text{total},t}^{(j)} < \sum_{t \in \mathcal{T}} P_{t} u_{t,\ell} \leq P_{\text{total},t}^{(j-1)}$$ (30)

$$\frac{1}{N} \sum_{n=j}^{k} (P_{\text{total},t}^{(k+1)} - P_{\text{total},t}^{(n)}) - \left( \alpha_{t} - \frac{j-1}{N} \right) \left( \sum_{t \in \mathcal{T}} P_{t} u_{t,\ell} - P_{\text{total},t}^{(k+1)} \right) \leq -\delta.$$ (31)

We denote $\Delta_{jk} \in \{0, 1\}$ for all $0 \leq j - 1 \leq k \leq N - 1$ such that $\Delta_{jk} = 1$ if we select $j$ and $k$ as the critical index pair; $\Delta_{jk} = 0$, otherwise. To impose constraint (31), we require

$$\sum_{j=1}^{N} \sum_{k=1}^{N-1} \left[ -\frac{1}{N} \sum_{n=j}^{k} (P_{\text{total},t}^{(k+1)} - P_{\text{total},t}^{(n)}) - \left( \alpha_{t} - \frac{j-1}{N} \right) \left( \sum_{t \in \mathcal{T}} P_{t} u_{t,\ell} - P_{\text{total},t}^{(k+1)} \right) \right] \Delta_{jk} \leq -\delta.$$ (32)

Constraint (32) is nonlinear due to two bilinear terms, i.e., $\alpha_{t} \Delta_{jk}$ and $u_{t,\ell} \Delta_{jk}$, and one trilinear term, $\alpha_{t} u_{t,\ell} \Delta_{jk}$. To linearize them, we define $\varepsilon_{jk} = \alpha_{t} \Delta_{jk}$, $\tau_{tjk} = u_{t,\ell} \Delta_{jk}$, and $o_{tjk} = \alpha_{t} u_{t,\ell} \Delta_{jk}$ for all $0 \leq j - 1 \leq k \leq N - 1$ and $1 \leq \ell \leq N_s$. Also, we introduce the McCormick inequalities (28e)–(28g).
To ensure the feasibility of the solution \((u_t, \alpha_t)\) associated with a \((j,k)\) pair (there can be multiple solutions associated with one \((j,k)\) pair), we need to further satisfy (29) and (30), which is equivalent to (28c) and (28d) Therefore, we conclude the proof. □

We remark that Theorem 5 also applies to any adjustable individual DR chance constraints with RHS uncertainty. The big-M free MILP reformulation of the adjustable DRCC formulation (18) under the \(D_t^2\) is

\[
\text{MILP 4: } \min_{u_t, \alpha_t} \left\{ c_{\text{sys}} \sum_{j=1}^{N_s} \beta_{t,j} + c_{\text{switch}} \sum_{j=1}^{N_s} u_{t,j} + c_t \alpha_t : (2), (3c) - (3e), (28a) - (28h), (18c) \right\}.
\]

5. Computation Setup

We consider a fleet of \(N_s = 100\) identical buildings and \(N_{PV} = 1\) PV panel for \(N_p = 53\) periods, every 10 minutes from 8:20 am to 17:00 pm over a day. We consider two typical weather conditions: a sunny day and a cloudy day. The PV power output data of \(P_{PV,t}\), as shown in Figure 1, is collected from a 13 kW PV panel located on the rooftop of the Distributed Energy Communication & Control (DECC) laboratory at Oak Ridge National Laboratory (ORNL) in Tennessee. We scale the PV output to be compatible with the aggregate of 100 residential HVAC systems (connected via a same step-down transformer).

![PV profile](image)

Figure 1    PV profile

For each building, we associate a random initial room temperature which is uniformly distributed between \(23.10^\circ\text{C}\) and \(23.15^\circ\text{C}\). The set-point \(x_{\text{ref}}\) is \(23.0^\circ\text{C}\) and the comfort band \([x_{\min}, x_{\max}]\)
is $[21.5, 24.5]^\circ$C. The building parameters are set as $A_j = 0.9914$, $B_j = -0.6767$, $G_j = (4.3e^{-5}, 0.0086)^\top$ for $j = 1, \ldots, N_s$. Each HVAC system consumes 3.5 kW when it is ON. The objective costs are set as follows: $c_{sys} = 1.0$, and $c_{switch} = 1.0$.

In our studies we consider the benchmark chance-constrained formulation and the two DRCC formulations, assuming unknown distributional information of the PV output generation. We refer the three models as:

1. CC: stochastic chance-constrained formulation;
2. DRCC-M: DRCC formulation under the moment-based ambiguity set;
3. DRCC-W: DRCC formulation under the Wasserstein ambiguity set.

We generate $N = 100$ i.i.d. samples (i.e., in-sample data) of $\bar{P}_{pV,t}$ following uniform distributions (Gaunt et al. 2017) with the mean $P_{pV,t}$ (shown in Figure 1) and half range of $0.15P_{pV,t}$ for $t = 1, \ldots, N_p$. We optimize the CC model and construct the ambiguity set of the DRCC-W model by using all $N = 100$ samples, and for DRCC-M, only 10 samples are randomly picked from the $N$ samples to calculate the empirical mean and covariance. With the optimal schedules obtained by solving different models, we generate 10 sets of $N' = 1000$ i.i.d. samples (i.e., out-of-sample data) from the same uniform distribution to evaluate the out-of-sample performance of each schedule.

All models are computed in Python 3.7.5 using Gurobi 9.0.0. The computations are performed on a Windows 10 Pro machine with Intel(R) Core(TM) i7-8700 CPU 3.20 GHz and 16 GB memory.

6. Studies on the DRCC Models

In the DRCC-M model, we set the parameters of the moment-based ambiguity set $(\gamma_1, \gamma_2) = (0, 1)$; in the DRCC-W model, we set the radius parameter $\delta_t = 0.02$. In particular, we solve the DRCC-W model by using the two MILP formulations derived in Section 3.3. The comparison of the computation time and optimality gaps is presented in Section 6.1. In Section 6.2, we present the solution details of the three models, including the tracking performance and resulting room temperatures. In Section 6.3, we present the out-of-sample performance of optimal solutions obtained. Furthermore, we study the sensitivity of the out-of-sample performance on the in-sample data size.
6.1. CPU Time and Optimality Gaps

In this section, we solve the DRCC-W model using both MILP1 and MILP2 formulations proposed in Section 3.3. We generate 10 in-sample sets of the sunny weather following the description in Section 5, with the size of $N = 100$ and $N = 500$, respectively. Each instance contains $N_p = 53$ periods. Given an initial room temperature of the first period, for each instance, we sequentially solve the remaining periods by using the resulting room temperature from previous periods as an initial room temperature. The CPU time limit is set as 100 seconds for each period. We test all 20 instances with risk parameter $1 - \alpha_t \in \{80\%, 90\%\}$ and Wasserstein radius $\delta_t \in \{0.01, 0.02\}$.

In Table 1, the total CPU time (of all 53 periods) of solving MILP2 is much shorter than MILP1 as MILP2 is more compact with fewer binary variables and constraints. For example, when $1 - \alpha_t = 80\%$, $\delta_t = 0.02$ and $N = 100$, the average CPU time of MILP1 is 285.17 seconds, while the average of MILP2 is less than 5 seconds. Column “# Limit” indicates the number of periods that cannot be solved when the time limit is reached. The average optimality gaps of these (unsolved) periods are presented in the next column “Gap.” For all the instances, MILP2 yields fewer unsolved periods and smaller optimality gaps. When $1 - \alpha_t = 80\%$, $N = 500$, with smaller $\delta_t$, the resulting problems are harder to solve, which is also observed in Chen et al. (2018) and Ho-Nguyen et al. (2020). We remark that, although both MILP1 and MILP2 reformulations solve the instances with the same initial room temperatures, they do not solve the same problems for all $N_p = 53$ periods as some periods do not solve to optimality and thus obtain suboptimal solutions, which can result in different initial temperatures for the remaining periods.

6.2. Tracking Performance and Room Temperatures

For all three models, we solve them for all $N_p$ periods under both the sunny weather and cloudy weather conditions, respectively. We note that the PV generation of the cloudy day is relatively lower than that of the sunny day (see in Figure 1). Consequently, when solving instances of the cloudy weather condition, we consider enrolling a subset of 35 residential HVAC units, of which the total energy consumption is compatible with the PV generation. Since the total number of
### Table 1  Comparison of CPU time (in seconds) and optimality gaps

<table>
<thead>
<tr>
<th>Instance</th>
<th>$1 - \alpha_1$</th>
<th>$\delta_1$</th>
<th>MILP1 CPU (seconds)</th>
<th>MILP1 # Limit</th>
<th>MILP1 Gap</th>
<th>MILP2 CPU (seconds)</th>
<th>MILP2 # Limit</th>
<th>MILP2 Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>N = 100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1029.16</td>
<td>6</td>
<td>1.96%</td>
<td>7.33</td>
<td>0</td>
<td>N/A</td>
<td>2551.71</td>
<td>17</td>
</tr>
<tr>
<td>2</td>
<td>1087.77</td>
<td>7</td>
<td>2.21%</td>
<td>6.76</td>
<td>0</td>
<td>N/A</td>
<td>2989.91</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>1148.70</td>
<td>6</td>
<td>1.73%</td>
<td>6.90</td>
<td>0</td>
<td>N/A</td>
<td>3232.25</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>1087.43</td>
<td>6</td>
<td>1.39%</td>
<td>7.44</td>
<td>0</td>
<td>N/A</td>
<td>2859.74</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>953.31</td>
<td>7</td>
<td>1.77%</td>
<td>6.97</td>
<td>0</td>
<td>N/A</td>
<td>3089.84</td>
<td>19</td>
</tr>
<tr>
<td>N = 500</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1364.95</td>
<td>8</td>
<td>1.30%</td>
<td>7.38</td>
<td>0</td>
<td>N/A</td>
<td>3258.24</td>
<td>21</td>
</tr>
<tr>
<td>7</td>
<td>1167.69</td>
<td>8</td>
<td>1.81%</td>
<td>6.75</td>
<td>0</td>
<td>N/A</td>
<td>3065.38</td>
<td>17</td>
</tr>
<tr>
<td>8</td>
<td>1124.80</td>
<td>7</td>
<td>1.74%</td>
<td>7.68</td>
<td>0</td>
<td>N/A</td>
<td>3033.64</td>
<td>14</td>
</tr>
<tr>
<td>9</td>
<td>1273.94</td>
<td>7</td>
<td>1.44%</td>
<td>7.28</td>
<td>0</td>
<td>N/A</td>
<td>2986.50</td>
<td>16</td>
</tr>
<tr>
<td>10</td>
<td>1022.04</td>
<td>6</td>
<td>1.59%</td>
<td>7.37</td>
<td>0</td>
<td>N/A</td>
<td>2988.51</td>
<td>16</td>
</tr>
</tbody>
</table>

Average: 1125.98, 7, 1.69% | 7.19 | 0 | N/A | 3045.97 | 17 | 3.33% | 84.68 | 0 | 1.89%

1.09% 2.27% 4.58 0 N/A 1627.63 9 3.45% 15.24 0 N/A
2.46% 5.05 0 N/A 2058.00 11 3.89% 12.11 0 N/A
3.20% 4.80 0 N/A 1991.55 9 2.52% 12.63 0 N/A
4.26% 4.42 0 N/A 1971.39 11 2.57% 12.04 0 N/A
5.24% 3.88 0 N/A 2049.40 13 3.19% 14.96 0 N/A
6.25% 4.65 0 N/A 2120.61 12 3.29% 14.14 0 N/A
7.24% 3.96 0 N/A 2049.14 11 4.14% 14.68 0 N/A
8.27% 4.60 0 N/A 1985.03 11 2.72% 11.62 0 N/A
9.28% 4.55 0 N/A 2125.29 11 2.84% 15.22 0 N/A

90% 0.02

1.56% 2.27% 4.58 0 N/A 1902.63 9 3.45% 15.24 0 N/A
2.56% 4.55 0 N/A 2058.00 11 3.89% 12.11 0 N/A
3.58% 4.80 0 N/A 1991.55 9 2.52% 12.63 0 N/A
4.62% 4.42 0 N/A 1971.39 11 2.57% 12.04 0 N/A
5.64% 3.88 0 N/A 2049.40 13 3.19% 14.96 0 N/A
6.66% 4.65 0 N/A 2120.61 12 3.29% 14.14 0 N/A
7.68% 3.96 0 N/A 2049.14 11 4.14% 14.68 0 N/A
8.70% 4.60 0 N/A 1985.03 11 2.72% 11.62 0 N/A
9.72% 4.55 0 N/A 2125.29 11 2.84% 15.22 0 N/A

90% 0.2

1.52% 2.27% 4.58 0 N/A 1902.63 9 3.45% 15.24 0 N/A
2.56% 4.55 0 N/A 2058.00 11 3.89% 12.11 0 N/A
3.58% 4.80 0 N/A 1991.55 9 2.52% 12.63 0 N/A
4.62% 4.42 0 N/A 1971.39 11 2.57% 12.04 0 N/A
5.64% 3.88 0 N/A 2049.40 13 3.19% 14.96 0 N/A
6.66% 4.65 0 N/A 2120.61 12 3.29% 14.14 0 N/A
7.68% 3.96 0 N/A 2049.14 11 4.14% 14.68 0 N/A
8.70% 4.60 0 N/A 1985.03 11 2.72% 11.62 0 N/A
9.72% 4.55 0 N/A 2125.29 11 2.84% 15.22 0 N/A

Distributionally Robust Chance-Constrained Building Load Control
Distributionally Robust Chance-Constrained Building Load Control

Figure 2  PV profile tracking under sunny and cloudy weathers

enrolled HVAC devices needs to be compatible with the magnitude of local solar PV generation
(Dong et al. 2018). That is, \( N_s = 35 \) under the cloudy weather. While, we consider enrolling all
the 100 HVAC units for the sunny day. We remark that our model is flexible to incorporate the
decision of the fleet size, which can be decided based on the nameplate capacity of HVAC devices
and local solar PV generation scales. The details can be found in Appendix E.

We present the overall tracking performance using certain number of ON/OFF HVAC devices,
i.e., \( \sum_{j=1}^{N_s} P_j u_{t,j}, \ t = 1, \ldots, N_p \) of the three models in Figure 2. The blue areas in the background is
the plot of 100 PV generation samples used for solving the models. Under both weather conditions,
al all three models track the PV generation well. Most of the periods, the two DRCC models provide
higher HVAC loads than the stochastic CC model given that the DRCC models take into account
ambiguous probability distributions and thus the solutions are more conservative. Between the
two DRCC models, the DRCC-M model yields higher HVAC loads as the DRCC-M model is
generally more conservative. In Figure 2b under the cloudy weather, around 16:00 pm, when the
PV generation is small, the optimal schedule of all three models do not track the PV generation
as close as before.

In Figure 3, we present the resulting room temperatures of all \( N_s = 100 \) buildings over \( N_p \) periods
for all three models. All the indoor temperatures are maintained within the desired comfort band
\([21.5, 24.5]^{\circ}C\). For both weather conditions, the DRCC models provide cooler room temperatures
for most buildings which is an immediate result of turning on more HVAC units as shown in Figure 2.

6.3. Out-of-Sample Performance

After solving all the models and obtaining the optimal schedules, we fix them in 10 out-of-sample data sets, each consisting of $N' = 1000$ samples. For each data set, an out-of-sample probability is calculated as the ratio of the number of scenarios, where the PV generation is consumed locally (that is, the total HVAC load is more than the PV generation), to the total number of scenarios $N'$. The performance is measured by the 95th percentile of the 10 probabilities of the 10 out-of-sample sets.

In Figure 4, the 95th percentile of probabilities are shown for all three models under the sunny and cloudy weather conditions. In both plots, the two DRCC models perform better than the CC models. Again, as the DRCC-M model is more conservative, the DRCC-M model achieves higher probability than the DRCC-W model.

To study the impact of the number of the samples used for solving the models, we consider two choices of the risk parameter $1 - \alpha_t$: 80% and 50% under the sunny weather condition. We solve the three models using 10 samples and 100 samples, respectively, with the two risk levels. The out-of-sample performance is presented in Figure 5. When the sample size is small $N = 10$, except for the DRCC-M model, both the DRCC-W and CC models fail to achieve the required risk level.
Figure 4  Probabilities of locally consuming PV generation under sunny and cloudy weather conditions

Figure 5  Probability of locally consuming PV generation under sunny weather

For example, in Figure 5a, the DRCC-W and the CC models perform below the required risk level $1 - \alpha_t = 50\%$ between 11 am and 12 pm when using only 10 samples. However, with more samples $N = 100$, the DRCC-W model is always above $1 - \alpha_t = 50\%$, while, the CC model sometimes still fails to achieve the required risk level. Overall, the DRCC models perform better than the CC model, which is consistent with the previous observations. The DRCC-W models are more sensitive to the number of samples used compared to the DRCC-M model.

7. Studies on the Adjustable DRCC Models

In this section, we focus on the adjustable variants of the two DRCC models, where we consider the risk parameter $\alpha_t$ as a variable than a known parameter. Specifically, we solve the DRCC-W models using the two MILP formulations proposed in Section 4.2. We present the comparison of
CPU time and optimality gaps in Section 7.1. The solution details and sensitivity analysis of the risk level are presented in Section 7.2.

7.1. CPU Time and Optimality Gaps

Following the in-sample data generation procedure in Section 5, we generate 10 instances (each of 10 samples) under the sunny weather condition and solve them using DRCC-W models by MILP3 and MILP4 reformulations, respectively. The CPU time limit for each period is 100 seconds. In Table 2, MILP4 solves all instances faster with an average of 1154.29 seconds than 3449.39 seconds of MILP3. Among the 53 periods solved, MILP3 has more periods not solved optimally. The average gap of the unsolved periods is up to 12.38%. While MILP4 only yields a gap of 0.73% as the MILP4 formulation provides a tighter linear relaxation than the MILP3.

<table>
<thead>
<tr>
<th>Instance</th>
<th>CPU Limit</th>
<th>Gap</th>
<th>CPU Limit</th>
<th>Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3410.35</td>
<td>33</td>
<td>12.35%</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3456.28</td>
<td>34</td>
<td>12.44%</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3460.71</td>
<td>33</td>
<td>12.84%</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3453.83</td>
<td>34</td>
<td>11.68%</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>3486.60</td>
<td>34</td>
<td>12.13%</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>3459.33</td>
<td>34</td>
<td>12.47%</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>3461.45</td>
<td>34</td>
<td>12.43%</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>3457.13</td>
<td>33</td>
<td>11.88%</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>3392.60</td>
<td>33</td>
<td>12.57%</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>3455.67</td>
<td>33</td>
<td>13.00%</td>
<td></td>
</tr>
</tbody>
</table>

Average: 3449.39 34 12.38% 1154.29 8 0.73%

7.2. Sensitivity Analysis of Risk-Level Coefficient Cost

In this section, we study the impact of the coefficient cost $c_t$ in the objective (18a) of risk level $\alpha_t$ for both DRCC-M and DRCC-W models. We vary the coefficient from 10 to 20 in increments of 2. For the adjustable models, we observe that the optimal risk levels for the periods between 9:30
am and 14:30 pm are relatively lower. That is, during these periods, it is harder to consume all PV generation, which is consistent with the observations in Section 6.3 for the non-adjustable models. Therefore, we focus on these periods in between 9:30 am and 14:30 pm of a day with an increment of one hour.

In Figure 6, we show the risk levels under different choices of coefficient $c_t$ for the DRCC-M and DRCC-W models. We see from Figures 6a and 6b the value $c_t$ of the risk level increases as larger coefficient is set. The graph in Figure 6a of the DRCC-M model is flatter as $c_t$ changes, because the performance of the DRCC-M model is less sensitive to the choice of $c_t$. While, the DRCC-W model has a very low risk level when $c_t$ is small and a higher risk level when $c_t$ is large.

In Figure 7, we show the objective costs for the various coefficient costs $c_t$. We see that both models yield higher objective costs as $c_t$ increases. We also see that the DRCC-M model yields higher objective costs under large $c_t$ even when the risk level is set lower than the DRCC-W model. For example, at 14:30 pm under $c_t = 20$, DRCC-M sets the risk level around 85% with an objective cost around 120, while DRCC-W achieves a higher risk level above 95% with an objective cost less than 90. This is expected as the DRCC-M model is more conservative and requires more HVAC units to run to achieve a similar risk level.

8. Conclusions

In this paper, we formulated a single-period BLC problem as a DRCC problem with uncertain PV generation under both the moment-based and Wasserstein ambiguity sets as DRCC-M and
DRCC-W, respectively. For the moment-based ambiguity set, we reformulated the DRCC problem as an MILP reformulation. For the Wasserstein ambiguity set, we provided an MILP1 reformulation and a more compact MILP2 reformulation by exploiting the RHS uncertainty. By considering the risk level as a decision variable, we proposed adjustable DRCC formulations that determine the optimal risk level of the chance constraint to balance the total cost and overall performance. For the moment-based ambiguity set, we developed an exact solution approach by solving two SOCP problems. For the Wasserstein ambiguity set, we derived a big-M MILP reformulation and a big-M free MILP reformulation using the RHS uncertainty. We note that all the results above can be applied to general individual chance constraints with RHS uncertainty. We conducted extensive computational studies on the non-adjustable and adjustable DRCC models under the two ambiguity sets. We find that the DRCC models achieve better out-sample performance while maintaining the indoor temperature within a desired comfort band. Specifically, the DRCC-M model requires fewer samples to achieve the required risk level. The DRCC-W model performs well when using enough many samples and only requires modest CPU time when solving the more compact reformulation MILP2. Furthermore, for the adjustable DRCC problem, we find that the DRCC-M is less sensitive to the choice of $c_t$ while it may require higher objective costs.

References


Appendices

A. Proof of Theorem 2

Proof of Theorem 2: Constraint (19) is equivalent to (20c) and

\[ (\gamma_2 - \gamma_1)\sigma_t^2 \leq \alpha_t \left[ \left( \sum_{j=1}^{N_s} P_j u_{t,j} - \theta_t - \sigma_t \sqrt{\gamma_1} \right)^2 + 2(\gamma_2 - \gamma_1)\sigma_t^2 \right]. \]  \hspace{1cm} (33)

The RHS of (33) is nonlinear. We let \( d = \left( \sum_{j=1}^{N_s} P_j u_{t,j} - \theta_t - \sigma_t \sqrt{\gamma_1} \right)^2 + 2(\gamma_2 - \gamma_1)\sigma_t^2. \) Then inequality (33) is equivalent to \( (\gamma_2 - \gamma_1)\sigma_t^2 \leq \alpha_t d \) which is equivalent to (20a). To linearize \( d \), we define \( g_{ij} := u_{t,i} u_{t,j} \) by McCormick inequalities: (20d)–(20g). We conclude the proof. \( \square \)

B. Proof of Theorem 3

Proof of Theorem 3: To show the equivalence, we need to show that (i) constraint (21) implies constraints (22a)–(22d) and (ii) constraints (22a)–(22d) imply constraint (21).

(i) (21) \( \rightarrow \) (22a)–(22d).

Given a solution \((u^*_t, \alpha^*_t)\) that satisfies (21), we let \( v^* = \sqrt{1/\alpha_t^*}, w^* = \sqrt{v^*}, q^* = 1/w^*. \) Then \((u^*_t, \alpha^*_t, v^*, w^*, q^*)\) is a solution to (22a)–(22d).

(ii) (22a)–(22d) \( \rightarrow \) (21).

We notice that (22b) is equivalent to

\[ \alpha_t v \geq q^2, \quad v \geq 0; \]  \hspace{1cm} (34)

and (22d) can be rewritten as

\[ q \geq \frac{1}{w}, \quad w \geq 0. \]  \hspace{1cm} (35)

Combining (34), (35), and (22c), we have \( \alpha_t v \geq \frac{1}{v} \), which is further equivalent to

\[ v \geq \sqrt{\frac{1}{\alpha_t}}. \]  \hspace{1cm} (36)

Combining (22a) and (36), we conclude that constraint (22a) implies (21). \( \square \)
C. Proof of Theorem 4

Proof of Theorem 4: In the adjustable DRCC model (18), we replace the adjustable DR chance constraint (18b) with the following convex reformulation

\[ \sum_{j=1}^{N_s} P_j u_{t,j} - \theta_t - \sigma_t \sqrt{\gamma_1} \geq \sigma_t \sqrt{\gamma_2 - \gamma_1 r} \quad (37) \]

\[ r \geq \sqrt{\frac{1 - \alpha_t}{\alpha_t}}. \quad (38) \]

The RHS of constraint (38) is convex when \(0 \leq \gamma_1/\gamma_2 \leq \alpha_t \leq 0.75\) as the second-order derivative \((3 - 4\alpha_t)(1 - \alpha_t)^{-3/2}\alpha_t^{-5/2}/4\) is non-negative. We can further construct an outer approximation of the reformulation (37)–(38) by replacing (38) with

\[ 2r \geq \sqrt{\frac{1}{\alpha_t}}. \quad (39) \]

Constraint (39) is implied by (38) when \(\alpha_t \leq 0.75\). Using a similar proof of Theorem 3, we can show that (39) is equivalent to constraints (23a) – (23d). □

D. Proof of Proposition 1

Proof of Proposition 1: To show that \(Z = Z_1\), we need to show that \(Z \subseteq Z_1\) and \(Z_1 \subseteq Z\).

(i) \(Z \subseteq Z_1\).

Given \(u_t \in Z\), there exists \(\gamma \geq 0\) such that \((u_t, \gamma)\) satisfies (9a) and (9b). If \(\gamma > 0\), let \(\lambda = 1/\gamma\).

It is easy to see that \((u_t, \lambda)\) satisfies (24a) and (24b). For the case \(\gamma = 0\), (9a) is equivalent to

\[ \left\{ u_t : \delta_t \leq \frac{1}{N} \sum_{n=1}^{N} \min \left\{ 0, \max \left[ \sum_{j=1}^{N_s} P_j u_{t,j} - P_{n,\text{total},t}, 0 \right] \right\} \right\} = \{ u_t : \delta_t \leq 0 \}. \]

Since \(\delta_t > 0\), the left-hand side of (D) is equivalent to an empty set.

(ii) \(Z_1 \subseteq Z\).

Given \(u_t \in Z_1\), there exists \(\lambda \geq 0\) such that \((u_t, \lambda)\) satisfies (24a) and (24b). Similarly, if \(\lambda > 0\), we let \(\gamma = 1/\lambda\), which satisfies (9a) and (9b). In the case \(\lambda = 0\), (24) is equivalent to

\[ \{ u_t : \alpha_t \geq 1 \} \]

which is empty. □
E. DRCC Model with the Decision of Fleet Size

In this section, we present a DRCC model that incorporates the decision of the fleet size of residential HVAC units, \( N_s \), which is a given parameter in previous models. We associate \( N_s \) with a unit penalty cost \( c_{N_s} \) in the objective coefficients. We denote \( N_{U_s} \) the maximum number of the HVAC units we can deploy to consume the PV generation. For HVAC unit \( j, j = 1, \ldots, N_{U_s} \), we introduce a logical binary variable \( \zeta_j \) such that \( \zeta_j = 1 \), if unit \( j \) belongs to the fleet, and 0 otherwise. The DRCC model is formulated as follows.

\[
\begin{align*}
\min_{N_s, u_t, t=1,\ldots, N_p} & \quad \sum_{t=1}^{N_p} \left[ c_{\text{sys}} \sum_{j=1}^{N_{U_s}} \beta_{t,j} + c_{\text{switch}} \sum_{j=1}^{N_{U_s}} u_{t,j} \right] + c_{N_s} N_s \\
\text{s.t.} & \quad (3c) - (3e) \text{ for } t = 1, \ldots, N_p \\
& \quad \inf_{f \in \mathcal{D}} \mathbb{P} \left( \sum_{j=1}^{N_{U_s}} P_j u_{t,j} - \sum_{i=1}^{N_{PV}} \tilde{P}_{PV,t,i} \geq 0 \right) \geq 1 - \alpha_t \text{ for } t = 1, \ldots, N_p \\
& \quad \zeta_j \leq \sum_{t=1}^{N_p} u_{t,j} \leq N_p \zeta_j, \ j = 1, \ldots, N_{U_s} \\
& \quad \sum_{j=1}^{N_{U_s}} \zeta_j \leq N_s \\
& \quad x_{1,j} = A_j x_{0,j} \zeta_j + B_j u_{1,j} + G_j v_j \zeta_j + (1 - \zeta_j) x_{\text{ref}}, \ j = 1, \ldots, N_{U_s} \\
& \quad x_{t,j} = A_j (x_{t-1,j} - x_{\text{ref}} + x_{\text{ref}} \zeta_j) + B_j u_{t,j} + G_j v_j \zeta_j + (1 - \zeta_j) x_{\text{ref}}, \ t = 2, \ldots, N_p, \ j = 1, \ldots, N_{U_s} \\
& \quad 0 \leq N_s \leq N_{U_s} \\
& \quad \zeta \in \{0, 1\}^{N_{U_s}}.
\end{align*}
\]

Constraint (40c) requires all \( u_{t,j} \)'s being zeros if \( \zeta_j = 0 \) and thus the HVAC unit \( j \) is not in the fleet. Constraints (40e) and (40f) ensure that for HVAC unit \( j \) not in the fleet, i.e., \( \zeta_j = 0 \), the indoor temperatures \( x_{t,j} \) over all \( N_p \) periods are imposed to be \( x_{\text{ref}} \) and thus contribute zero to the objective value. We remark that (40) is a multiperiod model over all \( N_p \) periods with individual DR chance constraints to guarantee the utilization of the PV generation for each period. All the solution methods and modeling techniques in Sections 3 and 4 can still be applied to (40).