An exact solution approach for an electric bus dispatch problem

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Abstract

We study how to efficiently plan a bus dispatch operation within a public transport terminal working with electric buses. This requires to formulate and solve a novel variant of the Vehicle Scheduling Problem model, in which one has to assign a daily trip itinerary and a battery charge plan to each vehicle within a fleet to cover a set of demanded trips and simultaneously sequence all charging tasks, each with varying duration, at a charging station within a terminal with limited parallel charging infrastructure. To optimally solve our problem, we decompose decisions into two stages; (1) assigning trip itineraries to each vehicle and (2) sequencing battery charging tasks to each station given a set of vehicle itineraries. Our decomposition is naturally suited for a customized variant of the integer L-shaped method, in which feasibility cuts are dynamically injected to the branch–and–bound tree to remove itinerary assignments leading to an infeasible second stage. The effectiveness of our solution is tested in computational experiments inspired by a bus operator from Santiago, Chile. We assess the benefits over a single-stage optimization approach and provide managerial insights for planners, such as the marginal benefits per additional charging station and electric bus. Also, we study the flexibility added to the dispatch operation when combining electric and conventional buses.
1 Introduction

Nowadays, it is urgent to replace fossil fuels by renewable energy sources. According to [20], air pollution caused nearly three million premature deaths in 2010 and might reach up to 6-9 million deaths per year in 2060. Among all air pollutant activities, the urban transportation sector is responsible for approximately 54% of CO and 14% of CO2 emissions worldwide [28]. To reduce pollution, some public transport authorities have transitioned to the use of cleaner vehicles, such as electric buses [30]. For example, the adoption of electric vehicles in China’s public transport systems reached roughly 400 thousand units in 2019 and represented 99% of the world’s electric bus fleet [18]. According to [27], over 5,000 electric buses will be delivered yearly to Latin America by 2025.

Also, a transition towards an electric fleet can be profitable in the long run. According to [14], the total cost of ownership for electric city buses could equate that of diesel ones as early as 2023. Compared to a diesel vehicle, the purchase of an electric bus is approximately 63% more expensive, but fuel and maintenance costs per vehicle-mile are approximately 3.3 and 2 times cheaper, respectively [23].

Operating electric buses cost-efficiently is also crucial to achieve long-run profitability and pollution reduction. Compared to conventional vehicles, this technology has limited driving ranges and relatively long battery charging times [22]. If these restrictions are left out from the operations planning process, one may overestimate the potential usage of an electric fleet at a public transport terminal (depot).

1.1 Literature Review

Recently, several research efforts within the Transportation Science and Logistics literature have studied strategic and tactical decisions related to the effective and efficient management of electric vehicle fleets and their charging infrastructure. Decisions studied involve the definition of battery capacities [15], charging infrastructure investments [15, 26, 31, 33], vehicle purchase decisions [17, 26], and charging station location [15, 31, 33], among others.

The daily bus dispatch operation at a terminal can be modeled as a Vehicle Scheduling Problem (VSP) introduced by [12], which plans the assignment of a sequence of trips (trip itinerary) to be covered by each vehicle. Each bus can serve one trip at-a-time and each trip is encoded as a time window that must be assigned to one vehicle without preemption. The single-depot VSP for conventional buses is a special case of the Maximum Flow Problem and is thus efficiently solvable. The authors in [4] provide a survey on the VSP and its extensions. More research on the VSP is discussed in [6, 8, 9, 13].
To improve the utilization of electric buses, one has to integrate battery charging activities into the bus dispatch planning. This gives rise to the Electric Vehicle Scheduling Problem (E-VSP) introduced by [32]. Besides assigning trip itineraries to buses, a feasible solution to the E-VSP should schedule bus charging tasks with variable duration each and guarantee that the state of charge (SoC) of each bus’ battery is kept within its acceptable range (minimum and maximum) at all times, since a battery usage out of this range significantly reduces its lifespan [19]. So, a feasible trip itinerary for each bus must be complemented with a **charge itinerary**, which is a sequence of battery charging tasks at the depot keeping feasible battery SoC levels.

Moreover, the solution to the E-VSP is harder when there is a limit on the number of vehicles that can be charged simultaneously. This setting arises in practice due to limited grid power capacity and parallel infrastructure installed at a depot [11, 21]. Here, a feasible solution must also select and sequence the bus charging tasks assigned to each charging station, *i.e.*, **charging station itinerary**.

Therefore, the E-VSP with limited charging infrastructure can be modeled as an optimization problem with three interlocked and simultaneous decisions: (1) planning feasible trip itineraries for each bus covering all trips; (2) designing feasible charge itineraries for each bus; and (3) assigning a feasible itinerary to each charging station. Similar synchronization constraints are studied for the Vehicle Routing Problem in [10].

Problems exhibiting a hierarchical structure among decisions might be amenable to decomposition approaches such as Benders’ decomposition [3]. This method has been successful in solving problems arising in a wide range of applications [24], in particular routing and logistics. Moreover, it is possible to extend this approach to a class of problems with discrete variables. As we will later see, the integer L-shaped method [16] precisely fits our setting.

The scarce literature regarding the E-VSP is summarized in Table 1. The authors in [1] study a VSP for alternative fuel vehicles, in which energy-feasibility constraints are considered for each vehicle. They propose an exact column generation approach and a heuristic method based on [5] to solve larger instances. In their study, charging events are considered instantaneous and thus not realistic for electric buses. The research of [32] addresses an E-VSP that allows partial battery charges and considers charge times as a linear function of the energy injected to the battery. They propose a mixed-integer linear programming (MILP) formulation, as well as an adaptive large neighbourhood search heuristic to solve the problem. However, their formulation has no limits on parallel charging capacity. Constraints that model limited parallel charging infrastructure are considered in [26] to address a variant of the E-VSP.
that also addresses charging station investment decisions. This problem is solved without allowing partial charges and heuristically via a genetic algorithm. Limited parallel charging is also modeled in the E-VSP presented by [25], which is solved as a discrete time model in which full battery charges take at most one time period. Finally, the authors of [29] propose a robust vehicle scheduling strategy considering trips with stochastic start and end times. They represent each vehicle itinerary in a path-based formulation solved via column generation. However, they limit battery charge operations to a full SoC with constant duration.

To the best of our knowledge, no research effort regarding the E-VSP simultaneously considers energy limitations, charge duration decisions, partial charges, and a limited charging capacity in a continuous time model. Furthermore, no two-stage decomposition approaches, such as a Benders’ decomposition and the integer L-shaped method, have been proposed for this problem.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Charge time decisions</th>
<th>Partial charges</th>
<th>Limited parallel charging</th>
<th>Continuous time model</th>
<th>Solution approach(es)</th>
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Table 1: Brief summary of related literature to the E-VSP.

1.2 Scope and Contribution

In this article, we present a novel study of bus dispatch operational decisions for a mixed fleet of conventional and electric vehicles. We propose a model that extends the existing E-VSP literature for the case in which there is a limited parallel charging capacity at a single depot and battery charge duration is a decision variable. The objective of our problem is set to hierarchically minimize (1) the
number of diesel buses pulled out from the terminal each day subject to a fixed fleet of electric vehicles and (2) their total energy consumed over the day. This choice is made to represent the current need of minimizing pollution.

Our main contributions are summarized below:

• We model an E-VSP with limited parallel charging infrastructure which simultaneously plans trip, battery charge and charging station itineraries. This model explicitly considers operational conditions, such as constrained battery energy levels and charging operations with variable duration.

• To optimally solve our problem, we propose a novel MILP formulation for the E-VSP which exploits the model’s structure and decomposes it into two natural decision stages to reduce its difficulty and adapt the Integer L-shaped Method [16, 2] to it. In the first stage, we plan trip and charge itineraries for each bus. In the second, we solve an auxiliary second-stage problem that plans charging station itineraries.

• The effectiveness of our solution method is assessed and compared to a single-stage model in computational experiments inspired by a bus operator from Santiago, Chile.

• Tactical fleet planning can also benefit from our electric bus dispatch model considering a limited charging infrastructure at the depot. We provide several managerial insights, such as estimating the marginal benefits provided by an additional charging station and electric bus. Such values could help managers in fleet planning and charging infrastructure investment decisions. Also, we discuss how investing in diesel vehicles or increasing battery sizes can help in adding operational flexibility to the electric fleet and improve its utilization.

The remainder of this article is organized as follows. Section 2 describes the problem to be addressed and the main assumptions considered. Section 3 presents the mathematical formulation of the exact two-stage approach. Section 4 briefly proposes a single-stage formulation, introduces notation, describes the numeric studies conducted, and explains the main managerial insights obtained. Finally, Section 5 concludes the study and proposes future research.

2 Problem statement

We consider the bus dispatch operation of a single public transport terminal, a.k.a., depot, over a 24-hour time period. Before the operation starts, the terminal’s dispatcher is given a set $T$ of trip
services to be executed throughout the day, where each trip $j \in T$ specifies its start and end time at the
depot given by $t_j^{\text{start}}$ and $t_j^{\text{end}}$, respectively. The dispatcher has to plan a feasible coverage of trips and
guarantee that each trip is executed by exactly one bus from the terminal’s available fleet represented
by a set $V$. Each bus is electric or conventional and serves at most one trip at-a-time. We assume that
all electric vehicles are homogeneous models with homogeneous batteries, but can start the day with
heterogeneous battery charge levels. Thus, let $V_0$ be the set of all available electric buses and $V_1 = \{v_1\}$
be a singleton set representing all $\tilde{d}^{\max}$ conventional buses available, i.e., $V := V_0 \cup V_1$.

Each vehicle $i \in V_0$ starts the day with a battery SoC level $\bar{e}_i$, must operate within a feasible range
given by $[\bar{e}_{\text{min}}, \bar{e}_{\text{max}}]$, and must end the day with a SOC level no smaller than $\bar{e}_{\text{end}}$. Executing trip $j$
consumes $\bar{e}^j$ SoC units from an electric bus battery; this parameter is not necessarily proportional to
trip distance or duration since other effects, such as traffic congestion and road degree, might be affect
consumption.

An electric bus can charge its battery at one of the homogeneous charging stations available at the
depot represented by a set $C$. Each station $c \in C$ can connect to at most one bus at-a-time, charge its
battery at a rate of $f$ SoC units per minute, and operate within time window $[p^{\text{start}}, p^{\text{end}}]$, which may be
the whole day or a sub-period where electricity is cheaper at the terminal. We consider that charging
time is a decision variable, but assume that each charge operation must take at least $\hat{t}_{\text{min}}$ minutes to
avoid a large number of short charges per vehicle which can be hard to execute in practice. For similar
reasons, we also assume that each bus can only be charged at most once between the execution of
two consecutive trips. Also, we assume a constant SoC charging rate as a function of battery charging
time, which is a first step to solve this hard operational problem. The potential extension to non-linear
charging rates is discussed in Section 5. Finally, we assume that conventional buses start with sufficient
fuel to cover their daily operation or, equivalently, can refuel instantaneously. Also, denote by $\gamma_j$ the
amount of fuel consumed when a conventional bus executes trip $j \in T$.

The problem studied in this work aims to simultaneously plan trip, bus battery charge and charging
station itineraries meeting trip coverage, operational bus constraints, parallel charging capacities and
feasible SoC patterns for each electric bus. To reduce costs and pollution, we set as an objective to
hierarchically minimize the number of conventional buses pulled out from the depot first and later the
total consumption of conventional buses.
3 Two-stage model formulation

Now, we present a two-stage MILP formulation that models the electric bus dispatch problem stated in Section 2.

A more direct approach is a single-stage MILP formulation, which is stated in Appendix C. According to [10], such a model imposes task and resource synchronization constraints, since vehicles must cover all trip tasks while they compete for scarce resources. As empirically discussed in Section 4, the computational time of a single-stage formulation rapidly explodes as a function of the number of vehicles, chargers and trips and, thus, becomes intractable for medium-sized instances.

Alternatively, we consider a two-stage MILP formulation, which is solved by a variant of the Integer L-shaped method [16]. In our first-stage model, we plan feasible trip and bus charge itineraries, but relax difficult charger sequencing constraints and replace them with aggregated restrictions limiting the maximum power allowed to be delivered per charger at specific time windows. In turn, this implies that the solution obtained only approximates a charge itinerary and might not be realizable.

To check feasibility of a given trip itinerary plan obtained from a first-stage solution, we try to identify a complementary pair of feasible bus charge and charging station itineraries in a second-stage problem. If no pair exists, then a feasibility cut is added to the first-stage model to cut-off the specific trip itinerary plan from the search space. Figure 1 presents a scheme of the proposed two-stage solution approach.

![Figure 1: Scheme of the proposed two-stage solution approach.](image)

The details of each model are given as follows.
3.1 First-stage model

We encode our first stage solution in a digraph $G = (N, A)$, where the set of nodes $N := T \cup V \cup \{0\}$ is formed by all trips, a source for each electric bus, one source for all conventional buses, and a sink, respectively. Set $A$ contains arc $(j_1, j_2)$ if trips $j_1, j_2 \in T$ can be executed by a single bus (i.e., $t_{j_1}^{\text{end}} \leq t_{j_2}^{\text{start}}$), arc $(i, j)$ for each $i \in V$ and $j \in T$, and arc $(j, 0)$ for each $j \in T$. So, a path in $G$ starting at node $i \in V$ and defined by $\{i, j_\sigma_1, j_\sigma_2, \ldots, j_\sigma_p, 0\}$ represents a daily trip itinerary for a bus executing $p$ trips $j_\sigma_1, j_\sigma_2, \ldots, j_\sigma_p$.

To distinguish vehicle types, define set $L := \{0, 1\}$ where type 0 represents electric buses and type 1 conventional ones. Then, define set $A_\ell := \{(u, v) \in A : u \in T \cup V\}$ as the subset of arcs that buses of type $\ell \in L$ can use. Also, define the set of arcs in $A_\ell$ leaving and entering node $i \in N$ as $\delta_\ell^+(i) := \{(i, j) \in A_\ell : j \in N\}$ and $\delta_\ell^-(i) := \{(j, i) \in A_\ell : j \in N\}$, respectively.

Each arc $a := (u, v) \in A_0$ also defines a time interval $[\alpha_a, \beta_a]$ when an electric bus covering consecutive trips $u$ and $v$ may charge in between these trips. Accordingly, $\alpha_a$ is set equal to $\max(p^{\text{start}}, t_u^{\text{end}})$ if $u \in T$ and to $p^{\text{start}}$ otherwise, while $\beta_a$ is set equal to $\min(p^{\text{end}}, t_v^{\text{start}})$ if $v \in T$ and to $p^{\text{end}}$ otherwise. So, it is possible to charge such a bus if and only if this interval duration is greater than or equal to the minimum charging time; let $\bar{A}_0 := \{a \in A_0 : \beta_a - \alpha_a \geq \bar{t}^\text{min}\} \subseteq A_0$ be the set of such charge arcs. Also define $I_{\text{start}} := \{\alpha_a : a \in \bar{A}_0\}$ and $I_{\text{end}} := \{\beta_a : a \in \bar{A}_0\}$ as the sets of all interval start and end times, respectively. Finally, let $\bar{\delta}_0^+(i) := \delta_0^+(i) \cap \bar{A}_0$ for $i \in V_0 \cup T$.

**Decision Variables**

We can now define first-stage decision variables. Let $x_\ell^a \in \{0, 1\}$ be a binary variable defined for each $\ell \in L$ and $a \in A_\ell$ representing a vehicle of type $\ell$ traversing arc $a$. Decision vector $\mathbf{x}$ encodes a set of at most $|V_0| + \bar{c}^{\text{max}}$ bus itineraries as paths starting from bus nodes in $V$, ending in node 0 and visiting a subset of trip nodes in $T$.

For each electric bus arc $a = (u, v) \in A_0$, we also define a continuous variable $e_a \geq 0$ representing the battery’s SoC just after possibly charging in between nodes $u$ and $v$ if $a \in \bar{A}_0$, or after leaving node $u$ if $a \notin \bar{A}_0$. So, decision vector $\mathbf{e}$ encodes battery SoC levels for electric buses.

To model charging decisions at each arc $a = (u, v) \in \bar{A}_0$, we define a binary variable $y_a^c \in \{0, 1\}$ and continuous variable $g_a^c \geq 0$ modeling whether there is a charge and charged SoC level at station $c \in C$ for an electric bus in between nodes $u$ and $v$, respectively. Therefore, decision vectors $\mathbf{y}$ and $\mathbf{g}$ represent charger itineraries.

A graphic example of a first-stage problem graph is depicted in Figure 2.
Figure 2: Example of a first-stage problem graph for an instance with two trips, two electric buses and one or more conventional buses.

**Objective function**

We minimize two objectives in a hierarchical order. The first objective function is to minimize the total number of conventional buses used in the operation as stated in (3.1a). The second objective function is to minimize the total energy consumed by all conventional buses (3.1b). Nonetheless, our decomposition approach works with any linear function on variables $x$.

\[
\begin{align*}
\min & \sum_{a \in \delta_T^+ (v_1)} x_a, \\
\min & \sum_{a=(u,v) \in A_1: v \neq 0} \gamma_v \cdot x_a. 
\end{align*}
\]  

(3.1a) (3.1b)

**First-stage feasibility constraints**

Any value of $(x, e, y, g)$ satisfying system (3.2) is feasible to the first-stage model.

\[
\sum_{\ell \in L} \sum_{a \in \delta^-_\ell (j)} x_{a} = 1, \quad \forall j \in T, 
\]  

(3.2a)
\[
\begin{align*}
\sum_{a \in \delta^+_0(i)} x^0_a & \leq 1, & \forall i \in V_0, & \quad (3.2b) \\
\sum_{a \in \delta^+_d(v_1)} x^1_a & \leq d^\text{max}, & \quad (3.2c) \\
\sum_{a \in \delta^+_t(j)} x^\ell_a - \sum_{a \in \delta^-_t(j)} x^\ell_a & = 0, & \forall j \in T, \ \ell \in L, & \quad (3.2d) \\
e_a - \bar{e}_i \cdot x^0_a & = \sum_{c \in C} g^c_a, & \forall i \in V_0, a \in \tilde{\delta}^+_0(i), & \quad (3.2e) \\
e_a - \bar{e}_i \cdot x^0_a & = 0, & \forall i \in V_0, a \in \tilde{\delta}^+_0(i) \setminus \tilde{\delta}^+_0(i), & \quad (3.2f) \\
\sum_{a \in \delta^+_0(j)} e_a - \sum_{a \in \delta^-_0(j)} e_a & = \sum_{c \in C} \sum_{a \in \delta^+_0(j)} g^c_a - \bar{e}_j \cdot \sum_{a \in \delta^-_0(j)} x^0_a, & \forall j \in T, & \quad (3.2g) \\
(e^\text{min} + \bar{e}_j) \cdot x^0_a & \leq e_a, & \forall j \in T, a \in \delta^+_0(j), & \quad (3.2h) \\
e^\text{end} \cdot x^0_a & \leq e_a, & \forall a \in \delta^+_0(0), & \quad (3.2i) \\
e_a & \leq \bar{e}_a \cdot x^0_a, & \forall a \in A_0, & \quad (3.2j) \\
\sum_{c \in C} y^c_a & \leq x^0_a, & \forall a \in \bar{A}_0, & \quad (3.2k) \\
y^c_a \cdot f \cdot \bar{t}^\text{min} & \leq g^c_a \leq \min(f \cdot (\beta_a - \alpha_a), \bar{e}^\text{max} - \bar{e}^\text{min}) \cdot y^c_a, & \forall a \in \bar{A}_0, c \in C, & \quad (3.2l) \\
\sum_{a \in \bar{A}_0 : [\alpha_a, \beta_a] \subseteq [t_1, t_2]} g^c_a & \leq f \cdot (t_2 - t_1), & \forall c \in C, t_1 \in I^\text{start}, t_2 \in I^\text{end}: t_1 < t_2. & \quad (3.2m)
\end{align*}
\]

Constraints (3.2a) guarantee that all trips are executed by exactly one vehicle. Constraints (3.2b), (3.2c) and (3.2d) enforce vehicle flow conservation constraints for each vehicle starting from node \(i \in V\). Constraints (3.2e), (3.2f) and (3.2g) impose battery SoC flow conservation for each node. Constraints (3.2h), (3.2i) and (3.2j) guarantee that battery SoC is kept within feasible ranges. Constraints (3.2k) allow charges only in arcs traversed by electric vehicles, whereas constraints (3.2l) impose feasible charging ranges. Finally, constraints (3.2m) guarantee that the total power charged from each station within all possible time intervals is always less than or equal to its power capacity. Observe that not all constraints (3.2m) must be considered, as one constraint may be implied by one or more constraints having smaller time intervals. In particular, if for \(t_1, t'_1 \in I^\text{start}\) and \(t_2, t'_2 \in I^\text{end}\), we have that \([t_1, t_2] \subseteq [t'_1, t'_2]\) and that the sets of time intervals \([\alpha_a, \beta_a]\) for \(a \in \bar{A}_0\) contained in each of \([t_1, t_2]\) and \([t'_1, t'_2]\) are the same, then the constraint imposed by \([t'_1, t'_2]\) is redundant.

Constraints (3.2m) meet charger station power capacities, but do not guarantee minimum charge duration and non-preemptive bus-to-charger assignments. Therefore, it may be feasible that the first-
stage solution yields an infeasible solution to the overall problem. Figures 3 and 4 illustrate an example where constraints (3.2m) produce a loose relaxation of the first-stage search space. In Figure 3, we observe that these constraints allow to preemptively schedule a particular bus charging operation interrupting it by another operation and then resuming it. Under these conditions, it is feasible for buses 1 and 2 to charge 120 and 60 minutes, respectively. However, a solution is only feasible to the second stage if each charging task is uninterruptedly performed. As depicted in Figure 4, if the charge of bus 2 is scheduled between 14:00 and 15:00, then bus 1 cannot charge more than 60 continuous minutes, making a 120 minute charge impossible.

Figure 3: Example of the charge time window availability of a feasible solution to the first-stage problem for an instance with four trips, one charger and two electric vehicles.
3.2 Second-stage model

We now present a model to identify if a given set of bus itineraries designed via a first-stage solution can be complemented with a feasible battery charge itinerary for each bus and a feasible itinerary for each charging station. Although a solution to the first stage model also identifies battery charge itineraries for each bus, we choose to only use vector $\mathbf{x}$ as input for the second stage to certify multiple first-stage solutions with identical values for $\mathbf{x}$ in a single second-stage computation. However, this approach imposes that the first-stage objective should only be dependent on $\mathbf{x}$ as it now occurs.

As conventional vehicles are not charged, this problem only applies to electric vehicles with at least one trip assigned. As parameters, we have a fleet of electric buses, each with previously assigned trips, and the objective is to identify feasible bus charge itineraries such that charging tasks can be feasibly sequenced in the available charging infrastructure.

For each electric bus $i$, define $\hat{d}_i$ as the number of possible charging events available within its itinerary. Recall that $\hat{d}_i$ is equal to the number of arcs in $A_0$ traversed by bus $i$ in the first-stage solution. Each possible charge defines an interval $[\hat{\alpha}_{i,d}, \hat{\beta}_{i,d}]$ where charging may occur. Also, let $\rho_{i,d}$ be the cumulative SoC level consumed by bus $i$ minus its initial SoC immediately before its $(d+1)$-th
possible charge or the end of its itinerary.

Our second-stage model is built over a digraph \( H = (\Omega, \Gamma) \), where the set \( \Omega \) contains nodes \((i, d)\), each representing the \( d \)-th possible charging event for bus \( i \), a source node \( S \) and a sink \( \bar{S} \). In the set \( \Gamma \), we add an arc \(((i_1, d_1), (i_2, d_2))\) for each pair of different charging events \((i_1, d_1) \neq (i_2, d_2)\) such that both possible charging events, namely the \( d_1 \)-th of bus \( i_1 \) and the \( d_2 \)-th of bus \( i_2 \), can be carried out in that order in one charging station for at least the minimum charging time each, i.e., \( \{((i_1, d_1), (i_2, d_2) : (i_1, d_1) \neq (i_2, d_2), \hat{\beta}_{i_2,d_2} - \hat{\alpha}_{i_1,d_1} \geq 2 \cdot \hat{t}_{\min}\} \). Additionally, we add to \( \Gamma \) source arc \((S, (i, d))\) and sink arc \(((i, d), \bar{S})\) for each \((i, d) \in \Omega\). Also, define \( \Gamma^+_d \) and \( \Gamma^-_d \) as auxiliary subsets containing the arcs entering and leaving node \((i, d)\), respectively, and define \( \phi_{(i,d), (i',d')} := \min\{(\min\{\hat{\beta}_{i',d'} - \hat{t}_{\min}, \hat{\beta}_{i,d}\} - \hat{\alpha}_{i,d}) \cdot f, (\hat{e}_{\max} - \hat{e}_{\min})\} \) as the maximum charge allowed for task \((i, d)\) if it is scheduled at a charger station immediately before task \((i', d')\). Figure 5 shows an example of how charging events and parameters \( \hat{\alpha}, \hat{\beta} \) and \( \rho \) are defined for a particular vehicle and trip itinerary.

![Diagram](image)

**Figure 5:** Example of the definition of charging events in the second-stage problem for an electric vehicle trip itinerary with three trips.

**Decision Variables**
We define continuous variables $v_{i,d} \geq 0$ and $b_{i,d} \geq 0$ representing the charged amount and start time for the $d$-th possible charging event of bus $i$, respectively. Also, we set a binary variable $z_a$ for each arc $a \in \Gamma$, representing that two possible charging events are consecutively executed in the same station.

Our model assumes that each station has the same charging power. However, it can be extended to consider heterogeneous charging infrastructure by creating one source node per charger, adding an index $c$ to $z$ variables to differentiate each charger, and slightly reformulating the first- and second-stage models.

**Second-stage feasibility constraints**

The second-stage model does not have an objective function. The constraints in formulation (3.3) define a feasible second-stage solution.

\[
\sum_{d' \leq d} v_{i,d'} - \rho_{i,d} \geq \max(e^{\text{min}}, I(d = \hat{d}_i) \cdot e^{\text{end}}), \quad \forall (i, d) \in \Omega, \tag{3.3a}
\]

\[
\sum_{d' \leq d} v_{i,d'} - \rho_{i,d-1} \leq e^{\text{max}} \quad \forall (i, d) \in \Omega, \tag{3.3b}
\]

\[
v_{i,d} \leq \sum_{a \in \Gamma^{+}_{(i,d)}} \phi_a \cdot z_a, \quad \forall (i, d) \in \Omega, \tag{3.3c}
\]

\[
f \cdot \hat{t}^{\text{min}} \cdot \sum_{a \in \Gamma^{+}_{(i,d)}} z_a \leq v_{i,d}, \quad \forall (i, d) \in \Omega, \tag{3.3d}
\]

\[
\sum_{a \in \Gamma^{+}_{(i,d)}} z_a = \sum_{a \in \Gamma^{-}_{(i,d)}} z_a \leq 1, \quad \forall (i, d) \in \Omega, \tag{3.3e}
\]

\[
\sum_{a \in \Gamma^{+}_{(i,d)}} z_a \leq |C|, \tag{3.3f}
\]

\[
b_{i,d} + \frac{1}{f} \cdot v_{i,d} \leq b_{i',d'} + (\hat{\beta}_{i,d} - \hat{\alpha}_{i',d'}) \cdot (1 - z_{(i,d),(i',d')}), \quad \forall ((i, d), (i', d')) \in \Gamma, \tag{3.3g}
\]

\[
\hat{\alpha}_{i,d} \leq b_{i,d}, \quad \forall (i, d) \in \Omega, \tag{3.3h}
\]

\[
b_{i,d} + \frac{1}{f} \cdot v_{i,d} \leq \hat{\beta}_{i,d}, \quad \forall (i, d) \in \Omega, \tag{3.3i}
\]

\[
b_{i,d} + \hat{t}^{\text{min}} \leq \hat{\beta}_{i,d}, \quad \forall (i, d) \in \Omega. \tag{3.3j}
\]

Constraints (3.3a) and (3.3b) guarantee that each electric battery SoC remains within a feasible range. Constraints (3.3c) allow to charge at task $(i, d)$ only if there exists a corresponding charging station itinerary containing that task, while constraints (3.3d) impose minimum charge time. Constraints (3.3e) and (3.3f) impose charging station itinerary flow conservation of at most $|C|$ paths starting at
the source node. Constraints (3.3g) guarantee that no overlap exists between consecutive charging tasks executed by a station. Finally, constraints (3.3h), (3.3i) and (3.3j) impose charging amounts and start times for each node to be carried out within feasible ranges.

The following proposition allows us to eliminate arcs from set $\Gamma$ without loss of generality.

**Proposition 1** Let $q, q' \in \Omega$ be two possible charging events such that $\hat{\alpha}_q \leq \hat{\alpha}_{q'}$, $\hat{\beta}_q \leq \hat{\beta}_{q'}$. Then, if there exists a feasible solution to (3.3) such that $z_{(q',q)} = 1$, then there exists another feasible solution such that $z_{(q',q)} = 0$.

The proof is given in Appendix A. It lies on the fact that if $q'$ directly precedes $q$ on the same charger, then both can be interchanged to obtain another feasible solution. This result allows us to reduce the search space by removing all such arcs $(q',q)$ from $\Gamma$.

**Dynamic generation of feasibility cuts**

When a particular solution $\bar{x}$ is proven infeasible for the second stage, then it must be eliminated from the search space of the first-stage model. To do so, we add a “no-good cut” to formulation (3.2) to remove solution $\bar{x}$ following the Combinatorial Benders’ (CB) approach proposed by [7].

Define sets $X := \{(a, \ell) : \ell \in L, a \in A_\ell, \bar{x}_a^\ell = 1\}$ and $\bar{X} := \{(a, \ell) : \ell \in L, a \in A_\ell, \bar{x}_a^\ell = 0\}$. A CB cut is defined as

$$\sum_{(a,\ell) \in X} (1 - x_a^\ell) + \sum_{(a,\ell) \in \bar{X}} (x_a^\ell) \geq 1, \quad (3.4)$$

meaning that a solution $x$ satisfying cut (3.4) must differ in at least one component to $\bar{x}$.

The path structure of vector $x$ allows us to lift the cut further to

$$\sum_{(a,0) \in X} (1 - x_a^0) \geq 1, \quad (3.5)$$

which is dynamically added in a Branch & Cut fashion to (3.2).

**4 Numerical experiments**

We test our model and solution in a family of computational experiments based on a bus operator in Santiago, Chile. Our goal is to assess the computational performance of our approach and obtain managerial insights of an efficient bus dispatch operation.

The bus dispatch optimization program was implemented in Python 3.7 with the Gurobi 9.0.1 solver interface. All experiments were executed on a computer with a 2.5 GHz Intel Core i5 processor and 4GB of RAM.
4.1 Dataset and experimental design

We designed a family of problem instances based on real data provided by a bus operator of Santiago’s public transportation system. An instance’s key parameters are presented in Table 2. The

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n^T$</td>
<td>Number of trips to serve</td>
</tr>
<tr>
<td>$n^C$</td>
<td>Number of available charging stations</td>
</tr>
<tr>
<td>$n^{D\text{bound}}$</td>
<td>Minimum pull-out of diesel vehicles required to cover all trips in absence of the electric fleet</td>
</tr>
<tr>
<td>$%E$</td>
<td>Target percentage of the fleet to be composed by electric vehicles</td>
</tr>
<tr>
<td>$n^E_A$</td>
<td>Number of available electric vehicles. Calculated as $\lceil %E \cdot n^{D\text{bound}} \rceil$</td>
</tr>
<tr>
<td>$BR$</td>
<td>Percentage of usable battery SoC range, i.e., $BR := \bar{e}<em>{\text{max}} - \bar{e}</em>{\text{min}}$</td>
</tr>
<tr>
<td>$rN$</td>
<td>Realization number for each instance</td>
</tr>
</tbody>
</table>

Table 2: Definition of main operational parameters.

minimum pull-out of only diesel vehicles $n^{D\text{bound}}$ required to cover all trips can easily be obtained by computing the maximum number of trips that occur simultaneously. Then, we set the number of available electric vehicles as $n^E_A := \lceil n^{D\text{bound}} \cdot \%E \rceil$ depending on the target percentage of electric vehicles ($\%E$). Clearly, $(n^{D\text{bound}} - n^E_A)$ and $n^{D\text{bound}}$ represent lower and upper bounds, respectively, for diesel vehicle pullouts.

All instances tested share the same base set of trips, each with a time window, trip duration and consumption information. Trips start and end times were gathered from historic data of a bus service dispatching more than 300 trips per weekday. The SoC consumption per trip was obtained from historic averages measured by an Automatic Vehicle Location (AVL) system. The scatter plot in Figure 6 illustrates the historic relation between SoC consumption and trip duration. In our data set, an average trip consumes 20.2% of the battery’s SoC and takes 141 minutes.

Figure 7 presents the number of trips required to be under execution for each minute over the operating horizon. As depicted, the demand pattern has clear morning and afternoon peaks, as is it usual in most public transportation systems. As fewer trips were needed for having instances of tractable sizes, subsets of trips were drawn from the dataset aiming to maintain the dispersion of trips to be served over time.
Battery SoC feasible ranges were set to $e_{\text{min}} = 20\%$ and $e_{\text{max}} = 100\%$, respectively, and therefore a vehicle with a full battery can serve roughly four trips, on average, before a charge is needed. Initial SoC levels $e_i$ were drawn from a uniform distribution between 20% and 30%. This reflects an operation in which the starting energy of the planning horizon is linked to the end of the previous day. Accordingly, parameter $e_{\text{end}}$ is set to 25%, i.e., the average between 20% and 30%. Also, each charging station power $f$ was set to 0.63 $\%$ minute$^{-1}$, meaning that a full battery charge (80% of SoC increase) takes roughly over 2 hours. Parameters $p_{\text{start}}$ and $p_{\text{end}}$ were set to 0 and 1140 minutes, respectively, to reflect an impossible charge in peak afternoon hours, where electricity is expensive and/or may not be available.

We generated different parameter combinations by setting different values of $n^T$, $n^C$ and $n^E$. From the base set of trips, we generated instances having 50, 60, 70, 80 and 90 trips each. For each value of
The number of chargers available \( n^C \) was set to 1, 2 and 3, and the number of electric vehicles \( n^E \) was set to four different levels depending on \( \%^E \in \{0.25, 0.5, 0.75, 1\} \). For each setting of parameters, we also simulated three scenarios of initial battery SoC levels for each bus, which were drawn from the same uniform distribution earlier mentioned.

The AVL system did not report fuel consumption data for diesel buses. Since we are interested in minimizing diesel fuel consumption, we consider the total travel time of diesel buses as a proxy to minimize, \( \gamma_v = (t_{v_{\text{end}}} - t_{v_{\text{start}}}) \) for each trip \( v \in T \). If available, we may easily equip our model with true fuel consumption information.

### 4.2 Performance Indicators

For each instance realization, we computed a set of Key Performance Indicators (KPIs) classified into computational and solution performance indicators presented in Tables 3 and 4, respectively.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( exT_1, exT_2 )</td>
<td>Total computation time (in seconds) for the first and second objective, respectively</td>
</tr>
<tr>
<td>( gap_1, gap_2 )</td>
<td>Absolute optimality gap for both objectives</td>
</tr>
<tr>
<td>( gap_1^%, gap_2^% )</td>
<td>Percentage optimality gap for both objectives</td>
</tr>
<tr>
<td>( cbT_1, cbT_2 )</td>
<td>Total time spent in callbacks (second-stage solution) for both objectives</td>
</tr>
<tr>
<td>( cb_1, cb_2 )</td>
<td>Number of CB cuts injected for both objectives</td>
</tr>
</tbody>
</table>

Table 3: Definition of KPIs measuring computational performance for each instance realization.

The values of \( n^D_H \) are obtained using a simple sequential bus dispatch policy similar to the one implemented by the public transit operator studied. The solution obtained with this policy is also used as an initial incumbent solution for both the single- and two-stage formulations. This policy is explained in detail in Appendix B.

In Table 5, we add additional KPIs measuring aggregated computational performance.

### 4.3 Results on computational performance

We first present computational performance results comparing our two-stage solution to a single-stage benchmark model presented in Appendix C and based on models proposed by [26, 32]. For the
KPI Definition

\( n^H_D \) Number of diesel vehicles pulled-out by our sequential heuristic

\( n^D, n^E \) Number of diesel and electric vehicles pulled-out in our solution, respectively

\( n^V \) Total number of diesel vehicles required + available electric vehicles, i.e., \( n^D + n^E_A \)

\( T^D, T^E \) Total travel time of diesel and electric vehicles, respectively

\( C^T, I^T \) Total charging and charging station idle time, respectively

\( U \) Average percentage of charger utilization \( \left( \frac{C^T}{C^T + I^T} \right) \)

<table>
<thead>
<tr>
<th>Parameter/metric</th>
<th>Definition</th>
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</thead>
<tbody>
<tr>
<td>( TO_1, TO_2 )</td>
<td>Number of instance realizations that timed-out for each objective</td>
</tr>
<tr>
<td>( CB_1, CB_2 )</td>
<td>Number of instance realizations in which one or more cuts were injected</td>
</tr>
</tbody>
</table>

Table 4: Definition of KPIs measuring solution quality for each instance realization.

<table>
<thead>
<tr>
<th>Parameter/metric</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( TO_1, TO_2 )</td>
<td>Number of instance realizations that timed-out for each objective</td>
</tr>
<tr>
<td>( CB_1, CB_2 )</td>
<td>Number of instance realizations in which one or more cuts were injected</td>
</tr>
</tbody>
</table>

Table 5: KPIs measuring aggregated computational performance.

first- and second-level objectives, we set 0% and 3% target relative gaps and 60 and 20 minutes time limits, respectively.

Table 6 presents the number of small-instance realizations optimally solved within different computation time intervals for the single- and two-stage formulations. We further classify each solution into two subsets, whether it was or not solved by the initial sequential heuristic.

Empirically, we observe that our two-stage approach optimally solves all small-instances and yields significant solution speed-ups when compared to the single-stage model. Moreover, the single-stage model struggles with some instance realizations and cannot solve them within the 1-hour time limit. In some cases, this formulation optimally solves them only because it is initially equipped with the heuristic solution but fails to close the optimality gap.

Table 7 presents the number of medium- and large-sized instance realizations optimally solved by the two-stage formulation within different solution time intervals, and average solution times over different instance sizes and initial heuristic solution optimality conditions. Compared to instance realizations in which the heuristic is not optimal, our model closes the gap of all instance realizations optimally solved by the heuristic within 30 seconds with significantly smaller average solution times.
Table 6: Number of realizations optimally solved per time interval for the first-level objective for the single and two-stage formulations. Small instances with up to 60 trips.

<table>
<thead>
<tr>
<th>$n_T$</th>
<th>$n^D - n^D_H$</th>
<th>Single-stage approach</th>
<th>Two-stage approach</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>&lt;30</td>
<td>30-3600</td>
<td>&gt;3600</td>
</tr>
<tr>
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Table 7: Number of realizations optimally solved per time interval for the first-objective for the two-stage formulation. Medium and large instances with up to 90 trips.

<table>
<thead>
<tr>
<th>$n_T$</th>
<th>$n^D - n^D_H$</th>
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<th>30-3600</th>
<th>&gt;3600</th>
<th>$time_1$</th>
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<td>70</td>
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<td>0</td>
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</tr>
<tr>
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<td>0</td>
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</tr>
<tr>
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<td>3</td>
<td>0</td>
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<td>0</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>10.8</td>
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</table>
To test our model over non-trivial instance realizations, in Table 8 we present its average performance over all realizations left unsolved by the heuristic (#) per instance setting \((n^T, n^C, n^E_A)\). For the main objective, our model optimally solves on average each instance with up to 80 trips within a minute. It also solve all instance realizations with 90 trips. In case of the secondary objective, our model fails to reach the target gap within 20 minutes even for some 70 trip instances. Empirically, the model’s performance for this objective is dichotomic, as it either reaches the target within 30 seconds or times out.

As expected, average solution times for both objectives significantly increase as a function of \(n^T\). The impact of \(n^A_E\) in solution times is unclear; replacing diesel vehicles with electric ones has more variables, but also restricts the feasible routes for each vehicle and potentially reduces the search space. Similarly, the impact of \(n^C\) in solution performance is unclear; adding more chargers increases the number of variables and constraints, but may deactivate parallel charging capacity constraints making the problem easier to be solved without CB cuts.

Cuts are injected in relatively few instance realizations when solving both the first- and second-level objectives, which empirically tell us that our first-stage model is a good approximation of the single-stage model.

### 4.4 Results on solution structure and performance

We now present results related to our solution’s structure and performance. To do so, we consider all previous instances with 70 trips and empirically study the optimal solution structure and value as a function of instance settings like the available number of charging stations and electric buses. If no electric vehicles are used, all instances with \(n^T = 70\) can be covered by exactly 12 diesel vehicles. Table 9 presents average results over the three realizations for each instance.

We see that 17 buses (12 electric & 5 diesel) are involved for a highly constrained electric vehicle operation, *e.g.*, 12 electric buses sharing one charger. This represents an increase of 5 buses with respect to our diesel bound. Moreover, some electric buses are left unused due to scarce battery charging times (99.5% utilization). Conversely, all instances with 3 chargers use all electric vehicles and use the same number of buses as if all were conventional ones. Empirically, we see how the number of available electric vehicles and chargers can significantly impact in bus utilization.

Figures 8 and 9 present the average difference in diesel buses pulled out from the depot between the heuristic and the exact model, and charging station utilization percentage for the exact solution,
Table 8: Average computational performance metrics for each instance setting over realizations left unsolved by the heuristic.

<table>
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<tr>
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<th>$n^C$</th>
<th>$n^E$</th>
<th>#</th>
<th>time$_1$</th>
<th>cbT$_1$</th>
<th>gap$_1$</th>
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<th>cb$_1$</th>
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Table 8: Average computational performance metrics for each instance setting over realizations left unsolved by the heuristic.
<table>
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<tr>
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<th>$n^E_A$</th>
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<th>$n^D$</th>
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<th>$n^D_H$</th>
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Table 9: Operational metrics for instances with 70 trips. Average over 3 realizations per combination of parameters.
respectively, for the different settings of available electric vehicles and charging stations.

![Figure 8](image)

**Figure 8:** Average difference of diesel bus pullouts between the heuristic and exact solution for different settings of available electric vehicles and charging stations. Instances with 70 trips.

![Figure 9](image)

**Figure 9:** Charger utilization (%) in exact solution for different settings of available electric vehicles and charging stations. Instances with 70 trips.

As noticed, the difference in pullouts is small or null for a relatively low congested charging infrastructure (small $U$), and it increases as chargers become utilized closer to their capacity. We conclude that the value of an optimal solution is relatively higher for a congested charging infrastructure.

Figure 10 presents the total number of vehicles involved ($n^V$) in the optimal solution for each value of $n^C$ and $n^A_E$. Figure 11 details the total charging time ($CT$) and charger idle time ($IT$) for each value of $n^C$ and $n^A_E$. Combined, both figures empirically show that bus requirements directly depend on the charger congestion effects produced by both $n^C$ and $n^A_E$. For example, consider the instance with 12
electric vehicles and 1 charger available. We can reduce $n^V$ from 17 to 12 if we add two more chargers or, alternatively, we replace six electric vehicles with diesel ones. Nonetheless, the marginal reduction in $n^V$ is 0 if we replace six electric vehicles with diesel ones when three chargers are already available; the value of an additional charger is also zero when the available number of electric buses is 6 and only one charger is available. For managers, it becomes necessary to analyze the congestion levels of the current charging infrastructure before investing in additional chargers or electric vehicles.

Figure 10: Number of buses involved in optimal solution ($n^V$) for each setting of $n^C$ and $n^A_E$. Instances with 70 trips.
Figure 11: Accumulated time usage of charging infrastructure. Instances with 70 trips.

Regarding the secondary objective, Figure 12 presents the total travel times of electric \(T^E\) and diesel \(T^D\) buses as a function of parameters \(n^A_E\) and \(n^C\). We empirically observe that the marginal reduction in \(T^D\) produced by adding a new charger is relatively higher in instances with a higher number of electric vehicles and a smaller number of charging stations \(i.e.,\) instances in which chargers are more congested). Also, the marginal reduction in \(T^D\) by adding more electric vehicles to the fleet is higher for less congested instances.
Figure 12: Average travel time executed by electric and diesel buses for each setting of $n_E^A$ and $n_C^C$. Instances with 70 trips.

4.5 Sensitivity analysis on battery capacity

In this subsection, we further analyze the impact on solution quality over varying driving ranges of the electric fleet represented by $BR$; it may be understood as an investment in higher capacity batteries or using batteries below 20% SoC levels. Figure 13 presents the average number of diesel and electric buses used in a fleet with 12 electric buses available for different values of $n_C^C$ and $BR$, while Figure 14 details the total charging time ($CT$) and charger idle time ($IT$) for the same values. These figures show how the effect of larger battery sizes directly depends on charger congestion. The marginal reduction in diesel bus pullouts marginally decreases as $BR$ is larger and is significantly larger when more charging stations are less congested; probably because more charging options are available for larger battery sizes when charging stations are not congested.
Figure 13: Number of vehicles used in best solution found for different battery ranges and number of chargers. Instances with 70 trips and 12 electric vehicles available.

Figure 14: Accumulated time usage of charging infrastructure. Instances with 70 trips and 12 electric vehicles available.
For example, consider the instance with 1 charger and 40% of battery range. As the charging stations are congested, increasing battery sizes barely reduces $n^V$, as there is not sufficient time for longer charges to occur. On the other hand, for the instance with 2 chargers and 40% of battery range, increasing battery sizes significantly reduces $n^V$. As charging stations are not congested, long charges may occur. In turn, this prevents batteries from being depleted during peak hours (see Figure 7). This behavior can be inferred from Figures 15 and 16, which show the usage of the electric and conventional fleet for instance realizations with 2 chargers and 12 electric vehicles available, for battery ranges of 40% and 80%, respectively.

![Figure 15: Electric and diesel vehicle fleet usage per minute. Instances with 70 trips, 2 chargers, 12 electric vehicles available and 40% of battery range.](image-url)
Figure 16: Electric and diesel vehicle fleet usage per minute. Instances with 70 trips, 2 chargers, 12 electric vehicles available and 80% of battery range.

The effect of battery capacity on $T^D$ is very similar. Figure 17 presents the total travel times of electric (TE) and diesel (TD) vehicles in a fleet with 12 electric buses available for different values of $n^C$ and $BR$. As can be seen in the figure, larger battery sizes have significant impacts when there are 2 or 3 chargers available, while effects are minor for the case with only one charger, as the charging station is already congested.
Figure 17: Total travel time by vehicle type for different battery ranges and number of chargers. Instances with 70 trips and 12 electric vehicles available.

5 Conclusions

We engage in solving a single-depot electric bus dispatch problem with limited parallel charging infrastructure at the depot and variable charging time duration per task.

As a single-stage formulation yields an intractable model for medium sized instances, we propose an exact two-stage decomposition solved via the Integer L-shaped Method. In this approach, the first-stage model identifies a feasible trip itinerary plan for each bus where the sequencing of charging tasks in each charger is relaxed. The overall feasibility of such an itinerary is tested in a second-stage problem which attempts to identify complementary and feasible pair of battery charge and charging station itineraries. If no such pair exists, then feasibility cuts are injected into the first-stage model to ban such first-stage solution.

As shown by numerical results, the proposed solution method significantly outperforms the single-stage model in terms of computational times. Also, it significantly outperforms a simple sequential dispatch heuristic in terms of solution quality, especially when the utilization of charging stations is relatively high.
We also empirically assess the marginal benefits provided by an additional electric charging station or an electric bus. Empirically, we suggest decision makers to invest in additional charging stations if relatively more electric buses per charger are available. Also, we suggest that the additional value of an electric vehicle is relatively higher in fleets having relatively less electric vehicles. For instance, if two bus terminals are managed by a single operator, then it may be convenient to set two mixed fleet operations in contrast to one fully electric and another fully conventional bus operations.

Lastly, we show that the impact of marginal changes in bus battery range (equivalently, driving range) is larger when charging stations are not highly congested, as it allows long charging events to occur.

The problem and methodology presented can be further extended in several directions. First, we could model non-linear SoC charging rates as a function of charge time. A first approach would be to implement a piece-wise linear relation to keep the model’s MILP structure. We could also study a robust and (possibly) dynamic decision planning model to represent a more realistic bus dispatch operation over stochastic travel time and energy consumption per trip. Finally, a multi-depot problem could be studied considering multiple bus services interacting within a public transit network.

Acknowledgment

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References


[18] Margolis, J, *China dominates the electric bus market, but the us is getting on board*, 2019.


Appendices

A Proof of proposition 1

Let \( q, q' \in \Omega \) with \( \hat{\alpha}_q \leq \hat{\alpha}_{q'} \) and \( \hat{\beta}_q \leq \hat{\beta}_{q'} \) such that, in a solution to the second-stage problem, \( z_{(q',q)} = 1 \). We will demonstrate that we can find a solution such that \( z_{(q',q)} = 0 \). Current variable values will be denoted with a 0 super-index and newly assigned values with a 1 super-index. Let \( p \) and \( s \) be the nodes that fulfil \( z^0_{(p,q')} = 1 \) and \( z^0_{(q,s)} = 1 \) and assume without loss of generality that \( (q',s) \in \Gamma \) and \( (p,q) \in \Gamma \). Trivially, current variable values fulfil \( \hat{\alpha}_q \leq \hat{\alpha}_{q'} \leq b^0_q \leq b^0_{q'} + v^0_q \leq \hat{\beta}_q \leq \hat{\beta}_{q'} \). Then, we can interchange nodes \( q \) and \( q' \) by setting \( z^1_{(p,q')} = z^1_{(q',q)} = z^1_{(q,s)} = 0 \), \( z^1_{(p,q)} = z^1_{(q,q')} = 1 \) and updating charge start times as \( b^1_q = b^0_q \) and \( b^1_{q'} = b^0_{q'} + v^0_q \), while maintaining charge durations. As \( \hat{\alpha}_q \leq b^0_{q'} = b^1_q \) and \( b^1_{q'} + v^1_q = b^0_{q'} + v^0_q \leq \hat{\beta}_q \), this yields feasible variable values for \( q \) and \( q' \), leaving other charging events unchanged.

B Sequential bus dispatch policy

We assume that, based on experience, the dispatcher knows the values of \( n^D_{bound} \) and \( n^E_A \), and, therefore, a lower bound \( n^D_H \). As she knows that at least \( n^D_H \) diesel vehicles are going to be used, the dispatcher prioritizes them (and every diesel vehicle already used) over electric vehicles when dispatching. Dispatches are assumed to be done sequentially by trip start time, using the following prioritization system over vehicles currently at the depot: (1) Diesel vehicle already used or prioritized; (2) Electric vehicle waiting at the depot for the longest time; (3) Diesel vehicle not used or prioritized. Furthermore, if an electric vehicle has time to be charged before its next assigned trip, it will charge as long as possible, and in the charger that was vacated earliest.

C Single-stage formulation

In this section we present a single-stage MILP model which is partly based on the models defined by [32] and [26]. The problem is represented in a digraph that models trip and battery charge itineraries, and a second digraph that plans charging station itineraries.

Sets \( V_0, V_1, V, T, C, L, \) sink 0 and parameters \( f, p^{\text{start}}, p^{\text{end}}, \bar{e}_{\text{min}}, \bar{e}_{\text{min}}, \bar{e}^\text{end}, \bar{e}_i, \bar{e}_d, t_j^{\text{start}} \) and \( t_j^{\text{end}} \) are defined as in Section 3.
We are going to make two charger copies for each trip $j \in T$. The first copy (denoted as $p$ copy) represents a charge happening directly from the source and immediately preceding trip $j$. The second copy (denoted as $r$ copy) represents a charge happening directly after serving trip $j$. Charger copy $n := (q,j)$ for $q \in \{p,r\}$ and $j \in T$ defines a time interval $[\alpha_n, \beta_n]$ when an electric bus visiting it may charge. Accordingly, $\alpha_n$ is set to $p^{\text{start}}$ if $q = p$ and to $t^{\text{end}}_{j}$ if $q = r$, while $\beta_n$ is set to $t^{\text{start}}_{j}$ if $q = p$ and to $p^{\text{end}}$ if $q = r$. So, it is possible to charge a bus if and only if this interval duration is greater than or equal to the minimum charging time. Let sets $P = \{(p, j) : j \in T, \beta_{(p,j)} - \alpha_{(p,j)} \geq t^{\text{min}}\}$ and $R = \{(r, j) : j \in T, \beta_{(r,j)} - \alpha_{(r,j)} \geq t^{\text{min}}\}$ be the sets containing $p$-type and $r$-type charger copies, respectively, for which it is feasible for a charge to occur. Then, let $F = P \cup R$ be a set containing all such charger copies.

We encode trip and battery charge itineraries in a digraph $G = (N, A)$, where the set of nodes $N := T \cup V \cup F \cup \{0\}$ is formed by all trips, a source for each electric bus, one source node for all conventional diesel buses, all charger copies, and a sink, respectively.

Set $A$ contains feasible arcs $(u, v)$ connecting nodes $u, v \in N$. Arcs are considered feasible according to the following rules: (1) An arc exists from each vehicle node $i \in V$ to each trip $j \in T$ and from each trip $j$ to the sink; (2) An arc exists from trip $j_1 \in T$ to trip $j_2 \in T$ if and only if $t_{j_2}^{\text{start}} \geq t_{j_1}^{\text{end}}$; (3) For charger copy $(p, j_1) \in P$, the only possible arcs are from vehicle nodes to the charger copy, and from the charger copy to trip $j_1$; (4) For charger copy $(r, j_1) \in R$, the only possible arcs are from trip $j_1$ to the charger copy, or from the charger copy to the sink or to a trip $j_2$ such that $t_{j_2}^{\text{start}} - t_{j_1}^{\text{end}} \geq t^{\text{min}}$.

Then, define sets $A_0 := \{(u, v) \in A : u \in T \cup V_0 \cup F\}$ and $A_1 := \{(u, v) \in A : u \in T \cup V_1, v \notin F\}$ as the subset of arcs that electric and conventional buses can use, respectively. Also, define the set of arcs in $A_\ell$ leaving and entering node $i \in N$ as $\delta^{\ell}_i(i) := \{(i, j) \in A_\ell : j \in N\}$ and $\delta^{-\ell}_i(i) := \{(j, i) \in A_\ell : j \in N\}$, respectively.

Charging station itineraries are encoded in a digraph $H = (D, B)$, where the set of nodes $D := \{S\} \cup F \cup \{\overline{S}\}$ is formed by all charger copies, a source $\underline{S}$, and a sink $\overline{S}$, respectively. Set $B$ contains arcs $(u, v)$ connecting time-feasible nodes $u, v \in D$, which models sequencing decisions. Let $\alpha_{\underline{S}} = \beta_{\underline{S}} = p^{\text{start}} - t^{\text{min}}$ and $\alpha_{\overline{S}} = \beta_{\overline{S}} = p^{\text{end}} + t^{\text{min}}$. Then, it is feasible for node $u$ to precede node $v$ if there is sufficient time for two charges to happen, i.e., $B = \{(u, v) : u \in \{\underline{S}\} \cup F, v \in F \cup \{\overline{S}\}, \beta_{v} - \alpha_{u} \geq 2 \cdot t^{\text{min}}\}$. Then, define the set of arcs in $B$ leaving and entering node $i \in D$ as $\Delta^+(i) := \{(i, j) \in B : j \in D\}$ and $\Delta^-(i) := \{(j, i) \in B : j \in D\}$, respectively.

Provided the above information, we are now ready to define all decision variables. Let $x^\ell_a \in \{0, 1\}$
be a binary variable defined for each $\ell \in L$ and $a \in A_\ell$ representing a vehicle of type $\ell$ traversing arc $a$. Decision vector $x$ is used to encode a set of at most $|V| + \bar{d}_{\text{max}}$ bus itineraries as paths from each node $i \in V$ to node 0 visiting each trip node in $T$ exactly once.

For each electric bus arc $a = (u, v) \in A_0$, we also define a continuous variable $e_a \geq 0$ representing the battery’s SoC just after leaving node $u \in N$. So, decision vector $e$ encodes battery SoC levels for electric buses. Also, we set a binary variable $w_a$ for each arc $a \in D$, representing that two possible charger copies are served by the same station in consecutive order. Finally, we define continuous variables $b_j \geq 0$ and $t_j \geq 0$ representing the charged amount and start time for the only possible charge in charger copy $j \in F$. Therefore, decision vectors $w$, $b$ and $t$ encode charging station itineraries. For trips $j_1, j_2 \in T$, we define big-M parameters $M_{j_1,j_2} = \beta_{j_1} - \alpha_{j_2}$, which are upper bounds for $(b_{j_1} + t_{j_1}) - b_{j_2}$.

**Objective function**

The objective function is equivalent to the one defined in Section 3.

\[
\min \sum_{a \in \delta^+_{\ell}(v_1)} x^1_a, \quad \text{(C.1a)} \\
\min \sum_{a:=(u,v)\in A_1;v\in T} \gamma_v \cdot x^1_a. \quad \text{(C.1b)}
\]

**Feasibility constraints**

\[
\sum_{\ell \in L} \sum_{a \in \delta^+_\ell(j)} x^\ell_a = 1 \quad \forall j \in T, \quad \text{(C.2a)} \\
\sum_{a \in \delta^-_\ell(j)} x^\ell_a - \sum_{a \in \delta^+_\ell(j)} x^\ell_a = 0 \quad \forall j \in T \cup F, \ell \in L, \quad \text{(C.2b)} \\
\sum_{a \in \delta^+_0(j)} x^0_a \leq 1 \quad \forall j \in V_0 \cup F, \quad \text{(C.2c)} \\
\sum_{a \in \delta^+_0(v_1)} x^0_a \leq \bar{d}_{\text{max}}, \quad \text{(C.2d)} \\
\sum_{a \in \Delta^-} w_a - \sum_{a \in \Delta^+} w_a = 0 \quad \forall j \in F, \quad \text{(C.2e)} \\
\sum_{a \in \Delta^+} w_a \leq |C|, \quad \text{(C.2f)} \\
\sum_{a \in \Delta^+} w_a - \sum_{a \in \delta^+_0(j)} x^0_a = 0 \quad \forall j \in F, \quad \text{(C.2g)}
\]
\( \alpha_j \leq b_j \quad \forall j \in F, \)  
\( b_j + t_j \leq \beta_j \quad \forall j \in F, \)  
\( b_{j_1} + t_{j_1} - \alpha_{j_1} \leq \sum_{j_2 \in T: (j_1, j_2) \in A_0} (t^{\text{start}}_{j_2} - \alpha_{j_1}) \cdot x^0_{(j_1, j_2)} \quad \forall j_1 \in R, \)  
\( b_{j_1} + t_{j_1} \leq b_{j_2} + M_a \cdot (1 - w_a) \quad \forall a = (j_1, j_2) \in B, \)  
\( \hat{\tau}_{\min} \cdot x^0_a \leq t_j \quad \forall j \in F, a \in \delta^+_0(j), \)  
\( (\sum_{a \in \delta^+_0(j)} e_a) + f \cdot t_j = (\sum_{a \in \delta^+_0(j)} e_a) \quad \forall j \in F, \)  
\( (\sum_{a \in \delta^+_0(j)} e_a) - \bar{e}^j \cdot \sum_{a \in \delta^+_0(j)} x^0_a = (\sum_{a \in \delta^+_0(j)} e_a) \quad \forall j \in T, \)  
\( e_a = x_a \cdot \bar{e}_i \quad \forall i \in V_0, a \in \delta^+_0(i), \)  
\( \bar{e}^\min \cdot x^0_a \leq e_a \quad \forall j \in T, a \in \delta^+_0(j), \)  
\( \bar{e}^\text{end} \cdot x^0_a \leq e_a \quad \forall a \in \delta^-_0(S), \)  
\( e_a \leq \bar{e}^\max \cdot x^0_a \quad \forall a \in A_0. \)  

Constraints (C.2a) guarantee that all trips are executed by exactly one vehicle. Constraints (C.2b), (C.2c) and (C.2d) enforce vehicle flow conservation constraints for each vehicle starting from node \( i \in V \). Constraints (C.2e) and (C.2f) imposes flow conservation constraints for nodes in \( D \), while (C.2g) ensures compatibility between digraphs \( G \) and \( H \). Constraints (C.2h), (C.2i), (C.2j) and (C.2k) impose time-feasibility constraints for consecutive nodes, while constraints (C.2l) sets a lower limit for charge times. Constraints, (C.2m) (C.2n) and (C.2o) ensure battery SoC flow conservation for each node. Finally, constraints (C.2p), (C.2q) and (C.2r) guarantee that battery SoC is kept within feasible ranges.

### D Results over medium and large instance realizations
Table 10: Performance metrics of the two-stage model for instances not solved by sequential dispatch heuristic. Instances with 70 trips.

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Table 11: Performance metrics of the two-stage model for instances not solved by sequential dispatch heuristic. Instances with 80 trips.
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Table 12: Performance metrics of the two-stage model for instances not solved by sequential dispatch heuristic. Instances with 90 trips.